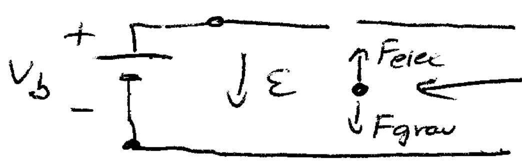
ELEG 867 Nanotechnology
HWK #5, due Monday, 22 Oct 2001

1- Millikan's Oil Drop Experiment



oil drop: $\rho_{mass} = 0.851\ gm/cm^3$

$r = 1.64\ \mu m$

$\vec{\mathcal{E}}_{balance} = 1.92 \times 10^5\ nT/coul$ $= \left(\frac{Volt}{meter}\right)$

$[Newton] = [kg \frac{m}{s^2}]$

for balance, $\Sigma F's = 0$

so $mg = Q\mathcal{E}$

mass of drop: $m = \frac{4}{3}\pi r^3 \times \rho_{mass} = \frac{4}{3}\pi (1.64 \times 10^{-6} m)^3$ or $(1.64 \times 10^{-4} cm)^3$ $\times 0.851 \frac{gm}{cm^3}$

$= 1.57 \times 10^{-11}\ gm = 1.57 \times 10^{-14}\ kg$

$$Q = \frac{mg}{\mathcal{E}} = \frac{1.57 \times 10^{-14}\ kg \times 9.8\ m/sec^2}{1.92 \times 10^5\ Nt/Coul} = 8.0 \times 10^{-19}\ Coul$$

$= 5 \times e$

(b) to balance electrons: $m_o = 9.11 \times 10^{-31}\ kg \approx \frac{m_{drop}}{10^{16}}$
the required $\mathcal{E}$ field would be 10^{16} lower →
molecular thermal motion would disrupt e^-
→ could perform experiment in vacuum, but observation of e^- would disrupt trajectory by Uncertainty Principle.

For the single electron structure the charging energy on the dot is:

$$E_{ch} = Q_d V_g + \frac{Q_d^2}{2C_{dot}} \quad , \text{where } V_g \equiv -\frac{Q_o}{C_{dot}}$$

when $Q_o = -(N+\frac{1}{2})e$, then E_c is the same for
$Q_{dot} = -Ne$ and $-(N+1)e$

$$E_{ch} = -\frac{Q_{dot}Q_o}{C_{dot}} + \frac{Q_{dot}^2}{2C_d}$$

notation: let $Q_{dot} \to Q$, $C_{dot} \to C$

$$E_{ch} = +\frac{Q(N+\frac{1}{2})e}{C} + \frac{Q^2}{2C}$$

let $Q = -ne$, solve for n

$$= -\frac{n(N+\frac{1}{2})e^2}{C} + \frac{n^2e^2}{2C}$$

$$= \frac{e^2}{C}\left[-n(N+\tfrac{1}{2}) + \frac{n^2}{2}\right]$$

if $n=N$:

$$E_c(n=N) = \frac{e^2}{C}\left[-N(N+\tfrac{1}{2}) + \frac{N^2}{2}\right] = -\frac{e^2}{C}\left[\frac{N^2}{2} + \frac{N}{2}\right] = -\frac{e^2}{2C}\left[N(N+1)\right]$$

$n=N+1$

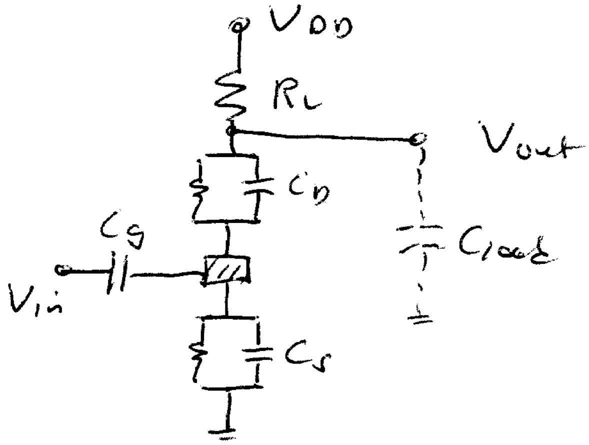
$$E_{ch}(n=N+1) = \frac{e^2}{C}\left[-(N+1)(N+\tfrac{1}{2}) + \frac{(N+1)^2}{2}\right]$$

$$= \frac{e^2}{C}(N+1)\underbrace{\left[-(N+\tfrac{1}{2}) + \frac{N+1}{2}\right]}_{-\frac{N}{2}} = -\frac{e^2}{2C}\left[N(N+1)\right]$$

$\Updownarrow$

so E_c same for $n=N$; $N+1$

3. Single Electron Transistor Inverter Circuit



$$C = \frac{Q}{V} \; ; \quad Q = CV \; ; \quad V = \frac{Q}{C}$$

for a single electron discharging the output;

$$\Delta V_{out} = \frac{e}{C_D \$ C_S} = \frac{e}{\left(\frac{1.2fF}{2\times 8}\right)}$$

for a single electron changing the input.

$$\Delta V_{in} = \frac{e}{C_g \$ C_S} = \frac{e}{1.2fF \$ \frac{1.2fF}{8}}$$

$$\text{so } A_v = \frac{\Delta V_{out}}{\Delta V_{in}} = \frac{\cancel{1.2fF}\; C_g \$ C_S}{C_D \$ C_S} = \frac{1.2fF \$ 1.2fF/8}{\frac{1.2fF}{16}}$$

a) $$A_v = \frac{1 \$ 1/8}{1/16} = 16 \$ 2 = \frac{16\times 2}{16+2} = \frac{32}{18} = 1.78$$

b) maximum T_{oper} such that $\Delta V_{out} > k_B T_{op}$

$$T_{op} < \frac{\Delta V_{out}}{k_B} = \frac{e/(1.2fF/16)}{k_B (= 0.02585 \text{ @ } 300K \text{ Volt})}$$

$$T_{op} < \frac{\frac{16\, e}{1.2fF}}{\left(\frac{0.02585\, V}{300K}\right)} = \frac{300K \times 16 \times 1.6\times 10^{-19}\,\text{Coul}}{1.2\times 10^{-15}\,F \times 0.02585\,V}$$

$$< 300K \times 0.0825 = 24.76\ K$$

3c. new A_v if V_{out} drives a load of $C_G \,\&\, C_S$

as before $\Delta V_{in} = \dfrac{e}{C_g \,\&\, C_s} = \dfrac{e}{\left[\dfrac{C_g \times C_s}{C_g + C_s}\right]}$

new $\Delta V_{out} = \dfrac{e}{[C_D \,\&\, C_S] \,||\, C_{load}}$ } net capacitance from node at V_{out} to ground.

$C_g = 1.2fF$

$C_S = C_D = 0.155F$

$$= \frac{e}{\left[\dfrac{C_D C_S}{C_D + C_S}\right] + C_L}$$

" $C_g \,\&\, C_s = \dfrac{C_g C_s}{C_g + C_s}$

so

$$A_v\Big|_{load} = \frac{\Delta V_{out}}{\Delta V_{in}} = \frac{\left[\dfrac{C_g C_s}{C_g + C_s}\right]}{\dfrac{C_D C_S}{C_D + C_S} + \dfrac{C_g C_s}{C_g + C_s}} = \frac{1}{\dfrac{(C_g + C_s)(C_D C_s)}{(C_g C_s)(C_D + C_s)} + 1}$$

$$= \frac{1}{\dfrac{(1.2 + 0.15)(\cancel{0.15} \times 0.15)}{(1.2 \times \cancel{0.15})(0.15 + 0.15)} + 1}$$

units cancel

$$= \frac{1}{\dfrac{(1.2 + 0.15)\,0.15}{1.2(0.15 + 0.15)} + 1} = \frac{1}{\dfrac{1.35 \times 0.15}{1.2 \times 0.3} + 1}$$

$$= \frac{1}{0.5625 + 1} = 0.64$$