ELEG 867 - Nano Hmk #4 Solution

1. $$E_i^{0D} = \frac{\hbar^2}{2m^*}\left[\frac{\alpha_{nl}}{R}\right]^2$$

from Abramowitz & Stegun

$\alpha_{10} = \pi$

$\alpha_{11} = 4.4934$

a) for $R = 5\,nm$: $m^* = m_0$

$$E_{10} = \frac{\hbar^2}{2m^* R^2}[\alpha_{10}]^2$$

$$= \frac{(1.054\times10^{-34}\,J\text{-}s)^2}{2\times9.109\times10^{-31}\,kg\times(5\times10^{-9}\,m)^2} \times \pi^2$$

$$= 2.44\times10^{-22}\,J\times\pi^2 = 2.407\times10^{-21}\,J$$

$$E_{10} = 0.015\,eV = 15\,meV$$

b) for $$\Delta E_{11\to10} = \frac{\hbar^2}{2m^*R^2}\left[\alpha_{11}^2 - \alpha_{10}^2\right] = 0.0258$$

$$\downarrow \qquad \downarrow$$

$$\left\{(4.4934)^2 - \pi^2\right\} = 10.32$$

so $$R^2 = \frac{\hbar^2}{2m^*\Delta E_{11,10}}\left[\alpha_{11}^2 - \alpha_{10}^2\right]$$

$$R = \frac{\hbar = 1.054\times10^{-34}\,J\text{-}s}{\sqrt{2\times9.11\times10^{-31}\,kg\times0.0258\,eV\times1.6\times10^{-19}\,J/eV}} \times \sqrt{10.32}$$

$$= 3.9\times10^{-9}\,m = 3.9\,nm$$

c)

c) Coulombic correction from Brus:

$$E(R=5nm) = E_g + \frac{\hbar^2\pi^2}{2R^2}\left[\frac{1}{m_e^*}+\frac{1}{m_h^*}\right] - \frac{1.8e^2}{4\pi\epsilon_0\epsilon R}$$

assume $E_g = 1eV$, $m_e^* = m_h^* = m_0$, $\epsilon = 1$

$$E(5nm) = 1eV + \frac{(1.054\times10^{-34}J\text{-}s)^2\pi^2}{\cancel{2}(5\ nm)^2}\ \frac{\cancel{2}}{\times 9.11\times10^{-31}kg}$$

$$- \frac{1.8\times(1.6\times10^{-19}C)^2}{4\pi\times8.85\times10^{-12}F/m\times1\times5\times10^{-9}m}$$

$= 1eV + 4.81\times10^{-21}$ Joule $- 8.29\times10^{-20}$ J

$= 1eV + 0.030$ eV $- 0.518$ eV

$= 0.512$ eV

2. Current in Quantum wire

dia = 5 nm

length = 100 nm

ΔV = 1 V.

The current in a single channel of a quantum wire is given by:

a) $I_n = \frac{2e^2}{h} V_{appl.} = \frac{2}{25882\,\Omega} \times 1V = 77.27\,\mu A$

$\frac{2e^2}{h} = \frac{2}{25.882\,K\Omega}$

b) $\frac{1\,mA}{77.27\,\mu A} = 12.94 \rightarrow 13$ channels needed

c) as the length increases, the propagation delay time increases but the number of electrons also increases so I = constant

also can interpret:

as the group velocity increases, the density of states decreases, with the current constant