7 April 2005

NAME

SOLUTION

Time Limit: 1 hour;
Closed Books and Notes;
However, you are permitted to use one page (both sides) of your own notes;
Please do not loan calculators to anyone.

Scoring: Short Questions 1-7 are worth 5 points each;
Long Problems 8-11 have the point values indicated.
Full credit requires giving the dimensions / units for all numerical quantities that you calculate.

I. Unless stated otherwise, assume:
   a) Silicon material at T = 300K,
   b) Steady state conditions,
   c) all carrier recombination lifetimes $\tau_{n,p} = 10^{-7}$ sec

II. Use appropriate value of mobility $\mu$, $D$, $L$, $m^*$, etc., for the given impurity concentrations (see data sheets).

III. Show all calculations.

IV. Accuracy to 2 significant figures is sufficient.

V. You may use either $n_i = 1 \times 10^{10}$ cm$^{-3}$, or, $1.5 \times 10^{10}$ cm$^{-3}$, for Si at room temperature (300K).

For your convenience, equation sheets and graphs are provided.
Short Questions (5 points each)

1. What is the algebraic statement of low level injection?

\[
\text{excess majority concentration} < \text{majority}
\]

\[
P_{n/\text{excess}} < N_n
\]

or

\[
N_p/\text{excess} < P_D
\]

2. A homogeneous sample of Ge has compensated doping with \(N_A = 5 \times 10^{17} \text{ cm}^{-3}\), and \(N_D = 1 \times 10^{17} \text{ cm}^{-3}\). What is the value of the carrier concentration?

\[
P = N_A - N_D = (5 - 1) \times 10^{17} \text{ cm}^{-3} = 4 \times 10^{17} \text{ cm}^{-3}
\]

\[
\Rightarrow n_i = 2.4 \times 10^{13} \text{ cm}^{-3} \quad \text{(so good approximation)}
\]

\[
n = \frac{n_i^2}{P} = \frac{(2.4 \times 10^{13})^2}{4 \times 10^{17} \text{ cm}^{-3}} = 1.44 \times 10^9 \text{ cm}^{-3}
\]


\[
P_n(0) = P_{no} e^{eV_t/\Phi T}
\]

or

\[
N_p(0) = N_{po} e^{eV_t/\Phi T}
\]

4. Why does the "reverse saturation current" \((I_s)\) of an ideal diode saturate?

\[
because \text{ for } V_R \gg \Phi T / e, \]

\[
P_n(0), \text{ or } N_p(0) \approx 0 \text{ so diffusion gradient is constant, so diffusion current is constant}
\]

\[
\overline{J} = 6D (0 - P_{no}) e^{-x/\lambda} = \frac{8D}{\lambda} P_{po} \approx \text{const.}
\]
5. True or False: The space charge region about the metallurgical junction is due to a pile up of electrons on the p-side and holes on the n-side. 

\[ \text{False} \]

6. True or False: For a $p^+\text{-}n$ step junction with $N_A(p\text{-}side) \gg N_D(n\text{-}side)$, then $x_p \ll x_n$.

\[ \text{True} \]

7. True or False: Zener breakdown typically occurs at a reverse voltage with a greater magnitude than that of avalanche breakdown.

\[ \text{False} \]
Long Problems 8 through 11 (point values indicated): Forward-and Reverse-Biased Si Diode:

An abrupt Si p+n junction diode has a cross sectional area of 1 mm$^2$, an acceptor concentration of 5 x $10^{18}$ boron atoms cm$^{-3}$ on the p-side, and a donor concentration of $10^{16}$ arsenic atoms cm$^{-3}$ on the n-side. The lifetime of holes in the n-region is 417 ns, and that of electrons in the p-region is 5 ns due to a greater concentration of impurities (recombination centers) on that side. Mean thermal generation lifetime in the depletion region ($\tau_g$) is about 1 $\mu$s. The lengths of the p- and n-regions are 5 and 100 microns, respectively.

8. (35 points) Calculate the minority diffusion lengths at the given doping concentrations, and determine if this diode is long or short base.

\[
\text{p side: } N_A = 5 \times 10^{18} \text{ cm}^{-3} \rightarrow \mu_n = 130 \text{ cm}^2/\text{V} \cdot \text{s} \\
D_n = \frac{\mu_n q}{\tau_n} = 0.0259 \times 130 \text{ cm}^2/\text{V} \cdot \text{s} = 3.37 \text{ cm}^2/\text{s} \\
L_n = \sqrt{D_n \tau_n} = \sqrt{3.37 \text{ cm}^2/\text{s} \times 5 \times 10^{-9} \text{s}} = 1.2 \times 10^{-4} \text{ cm} = 1.2 \mu\text{m} \\
\text{since } L_n < X_p = 5 \mu\text{m}
\]

\[
\text{n side: } N_D = 10^{16} \text{ cm}^{-3} \rightarrow \mu_p = 440 \text{ cm}^2/\text{V} \cdot \text{s} \\
D_p = \frac{\mu_p q}{\tau_p} = 0.0259 \times 440 \text{ cm}^2/\text{V} \cdot \text{s} = 11.4 \text{ cm}^2/\text{s} \\
L_p = \sqrt{D_p \tau_p} = \sqrt{11.4 \text{ cm}^2/\text{s} \times 417 \times 10^{-9} \text{s}} = 2.18 \times 10^{-3} \text{ cm} = 2.18 \mu\text{m} \\
\text{since } L_p < X_n = 100 \mu\text{m}
\]

8n < X_p \quad \text{so long base diode}
\]

8p < X_n \quad \text{lengths of neutral regions}
9. (15 points) What is the built-in potential across the junction?

\[
\phi_{bi} = \frac{RT}{e} \ln \frac{N_{A\,ND}}{n_i^2} = 0.0259 \ln \left[ \frac{5 \times 10^{18}}{10^{20}} \right] V
\]

\[
= 0.0259 \times 33.84 \quad (\text{or } 33.03)
\]

\[
= 0.88 \quad V \quad (\text{or } 0.859 \quad V)
\]

0.876 0.855
10. (25 points) What is the current at a forward bias of 0.6 V across the diode at 27°C? Assume that the current is by minority carrier diffusion. (Hint: You may use approximations if you justify them.)

Since $p^+ n \rightarrow$ consider only holes, which dominate

\[
I_p = qD_p \frac{n_i^2}{N_D L_p} e^{\frac{eV_F}{kT}}
\]

\[
= \frac{1.6 \times 10^{-9} \text{C} \times (1 \times 10^{10} \text{cm}^{-3})^2 \left(\frac{12.45 \text{ cm}^2}{5}\right)}{(2.18 \times 10^{-4} \text{ cm}) (10^{16} \text{ cm}^{-3})}
\]

\[
I_p = 8.36 \times 10^{-12} \text{ A/cm}^2
\]

\[
I_S = I_p \times 0.01 \text{ cm}^2 = 8.36 \times 10^{-14} \text{ A} = 0.084 \text{ pA}
\]

\[
I_F = I_S e^{\frac{eV_F}{kT}} = (8.36 \times 10^{-14} \text{ A}) \exp \left[\frac{0.6 \text{ V}}{0.0259 \text{ V}}\right]
\]

\[
= 0.96 \times 10^{-3} \text{ A} = 0.96 \text{ mA}
\]
11. (25 points) What is the reverse current due to thermal generation in the depletion region when the diode is reverse-biased by a voltage $V_R = 5$ V?

$$W_{dep} = \left[ \frac{2kT \ln (\frac{N_D}{N_b} + V_R)}{q N_D} \right]^{1/2}$$

$$= \left[ \frac{2 \times 11.9 \times 8.85 \times 10^{-12} \text{F/m} \times (0.877 + 5)}{1.6 \times 10^{-19} \text{C} \times 10^{22} \text{m}^{-3}} \right]^{1/2}$$

$$= 0.88 \times 10^{-6} \text{ m} = 0.88 \mu \text{m}$$

$$I_{gen} = \frac{q A W_{ni}}{2} = 1.6 \times 10^{-19} \text{C} \times 0.01 \text{cm}^2 \times \frac{10^{-6} \text{S}}{2}$$

$$= 1.6 \times 10^{-19} \text{C} \times 0.01 \text{cm}^2 \times 0.88 \times 10^{-4} \text{cm} \times 10^{10} \text{cm}^{-3}$$

$$= 1.41 \times 10^{-9} \text{A} = 1.4 \text{ nA}$$

($$\overline{J_R} = 1.41 \times 10^{-7} \text{A/cm}^2$$)

@ 0.7 nA
TABLE 4.2
Properties of Ge, Si and GaAs at 300K

<table>
<thead>
<tr>
<th>Property</th>
<th>Ge</th>
<th>Si</th>
<th>GaAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atomic/molecular weight</td>
<td>72.6</td>
<td>28.09</td>
<td>144.63</td>
</tr>
<tr>
<td>Density (g cm⁻³)</td>
<td>5.33</td>
<td>2.33</td>
<td>5.32</td>
</tr>
<tr>
<td>Dielectric constant</td>
<td>16.0</td>
<td>11.9</td>
<td>13.1</td>
</tr>
<tr>
<td>Effective density of states</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conduction band, Nc (cm⁻³)</td>
<td>1.04 × 10¹⁹</td>
<td>2.8 × 10¹⁹</td>
<td>4.7 × 10¹⁷</td>
</tr>
<tr>
<td>Valence band Nv (cm⁻³)</td>
<td>6.0 × 10¹⁸</td>
<td>1.02 × 10¹⁹</td>
<td>7.0 × 10¹⁸</td>
</tr>
<tr>
<td>Electron affinity (eV)</td>
<td>4.01</td>
<td>4.05</td>
<td>4.07</td>
</tr>
<tr>
<td>Energy gap, Eₔ (eV)</td>
<td>0.67</td>
<td>1.12</td>
<td>1.43</td>
</tr>
<tr>
<td>Intrinsic carrier concentration, ni (cm⁻³)</td>
<td>2.4 × 10¹³</td>
<td>1.5 × 10¹⁹</td>
<td>1.79 × 10⁶</td>
</tr>
<tr>
<td>Lattice constant (Å)</td>
<td>5.65</td>
<td>5.43</td>
<td>5.65</td>
</tr>
<tr>
<td>Effective mass</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density of states m*ₗ/mₗ</td>
<td>0.55</td>
<td>1.18</td>
<td>0.68</td>
</tr>
<tr>
<td>m*ₗ/mₗ</td>
<td>0.3</td>
<td>0.81</td>
<td>0.56</td>
</tr>
<tr>
<td>Conductivity mₗ/mₗ</td>
<td>0.12</td>
<td>0.26</td>
<td>0.09</td>
</tr>
<tr>
<td>m*ₗ/mₗ</td>
<td>0.23</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>Melting point (°C)</td>
<td>937</td>
<td>1415</td>
<td>1238</td>
</tr>
<tr>
<td>Intrinsic mobility</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electron (cm² V⁻¹ sec⁻¹)</td>
<td>3900</td>
<td>1350</td>
<td>8500</td>
</tr>
<tr>
<td>Hole (cm² V⁻¹ sec⁻¹)</td>
<td>1900</td>
<td>480</td>
<td>400</td>
</tr>
</tbody>
</table>

TABLE 4.2
IMPORTANT FORMULAS IN SEMICONDUCTOR PHYSICS

Complete ionization of impurities

\[ \rho = q(p - n + N_D - N_A) = 0 \]

Equilibrium condition

\[ pm = n_i^3 \]

Fermi-Dirac distribution function

\[ f(E) = \frac{1}{1 + e^{(E - E_f)/kT}} \]

Carrier concentrations in non-degenerate semiconductors:

\[ n = N_e e^{-(E_f - E_g)/kT} = n_i e^{(E_f - E_g)/kT} \]

\[ p = N_h e^{-(E_f - E_g)/kT} = n_i e^{(E_f - E_g)/kT} \]

In the extrinsic case,

\[ |N_D - N_A| \gg n_i: \]

\[ n = N_D - N_A \]

\[ p = N_A - N_D \]

\[ \rho = \frac{n_i^3}{N_D - N_A} \]

\[ \rho = \frac{n_i^3}{N_A - N_D} \]
\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \]
\[ \nabla^2 \phi = -\rho/\varepsilon_0 \]
\[ \frac{d\mathbf{E}}{dx} = \frac{q}{\varepsilon_0} [N_d - N_a + p_o - n_o + p_e - n_e] \]

**TABLE 6.1**

**IMPORTANT FORMULAS FOR ONE-SIDED STEP JUNCTIONS**

<table>
<thead>
<tr>
<th>Built-in voltage</th>
<th>( \phi_B = \frac{kT}{q} \ln \frac{N_a N_0}{n_i^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depletion region width</td>
<td>( W = \sqrt{\frac{2K_T \phi_B \pm</td>
</tr>
<tr>
<td>where (+:) reverse bias</td>
<td>(-:) forward bias</td>
</tr>
<tr>
<td>Maximum electric field</td>
<td>( \varepsilon_{\text{max}} = 2 \frac{\phi_B \pm</td>
</tr>
<tr>
<td>Capacitance per unit area</td>
<td>( C = \frac{K_T \varepsilon_0}{W} )</td>
</tr>
<tr>
<td>Reverse current</td>
<td>( I_R = I_{\text{rec}} + I_{\text{diff}} )</td>
</tr>
<tr>
<td>( I_{\text{gen}} = \frac{1}{2} q n_i^2 W A_f )</td>
<td>( I_{\text{diff}} = q D \frac{n_i^2}{C_B L} A_f )</td>
</tr>
<tr>
<td>Forward current</td>
<td>( I_F = I_{\text{rec}} + I_{\text{diff}} )</td>
</tr>
<tr>
<td>( I_{\text{rec}} = -\frac{1}{2} q n_i^2 W A_f \sqrt{V_f / 2kT} )</td>
<td>( I_{\text{diff}} = -q D \frac{n_i^2}{C_B L} \sqrt{V_f / 2kT} A_f )</td>
</tr>
<tr>
<td>Avalanche breakdown voltage</td>
<td>( BV = \frac{K_T \varepsilon_0}{2 q C_B} )</td>
</tr>
</tbody>
</table>

\( \phi_B \equiv N_D + N_A \)

**TABLE 5.1**

**IMPORTANT FORMULAS FOR SEMICONDUCTORS UNDER NON-EQUILIBRIUM CONDITIONS**

Midgap recombination-generation centers, i.e., \( E_i = E_l \)

Equal capture cross-sections, i.e., \( \sigma_p = \sigma_n = \sigma \)

<table>
<thead>
<tr>
<th>( n )-Type semiconductor</th>
<th>( p )-Type semiconductor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net bulk recombination rate per unit volume</td>
<td>( U = \frac{1}{\tau} (p_n - p_{n0}) )</td>
</tr>
<tr>
<td>Net surface recombination rate per unit area</td>
<td>( U_s = s[p_n(0) - p_{n0}] )</td>
</tr>
<tr>
<td>Lifetime</td>
<td>( \tau = \frac{1}{\sigma v_{th} N_t} )</td>
</tr>
<tr>
<td>Surface recombination velocity</td>
<td>( \frac{N_p}{s_o n_s + p_s + 2n_i} )</td>
</tr>
</tbody>
</table>