ELEG 646/ 446 - Nanoelectronic Device Principles – Spring 2005
Mid Term Examination

7 April 2005 NAME SOLUTION

Time Limit: 1 hour;

Closed Books and Notes;

However, you are permitted to use one page (both sides) of you own notes;

Please do not loan calculators to anyone

35
35
15
25
25
135

Scoring: Short Questions 1 -7 are worth 5 points each;
Long Problems 8-11 have the point values indicated.
Full credit requires giving the dimensions / units for all numerical quantities that you calculate.

I. Unless stated otherwise, assume:

a) Silicon material at T = 300K,

b) Steady state conditions,

c) all carrier recombination lifetimes $\tau_{n,p} = 10^{-7}$ sec

II. Use appropriate value of mobility μ, D, L, m^*, etc., for the given impurity concentrations (see data sheets).

III. Show all calculations.

IV. Accuracy to 2 significant figures is sufficient.

V. You may use either $n_i = 1 \times 10^{10}$ cm^{-3}, or, 1.5×10^{10} cm^{-3}, for Si at room temperature (300K).

For your convenience, equation sheets and graphs are provided

Short Questions (5 points each)

1. What is the algebraic statement of low level injection?

excess majority concentration < majority

$p_n|_{excess} < n_n$

or $n_p|_{excess} < p_p$

2. A homogeneous sample of Ge has compensated doping with $N_A = 5 \times 10^{17}$ cm^{-3} , and $N_D = 1 \times 10^{17}$ cm^{-3}. What is the value of the carrier concentration?

$p \simeq N_A - N_D = (5-1) \times 10^{17} \text{ cm}^{-3} = 4 \times 10^{17} \text{ cm}^{-3}$

$\gg n_i \; (= 2.4 \times 10^{13} \text{ cm}^{-3})$ so good approximation

$$n = \frac{n_i^2}{p} = \frac{(2.4 \times 10^{13})^2 \text{ cm}^{-6}}{4 \times 10^{17} \text{ cm}^{-3}} = 1.44 \times 10^{9} \text{ cm}^{-3}$$

3. Give the mathematical statement of the Law of the Junction.

$$p_n(0) = p_{no}\, e^{qV_F/kT}$$

or

$$n_p(0) = n_{po}\, e^{qV_F/kT}$$

4. Why does the "reverse saturation current" (J_s) of an ideal diode saturate?

because for $V_R \gg kT/q$,

$p_n(0)$, or $n_p(0) \simeq 0$ so diffusion gradient is constant, so diffusion current is constant

$$J = qD(0 - p_{no})e^{-x/L} = q\frac{D}{L}p_{no} = \text{const.}$$

5. True or False: The space charge region about the metallurgical junction is due to a pile up of electrons on the p-side and holes on the n-side.

False

6. True or False: For a p^+-n step junction with N_A(p-side) >> N_D(n-side), then $x_p << x_n$.

True

7. True or False: Zener breakdown typically occurs at a reverse voltage with a greater magnitude than that of avalanche breakdown.

False

Long Problems 8 through 11 (point values indicated):Forward-and Reverse-Biased Si Diode:

An abrupt Si p^+n junction diode has a cross sectional area of 1 mm^2, an acceptor concentration of 5 x 10^{18} boron atoms cm^{-3} on the p-side, and a donor concentration of 10^{16} arsenic atoms cm^{-3} on the n-side. The lifetime of holes in the n-region is 417 ns, and that of electrons in the p-region is 5 ns due to a greater concentration of impurities (recombination centers) on that side. Mean thermal generation lifetime in the depletion region (τ_g) is about 1 μs. The lengths of the p- and n-regions are 5 and 100 microns, respectively.

8. (35 points) Calculate the minority diffusion lengths at the given doping concentrations, and determine if this diode is long or short base.

p side: $N_A = 5\times10^{18}\ cm^{-3} \rightarrow \mu_n = 130\ cm^2/V\text{-}s$

$$D_n = \frac{kT}{q}\mu_n = 0.0259\ V \times 130\ \frac{cm^2}{V\text{-}s} = 3.37\ \frac{cm^2}{s}$$

$$L_n = \sqrt{D_n \tau_n} = \sqrt{3.37\ \frac{cm^2}{s} \times 5\times10^{-9} s} = 1.2\times10^{-4}\ cm$$

$= 1.2\ \mu m$

$<< x_p = 5\ \mu m$

n side $N_D = 10^{16}\ cm^{-3} \rightarrow \mu_p = 440\ cm^2/V\text{-}s$

$$D_p = \frac{kT}{q}\mu_p = 0.0259 \times 440\ \frac{cm^2}{V\text{-}s} = 11.4\ cm^2/s$$

$$L_p = \sqrt{D_p \tau_p} = \sqrt{11.4\ \frac{cm^2}{s} \times 417\times10^{-9} s} = 2.18\times10^{-3}\ cm$$

$= 21.8\ \mu m$

$<< x_n = 100\ \mu m$

$$\left.\begin{matrix} L_n < x_p \\ L_p < x_n \end{matrix}\right\} \text{ so long base diode}$$

$\underbrace{x_p,\ x_n}$ → lengths of neutral regions

9. (15 points) What is the built-in potential across the junction?

$$\phi_{bi} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} = \overset{0.0259}{\cancel{0.026}} \ln\left[\frac{5\times10^{18} \times 10^{16}}{10^{20}}\right] V$$

(or 2.25)

$$= \overset{0.0259}{\cancel{0.026}} \times 33.84 \quad (\text{or } 33.03)$$

$$= \underset{0.876}{\cancel{0.88}} \text{ V} \quad (\text{or } \underset{0.855}{\cancel{0.859}} \text{ V})$$

10. (25 points) What is the current at a forward bias of 0.6 V across the diode at 27°C? Assume that the current is by minority carrier diffusion. (Hint: You may use approximations if you justify them.)

since $p^+n \rightarrow$ consider only holes, which dominate

$$J_s^p = q D_p \frac{n_i^2}{N_D L_p} e^{qV_F/kT}$$

$$= \frac{1.6\times10^{-19}\,C \times (1\times10^{10}\,cm^{-3})^2 \left(\overset{11.4}{\cancel{12.95}}\,\frac{cm^2}{s}\right)}{(21.8\times10^{-4}\,cm)(10^{16}\,cm^{-3})}$$

$$J_s = 8.36\times10^{-12}\,A/cm^2$$

$$I_s = J_s \times 0.01\,cm^2 = 8.36\times10^{-14}\,A = 0.084\,pA$$

$$I_F = I_s\, e^{qV/kT} = (8.36\times10^{-14}A)\exp\left[\frac{0.6v}{0.0259v}\right]$$

$$= 0.96\times10^{-3}\,A = 0.96\,mA$$

11. (25 points) What is the reverse current due to thermal generation in the depletion region when the diode is reverse-biased by a voltage $V_R = 5$ V?

$$W_{dep} = \left[\frac{2 k_s \epsilon_0 (\phi_{bi} + V_R)}{q N_D} \right]^{1/2}$$

$$= \left[\frac{2 \times 11.9 \times 8.85 \times 10^{-12} F/m \times (0.877 + 5)}{1.6 \times 10^{-19} C \times 10^{22} m^{-3}} \right]^{1/2}$$

$$= 0.88 \times 10^{-6} \ m = 0.88 \ \mu m$$

$$I_{gen} = \frac{q A W n_i}{2 \tau_g} = 1.6 \times 10^{-19} C \times 0.01 cm^2 \times$$

$$= \frac{1.6 \times 10^{-19} C \times 0.01 cm^2 \times 0.88 \times 10^{-4} cm \times 10^{10} cm^{-3}}{10^{-6} s}$$

$$= 1.41 \times 10^{-9} A = 1.4 \ nA \qquad \left(J_R = 1.41 \times 10^{-7} \frac{A}{cm^2} \right)$$

or $0.7 \ nA$

TABLE 4.2
Properties of Ge, Si and GaAs at 300 K

Property	Ge	Si	GaAs
Atomic/molecular weight	72.6	28.09	144.63
Density (g cm^{-3})	5.33	2.33	5.32
Dielectric constant	16.0	11.9	13.1
Effective density of states			
Conduction band, N_C (cm^{-3})	1.04×10^{19}	2.8×10^{19}	4.7×10^{17}
Valence band N_V (cm^{-3})	6.0×10^{18}	1.02×10^{19}	7.0×10^{18}
Electron affinity (eV)	4.01	4.05	4.07
Energy gap, E_g (eV)	0.67	1.12	1.43
Intrinsic carrier concentration, n_i (cm^{-3})	2.4×10^{13}	1.5×10^{10}	1.79×10^{6}
Lattice constant (Å)	5.65	5.43	5.65
Effective mass			
Density of states m_e^*/m_o	0.55	1.18	0.068
m_h^*/m_o	0.3	0.81	0.56
Conductivity m_e/m_o	0.12	0.26	0.09
m_h/m_o	0.23	0.38	
Melting point (°C)	937	1415	1238
Intrinsic mobility			
Electron (cm^2 V^{-1} sec^{-1})	3900	1350	8500
Hole (cm^2 V^{-1} sec^{-1})	1900	480	400

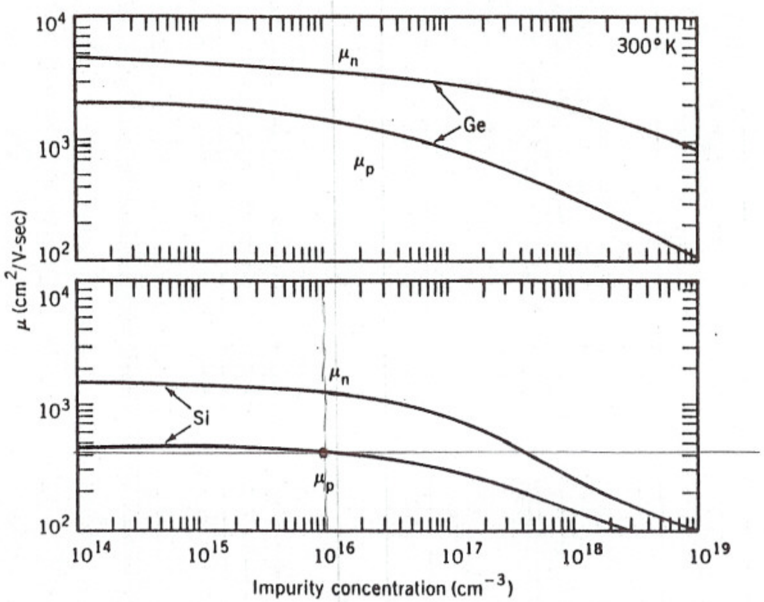

FIGURE 4.5 Electron and hole mobilities in germanium and silicon as a function of dopant impurity concentrations. (From S. M. Sze, reference 5, p. 29. Copyright © 1981. Reprinted by permission of John Wiley & Sons, Inc., New York.)

Quantity	Symbol	Value
Angstrom unit	Å	1 Å = 10^{-8} cm = 10^{-10} m
Avogadro number	N	6.023×10^{23}/mol
Boltzmann constant	k	8.620×10^{-5} eV/K = 1.381×10^{-23} J/K
Electronic charge	q	1.602×10^{-19} C
Electron rest mass	m_o	9.109×10^{-31} kg
Electron volt	eV	1 eV = 1.602×10^{-19} J
Gas constant	R	1.987 cal/mole-K
Permeability of free space	μ_o	1.257×10^{-6} H/m
Permittivity of free space	ε_o	8.850×10^{-12} F/m
Planck constant	h	6.626×10^{-34} J-s
Proton rest mass	m_p	1.673×10^{-27} kg
$h/2\pi$	$\hbar$	1.054×10^{-34} J-s
Thermal voltage at 300 K	V_T	0.02586 V
Velocity of light in vacuum	c	2.998×10^{10} cm/s
Wavelength of 1-eV quantum	λ	1.24 μm

$D = \frac{kT}{q} \mu$ $\qquad$ $\sigma = ne\mu_n + pe\mu_p$

TABLE 4.2
IMPORTANT FORMULAS IN SEMICONDUCTOR PHYSICS
Complete ionization of impurities
Thermal equilibrium

Charge neutrality	$\rho = q(p - n + N_D - N_A) = 0$
Equilibrium condition	$pn = n_i^2$
Fermi-Dirac distribution function	$f(E) = \dfrac{1}{1 + e^{(E-E_F)/kT}}$
Carrier concentrations in non-degenerate semiconductors:	$n = N_c e^{-(E_c-E_F)/kT} = n_i e^{(E_F-E_i)/kT}$ $p = N_v e^{-(E_F-E_v)/kT} = n_i e^{(E_i-E_F)/kT}$
In the extrinsic case, $\lvert N_D - N_A \rvert \gg n_i$:	$n_n \doteq N_D - N_A$ $\quad$ $p_p \doteq N_A - N_D$ $p_n \doteq \dfrac{n_i^2}{N_D - N_A}$ $\quad$ $n_p \doteq \dfrac{n_i^2}{N_A - N_D}$

$$\nabla \cdot \mathscr{E} = \frac{\rho}{\varepsilon_s} \qquad \nabla^2\phi = -\rho/\varepsilon_s$$

$$\frac{d\mathscr{E}}{dx} = \frac{q}{\varepsilon_s}[N_d - N_a + p_o - n_o + p_e - n_e]$$

$$\mathbf{J}_p = q\mu_p p\mathscr{E} - qD_p\nabla p$$

$$\mathbf{J}_n = q\mu_n n\mathscr{E} + qD_n\nabla n$$

$$\frac{\partial p_e}{\partial t} = G_L - U - \frac{1}{q}\nabla \cdot \mathbf{J}_p$$

$$\frac{\partial n_e}{\partial t} = G_L - U + \frac{1}{q}\nabla \cdot \mathbf{J}_n$$

TABLE 6.1

IMPORTANT FORMULAS FOR ONE-SIDED STEP JUNCTIONS: note $C_B \equiv N_D$ or N_A

Built-in voltage	$\phi_B = \frac{kT}{q}\ln\frac{N_A N_D}{n_i^2}$
Depletion region width	$W = \sqrt{\frac{2K_s\epsilon_0[\phi_B \pm \lvert V_J\rvert]}{qC_B}}$ where $\left.\begin{matrix}+\text{: reverse}\\-\text{: forward}\end{matrix}\right\}$ bias
Maximum electric field	$\mathscr{E}_{\max} = 2\frac{\phi_B \pm \lvert V_J\rvert}{W}$
Capacitance per unit area	$C = \frac{K_s\epsilon_0}{W}$
Reverse current	$I_R = I_{\text{gen}} + I_{\text{diff}}$ $I_{\text{gen}} = \frac{1}{2}q\frac{n_i}{\tau}WA_J$ $I_{\text{diff}} = qD\frac{n_i^2}{C_B L}A_J$
Forward current	$I_F = I_{\text{rec}} + I_{\text{diff}}$ $I_{\text{rec}} = -\frac{1}{2}q\frac{n_i}{\tau}We^{q\lvert V_F\rvert/2kT}A_J$ $I_{\text{diff}} = -qD\frac{n_i^2}{C_B L}e^{q\lvert V_F\rvert/kT}A_J$
Avalanche breakdown voltage	$BV = \frac{K_s\epsilon_0\mathscr{E}_{\text{crit}}^2}{2qC_B}$

$$\frac{\partial p_e}{\partial t} = G_L - U - \mu_p\mathscr{E}\frac{\partial p}{\partial x} - \mu_p p\frac{\partial\mathscr{E}}{\partial x} + D_p\frac{\partial^2 p}{\partial x^2}$$

$$\frac{\partial n_e}{\partial t} = G_L - U + \mu_n\mathscr{E}\frac{\partial n}{\partial x} + \mu_n n\frac{\partial\mathscr{E}}{\partial x} + D_n\frac{\partial^2 n}{\partial x^2}$$

TABLE 5.1

IMPORTANT FORMULAS FOR SEMICONDUCTORS UNDER NON-EQUILIBRIUM CONDITIONS

Midgap recombination-generation centers, i.e., $E_t = E_i$

Equal capture cross-sections, i.e., $\sigma_p = \sigma_n = \sigma$

	n-Type semiconductor	p-Type semiconductor
Net bulk recombination rate per unit volume	$U = \frac{1}{\tau}(p_n - p_{no})$	$U = \frac{1}{\tau}(n_p - n_{po})$
Net surface recombination rate per unit area	$U_s = s[p_n(0) - p_{no}]$	$U_s = s[n_p(0) - n_{po}]$
Lifetime	$\tau = \frac{1}{\sigma v_{th}N_t}$	$\tau = \frac{1}{\sigma v_{th}N_t}$
Surface recombination velocity	$s = s_o\frac{N_D}{n_s + p_s + 2n_i}$	$s = s_o\frac{N_A}{n_s + p_s + 2n_i}$
	$s_o \equiv \sigma v_{th}N_{st}$	