The given JFET has
\[ V_p = 3.5 \, \text{V}, \quad V_i = 0.8 \, \text{V} \quad \text{and} \quad G_o = 1.44 \times 10^{-2} \, \text{A/V} \]

\[ V_{Dsat} = 3.5 - (0.8 + 0) = 2.7 \, \text{V} \]

gives
\[ I_D = 1.44 \times 10^{-2} \left[ 2.7 - \frac{2}{3} \times 3.5 \left( \left( \frac{0.8+2.7}{3.5} \right)^{3/2} - \left( \frac{0.8}{3.5} \right)^{3/2} \right) \right] \]
\[ = 8.952 \times 10^{-3} \, \text{A} \]

The value of the load resistance at this current is
\[ R_D = \frac{V_D}{I_D} = \frac{2.7}{8.952 \times 10^{-3}} = 301.61 \, \Omega \]

(c) The transconductance in the saturation is given by
\[ g_m = 1.44 \times 10^{-2} \left[ 1 - \left( \frac{0.8 + 0}{3.5} \right)^{1/2} \right] = 7.515 \times 10^{-3} \, \text{A/V} \]

For a bipolar transistor
\[ g_m \approx \frac{|I_C|}{V_T} = \frac{8.952 \times 10^{-3}}{25.84 \times 10^{-3}} = 0.346 \, \text{A/V} \]

which is large compared to that of the JFET.
\[ I_D = q N_d Z \mu_n a \left[ 1 - \left( \frac{V_i - V_G + V(x)}{V_p} \right)^{1/2} \right] \left( \frac{\mu_n}{v_s} \right) \frac{dV}{dV + dx} \]

or

\[ I_D dx + I_D \frac{\mu_n}{v_s} dV = q N_d Z \mu_n a \left[ 1 - \left( \frac{V_i - V_G + V(x)}{V_p} \right)^{1/2} \right] dV \]

Integrating both sides from \( x = 0 \) to \( x = L \) we get

\[ I_D \left| x \right|_0^L + I_D \frac{\mu_n}{v_s} \left| V \right|_0^L = q N_d Z \mu_n a \left[ \left| V \right|_0^L - \frac{2}{3} V_p \left| \frac{V_i - V_G + V}{V_p} \right|_0^L \right] \]

and finally we obtain

\[ I_D \left[ L + \frac{\mu_n V_D}{v_s} \right] = q N_d Z \mu_n a \left[ V_D - \frac{2}{3} V_p \left( \frac{V_i - V_G + V}{V_p} \right)^{3/2} - \left( \frac{V_i - V_G}{V_p} \right)^{3/2} \right] \]
3.

\[ \phi_s = \frac{K T}{q} \ln \left( \frac{n_s}{n_o} \right) \]

where \( n_o = N_d = 5 \times 10^{15} \ \text{cm}^{-3} \)

When the surface becomes intrinsic \( n_s = n_i \) and

\[ \phi_s = 25.86 \times 10^{-3} \ln \frac{1.5 \times 10^{10}}{5 \times 10^{15}} = -0.329 \ \text{V} \]

For strong inversion at the surface \( p_s = n_o \) and \( n_s = \frac{n_i^2}{n_o} \).

Thus,

\[ \phi_s = 25.86 \times 10^{-3} \ln \left( \frac{(1.5 \times 10^{10})^2}{(5 \times 10^{15})^2} \right) = -0.658 \ \text{V} \]

4.

\[ V_{FB} = \phi_{ms} - \frac{q \varepsilon_{ox}}{C_{ox}} \]

For gold on n-type silicon we obtain

\[ \phi_{ms} = 0.58 \ \text{V} \] and

\[ C_{ox} = \frac{\varepsilon_{ox}}{d_{ox}} = \frac{8.85 \times 10^{-14} \times 3.9}{1.2 \times 10^{-5}} = 2.876 \times 10^{-8} \ \text{F/cm}^2 \]

Thus,

\[ V_{FB} = 0.58 - \frac{3 \times 10^{11} \times 1.6 \times 10^{-19}}{2.876 \times 10^{-8}} = -1.087 \ \text{V}. \]

We obtain the Fermi potential by substituting \( N_d \) for \( N_a \) in

\[ \phi_F = -\frac{k T}{q} \ln \left( \frac{N_d}{n_i} \right) = -25.86 \times 10^{-3} \ln \frac{10^{15}}{1.5 \times 10^{10}} = -0.287 \ \text{V} \]
\[ G_{BO} = \left(4\varepsilon_s q N_d \Phi_F \right)^{1/2} \]

\[ = (4 \times 8.85 \times 10^{-14} \times 11.9 \times 1.6 \times 10^{-19} \times 10^{15} \times 0.287)^{1/2} \]

\[ = 1.391 \times 10^{-8} \text{ C/cm}^2. \]

\[ V_{th} = -1.089 - 0.574 - \frac{1.391 \times 10^{-8}}{2.876 \times 10^{-8}} = -2.147 \text{ V}. \]

To get band bending:

solve Poisson's equation to get

\[ \frac{d \Phi(x)}{dx} = -\varepsilon(x) \]

\[ \Phi_S = \varepsilon_S \varepsilon_S \]

\[ V_{ok} = -\frac{\Phi_S}{\varepsilon_S} \]

\[ \Phi_S = (-V_{fb}) - V_{ok} \]

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FIG.P16.5

(c) The energy band diagram of the MOS system under thermal equilibrium is shown in Fig. P16.5(a). Note that the bands are bent downward because of electron accumulation at the surface. The energy band diagram at the onset of strong inversion is shown in Fig. P16.5(b). Now \( E_{FM} \) lies above the semiconductor Fermi level \( E_F \) and the bands are bent upward.
For this MOS capacitor we have

\[ C_{ox} = \frac{\varepsilon_{ox}}{d_{ox}} = \frac{8.85 \times 10^{-14} \times 3.9}{2 \times 10^{-5}} = 1.726 \times 10^{-8} \text{ F/cm}^2 \]

The semiconductor capacitance at flat-band condition is

\[ C_s(FB) = \frac{\varepsilon_s}{L_D} = \left[ \frac{\varepsilon_s q N_d}{V_T} \right]^{1/2} \]

\[ = \left[ \frac{8.85 \times 10^{-14} \times 11.9 \times 1.6 \times 10^{-19} \times 1.5 \times 10^{14}}{25.86 \times 10^{-3}} \right]^{1/2} \]

\[ = 3.126 \times 10^{-8} \text{ F/cm}^2 \]

Now,

\[ C_{FB} = \frac{C_{ox} C_s}{C_{ox} + C_s} = 1.112 \times 10^{-8} \text{ F/cm}^2 \]

In order to determine the zero bias capacitance we must know the flat-band voltage \( V_{FB} \). For Al gate on n-type silicon with \( N_d = 1.5 \times 10^{14} \text{ cm}^{-3} \) we obtain \( \phi_{ms} = -0.375 \text{ V} \) and since \( Q_{ox} = 0 \) we have \( V_{FB} = \phi_{ms} = -0.375 \text{ V} \). Thus, there is a positive voltage at the gate and the surface is under accumulation. To determine the surface potential we apply Gauss's law at the Si - SiO\(_2\) interface. This gives

\[ V_{ox} = (0.375 - \phi_s) \]

\[ = \frac{\varepsilon_s}{\varepsilon_{ox}} d_{ox} \phi_s = \frac{\varepsilon_s}{\varepsilon_{ox}} d_{ox} \left[ \frac{2q N_d \phi_s}{\varepsilon_s} \right] \]

\[ = \frac{11.9}{3.9} \times 2 \times 10^{-5} \left[ \frac{2 \times 1.6 \times 10^{-19} \times 1.5 \times 10^{14} \phi_s}{8.85 \times 10^{-14} \times 11.9} \right]^{1/2} \]

Solving this equation we obtain \( \phi_s = 0.194 \text{ V} \) and from
\[ n_s = 1.5 \times 10^{14} \exp \left( -\frac{0.194}{25.86 \times 10^{-3}} \right) = 2.717 \times 10^{17} \text{ cm}^{-3} \]

As an approximation if we substitute this concentration for \( N_d \) then \( C_s \approx 1.33 \times 10^{-6} \text{ F/cm}^2 \) and

\[
C(V_G = 0) = \frac{C_{ox} C_s}{C_{ox} + C_s} = \frac{1.726 \times 10^{-8} \times 1.33 \times 10^{-6}}{1.726 \times 10^{-8} + 1.33 \times 10^{-6}}
\]

\[ = 1.704 \times 10^{-8} \text{ F/cm}^2. \]

The maximum value of capacitance is

\[ C_{\text{max}} = C_{ox} = 1.726 \times 10^{-8} \text{ F/cm}^2 \]

and the minimum capacitance occurs at strong inversion.

For this capacitor

\[ \phi_F = -25.86 \times 10^{-3} \ln \frac{1.5 \times 10^{14}}{1.5 \times 10^{10}} = -0.238 \text{ V} \]
\[ W_m = \left[ \frac{4 \times 8.85 \times 10^{-14} \times 11.9 \times 0.238}{1.6 \times 10^{-19} \times 1.5 \times 10^{14}} \right]^{1/2} = 2.044 \times 10^{-4} \text{ cm}. \]

This width is much less than the epitaxial layer thickness of \( 2 \times 10^{-3} \text{ cm} \). Thus,

\[ C_s(\text{min}) = \frac{\varepsilon_s}{W_m} = \frac{8.85 \times 10^{-14} \times 11.9}{2.044 \times 10^{-4}} = 5.152 \times 10^{-9} \text{ F/cm}^2 \]

and

\[ C_{\text{min}} = \frac{1.726 \times 10^{-8} \times 5.152 \times 10^{-9}}{1.726 \times 10^{-8} + 5.152 \times 10^{-9}} = 3.968 \times 10^{-9} \text{ F/cm}^2. \]

Which gives

\[ \frac{C_{\text{max}}}{C_{\text{min}}} = 4.35 \]

The C-V plot can now be sketched.

Homework assignments will appear on the web at:
http://www.ece.udel.edu/~kolodzey/courses/eleg646s05.html

Note: On each homework and report submission, you must please give your name, the due date, assignment number and the course number.