ELEG 340 - Fall 08
Solid-State Electronics
Quiz 6

4 December 2008 NAME Solution

Time Limit: 30 minutes

Closed Books and Notes. You may use your own calculator, but may not loan or borrow one (ask proctor if you have questions). Put each expression in a final form as best you can.

Guidelines:
I. Full credit requires the dimensions/ units for all numerical quantities.

II. Show your work and calculations for full credit; accuracy to 2 significant figures is sufficient.

III. Assume that the semiconductor material is silicon at room temperature (300 K), unless otherwise stated.

IV. Data: at room temperature (300K): thermal energy $k_BT = 0.026$ eV; thermal voltage $k_BT/q = 0.026$ volts; silicon intrinsic concentration $n_i = 1$ (or 1.5) x 10^{10} cm^{-3}; recombination lifetimes: τ_n, $\tau_p = 1$ μsec; dielectric constants $\kappa_{Si} = 11.8$; $\kappa_{ox} = 3.9$; permittivity of free space $\varepsilon_o = 8.85 \times 10^{-14}$ F/cm; electron charge $|q| = |e| = 1.6 \times 10^{-19}$ Coul;

V. Equations: see list at end of quiz

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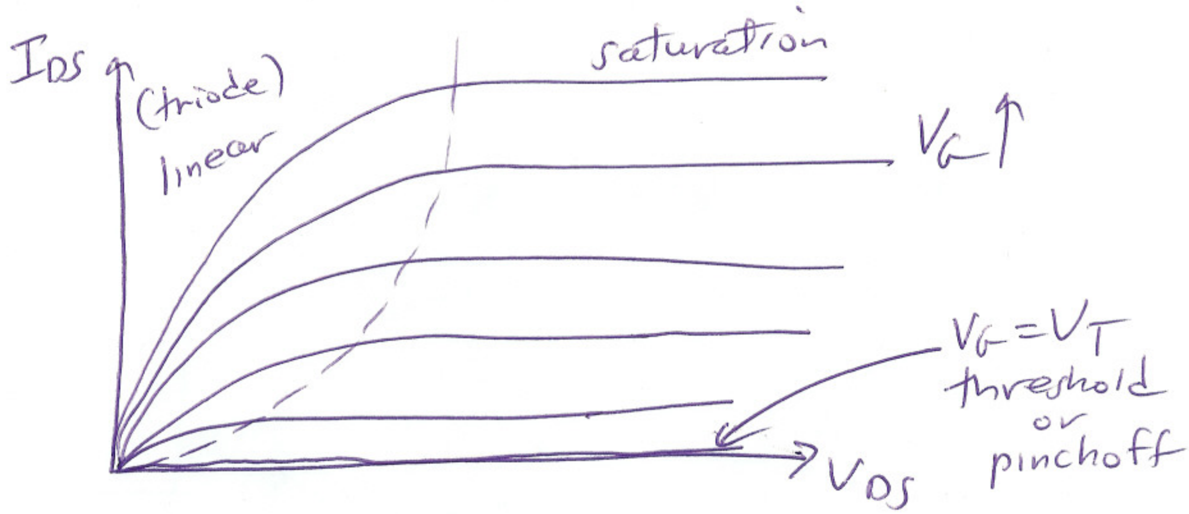
1. For an n-channel MOSFET, draw the output characteristics of drain current (I_{DS}) versus drain source voltage (V_{DS}), with the gate current (V_{GS}) as the parameter. Label the linear and the saturation regions of your plot. Indicate the threshold or pinch-off values of the drain current and the gate voltage.



$I_{DS} \rightarrow 0$ at threshold

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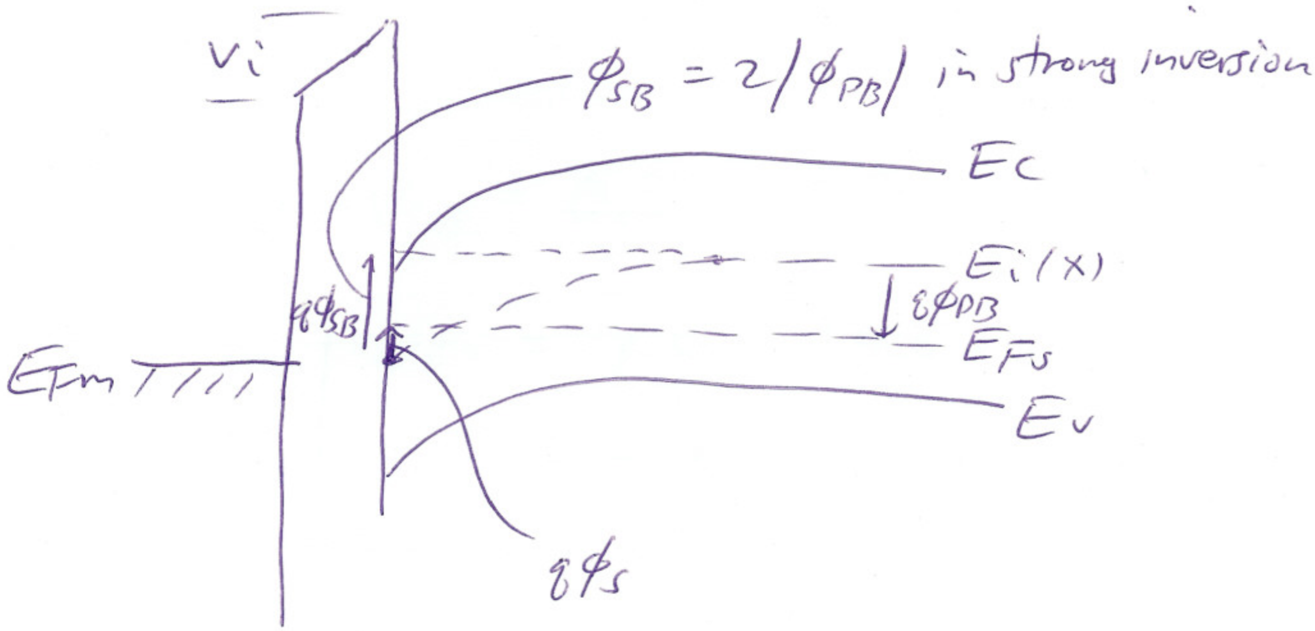
2. Draw the energy band diagram (E_C, E_V, E_F, and E_i) versus position for an n-channel MOSFET transistor. Show metal gate, dielectric insulator (e.g. oxide), and the semiconductor "surface" and bulk energies for the condition of *strong inversion.* Label the: Fermi level in the semiconductor (E_{Fs}), the intrinsic level ($E_i(x)$), the bulk Fermi potential φ_{pB}, the total band bending from surface to bulk (φ_{sB}), and the conductivity type of the substrate (i.e. *n* or *p*). Write down the mathematical relation between φ_{pB} and φ_{sB} at the onset of strong inversion.



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3. For the n-MOSFET of problem 2 above with an applied gate-to-body voltage $V_{GB} = 1.0$ volts, assume that the insulator voltage drop $V_i = 0.5$ volts, and the total band bending in the semiconductor $\varphi_{sB} = 0.7$ volts . What is the numerical value of the flat band voltage V_{FB}?

$$V_G - V_{FB} = V_i + \phi_{sB}$$

↑

$= V_{GB}$

$$1.0 - V_{FB} = 0.5v + 0.7v$$

$$V_{FB} = 1.0 - (0.5 + 0.7)\ v$$

$$= -0.2\ V$$

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V. Equations:

$p_{op} = i\hbar d/dx$ $f_{FD}(E) = 1/[1 + \exp(E–E_F)/k_BT]$ $E = Q\ V$

$n = n_i\exp[(E_F–E_i)/k_BT]$. $p = n_i\exp[(E_i–E_F)/k_BT]$ $n_op_o = n_i^2$ $np = n_i^2e^{(Fn\text{-}Fp)/kT}$

$J_n = q\mu_nn\mathcal{E} + qD_ndn/dx$ $J_p = q\mu_pp\mathcal{E} – qD_pdp/dx$ $\sigma_{elec} = q(n\mu_n + p\mu_p)$

$U_n = (n_p–n_{po})/\tau_n$ $U_p = (p_n–p_{no})/\tau_p$ $p' = p - p_o = g_{opt}\tau_p$ $n' = n - n_o = g_{opt}\tau_n$

$C_{dep} = \kappa_s\varepsilon_o\ A/W$ $C_{diff} = qI\tau/k_BT$ $D/\mu = k_BT/q$ $L = \sqrt{(D\tau)}$

$\partial p/\partial t = -1/q\ \partial J_p/\partial x – p'/\tau_p$ $\partial n/\partial t = -1/q\ \partial J_n/\partial x – n'/\tau_n$;

$\partial p/\partial t = D_p\ \partial^2p/\partial x^2 – p'/\tau_p$ $\partial n/\partial t = D_n\ \partial^2n/\partial x^2 – n'/\tau_n$

$\varphi_{bi} = k_BT/q\ \ln(N_AN_D/n_i^2)$ $W_{dep} = [2\kappa_s\varepsilon_o/q\ (1/N_A + 1/N_D)(\varphi_{bi} - V_F)]^{1/2}$

$p_n(x_{no}) = p_{no}(x_{no})e^{qVf/kT}$; $n_p(-x_{po}) = n_{po}(-x_{po})e^{qVf/kT}$

Diode current:

$I = qA(D_pp_n/L_p + D_nn_p/L_n)[e^{qV/kT} – 1] = I_o[e^{qV/kT} – 1]$; $I_o = I_{th} = qA(D_pp_n/L_p + D_nn_p/L_n)$

Solar Cell and Photodetector diodes:

$I_{tot} = I_o\ [e^{qV/kT} – 1] – I_{opt}$; $I_{opt} = qAg_{opt}(L_n + L_p + W)$

BJTs:

$I_E = I_C + I_B$ $I_C = \alpha I_E + I_{CBO}$ $I_C = \beta I_B + I_{CEO}$

$I_E \approx qAD_p/L_p\ \Delta p_E\ \text{ctnh}W_b/L_p$ $I_C \approx qAD_p/L_p\ \Delta p_E\ \text{csch}W_b/L_p$

$I_B \approx qAD_p/L_p\ \Delta p_E\ \tanh W_b/2L_p \ldots$ $\alpha = \gamma\ B_T = (\beta/1+\beta)$ $\beta = \tau_p/\tau_{tr} = (\alpha/1\text{-}\alpha)$

$I_E = I_{Ep} + I_{En}$. $I_{Ep} = \gamma I_E$ $I_{Cp} = B_T\ I_{Ep}\ldots$ $I_C = I_{Cp} + I_{CBO}$

$\gamma = I_{Ep}/(I_{Ep} + I_{En})$ $B_T = \text{sech}W_b/L_p \approx (1 – W_b^2/2L_p^2)$

Ebers-Moll:

$$I_E = I_{EN} + I_{EI} = I_{ES}(e^{qV_{EB}/kT} - 1) - \alpha_I I_{CS}(e^{qV_{CB}/kT} - 1)$$
$$I_C = I_{CN} + I_{CI} = \alpha_N I_{ES}(e^{qV_{EB}/kT} - 1) - I_{CS}(e^{qV_{CB}/kT} - 1)$$

$$I_C = \alpha_N I_{ES}\frac{\Delta p_E}{p_n} - I_{CS}\frac{\Delta p_C}{p_n} = \frac{I_{CS}}{p_n}(\alpha_I\Delta p_E - \Delta p_C)\ ,\quad I_E = I_{ES}\frac{\Delta p_E}{p_n} - \alpha_I I_{CS}\frac{\Delta p_C}{p_n} = \frac{I_{ES}}{p_n}(\Delta p_E - \alpha_N\Delta p_C)$$

JFET equations:

$V_P = qa^2N_D/2\mathcal{E}_s$ $G_o = 2aZq\mu_nN_D/L$

$I_D(sat) = I_{DSS}(1 + V_G/V_P)^2$ .. note: $V_G < 0$ for nJFET $g_m(sat) = \partial I_D(sat)/\partial V_G$

n-channel MOSFET equations:

$q\varphi = E_F - E_i$ $q\varphi_{pB} = -(kT_B/q)\ln(N_A/n_i) = E_F - E_i$ (in bulk); note $\varphi_{pB} < 0$ for p-substrate

$\varphi_{sB} = \varphi_s - \varphi_{pB}$ $\varphi_{pB} = -(kT_B/q)\ln(N_A/n_i)$ strong inversion: $\varphi_{sB} = -2\varphi_{pB} = 2|\varphi_{pB}|$

$V_{FB} = \Phi_{ms} - Q_i/C_i$ $V_T = V_{FB} - Q_{dep}/C_i - 2\varphi_{pB}$ $V_{GB} - V_{FB} = V_i + \varphi_{sB}$

$V_i = -Q_{semi}/C_i$ $C_i = \mathcal{E}_i/d_{ox} = \kappa_i\varepsilon_o/d_{ox}$ $x_{dep}(max) = [(2\kappa_s\varepsilon_o/qN_A)\, 2|\varphi_{pB}|]^{1/2}$

$Q_{semi} = Q_{dep} + Q_n$ $Q_{dep} = -qN_Ax_{dep}(max)$ $Q_n \approx -qC_i(V_{GB} - V_T)$

Text cover page equations:

Electron Momentum: $p = m^*v = \hbar k = \dfrac{h}{\lambda}$ Planck: $E = h\nu = \hbar\omega$

Kinetic: $E = \dfrac{1}{2}m^*v^2 = \dfrac{1}{2}\dfrac{p^2}{m^*} = \dfrac{\hbar^2}{2m^*}k^2$ (3–4) Effective mass: $m^* = \dfrac{\hbar^2}{d^2E/dk^2}$ (3–3)

Total electron energy $= P.E. + K.E. = E_c + E(\mathbf{k})$

Fermi-Dirac e^- distribution: $f(E) = \dfrac{1}{e^{(E-E_F)/kT} + 1} \cong e^{(E_F - E)/kT}$ for $E \gg E_F$ (3–10)

Equilibrium: $n_0 = \displaystyle\int_{E_c}^{\infty} f(E)N(E)dE = N_c f(E_c) = N_c e^{-(E_c - E_F)/kT}$ (3–15)

$N_c = 2\left(\dfrac{2\pi m_n^* kT}{h^2}\right)^{3/2}$ $N_v = 2\left(\dfrac{2\pi m_p^* kT}{h^2}\right)^{3/2}$ (3–16), (3–20)

$p_0 = N_v[1 - f(E_v)] = N_v e^{-(E_F - E_v)/kT}$ (3–19)

$n_i = N_c e^{-(E_c - E_i)/kT}$, $p_i = N_v e^{-(E_i - E_v)/kT}$ (3–21)

$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT} = 2\left(\dfrac{2\pi kT}{h^2}\right)^{3/2}(m_n^* m_p^*)^{3/4} e^{-E_g/2kT}$ (3–23), (3–26)

Equilibrium: $n_0 = n_i e^{(E_F - E_i)/kT}$, $p_0 = n_i e^{(E_i - E_F)/kT}$ (3–25) $n_0 p_0 = n_i^2$ (3–24)

Steady state: $n = N_c e^{-(E_c - F_n)/kT} = n_i e^{(F_n - E_i)/kT}$, $p = N_v e^{-(F_p - E_v)/kT} = n_i e^{(E_i - F_p)/kT}$ (4–15) $np = n_i^2 e^{(F_n - F_p)/kT}$ (5–38)

$\mathscr{E}(x) = -\dfrac{d\mathscr{V}(x)}{dx} = \dfrac{1}{q}\dfrac{dE_i}{dx}$ (4–26)

Poisson: $\dfrac{d\mathscr{E}(x)}{dx} = -\dfrac{d^2\mathscr{V}(x)}{dx^2} = \dfrac{\rho(x)}{\epsilon} = \dfrac{q}{\epsilon}(p - n + N_d^+ - N_a^-)$ (5–14)

$\mu \equiv \dfrac{q\bar{t}}{m^*}$ (3–40a) Drift: $v_d \cong \dfrac{\mu\mathscr{E}}{1 + \mu\mathscr{E}/v_s}\begin{cases} = \mu\mathscr{E} \text{ (low fields, ohmic)} \\ = v_s \text{ (high fields, saturated vel.)}\end{cases}$ (Fig. 6–9)

Drift current density: $\dfrac{I_x}{A} = J_x = q(n\mu_n + p\mu_p)\mathscr{E}_x = \sigma\mathscr{E}_x$ (3–43)

Conduction Current: (4–23)

$$J_n(x) = q\mu_n n(x)\mathscr{E}(x) + qD_n\frac{dn(x)}{dx}$$

drift diffusion

$$J_p(x) = q\mu_p p(x)\mathscr{E}(x) - qD_p\frac{dp(x)}{dx}$$

$$J_{\text{total}} = J_{\text{conduction}} + J_{\text{displacement}} = J_n + J_p + C\frac{dV}{dt}$$

Continuity: $\frac{\partial p(x,t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}$ $\quad \frac{\partial \delta n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$ (4–31)

For steady state diffusion: $\frac{d^2\delta n}{dx^2} = \frac{\delta n}{D_n\tau_n} \equiv \frac{\delta n}{L_n^2}$ $\quad \frac{d^2\delta p}{dx^2} = \frac{\delta p}{L_p^2}$ (4–34)

Diffusion length: $L \equiv \sqrt{D\tau}$ Einstein relation: $\frac{D}{\mu} = \frac{kT}{q}$ (4–29)

p-n JUNCTIONS

Equilibrium: $V_0 = \frac{kT}{q}\ln\frac{p_p}{p_n} = \frac{\text{kT}}{\text{q}}\ln\frac{N_a}{n_i^2/N_d} = \frac{\text{kT}}{\text{q}}\ln\frac{N_aN_d}{n_i^2}$ (5-8)

$\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{qV_0/kT}$ (5-10) $\quad W = \left[\frac{2\epsilon(V_0 - V)}{q}\left(\frac{N_a + N_d}{N_aN_d}\right)\right]^{1/2}$ (5-57)

One-sided abrupt p^+-n: $x_{n0} = \frac{WN_a}{N_a + N_d} \simeq W$ (5-23) $\quad V_0 = \frac{qN_dW^2}{2\epsilon}$

$\Delta p_n = p(x_{n0}) - p_n = p_n(e^{qV/kT} - 1)$ (5-29)

$\delta p(x_n) = \Delta p_n e^{-x_n/L_p} = p_n(e^{qV/kT} - 1)e^{-x_n/L_p}$ (5-31b)

Ideal diode: $I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)(e^{qV/kT} - 1) = I_0(e^{qV/kT} - 1)$ (5-36)

Non-ideal: $I = I_0'(e^{qV/\text{n}kT} - 1)$ $\quad (\text{n} = 1 \text{ to } 2)$ (5-74)

With light: $I_{\text{op}} = qAg_{\text{op}}(L_p + L_n + W)$ (8-1)

Capacitance: $C = \left| \frac{dQ}{dV} \right|$ (5–55)

Junction Depletion: $C_j = \epsilon A \left[\frac{q}{2\epsilon(V_0 - V)} \frac{N_d N_a}{N_d + N_a} \right]^{1/2} = \frac{\epsilon A}{W}$ (5–62)

Stored charge
exp. hole dist.: $Q_p = qA \int_0^\infty \delta p(x_n) dx_n = qA\Delta p_n \int_0^\infty e^{-x_n/L_p} dx_n = qAL_p \Delta p_n$ (5–39)

$I_p(x_n = 0) = \frac{Q_p}{\tau_p} = qA \frac{L_p}{\tau_p} \Delta p_n = qA \frac{D_p}{L_p} p_n(e^{qV/kT} - 1)$ (5–40)

$G_s = \frac{dI}{dV} = \frac{qAL_p p_n}{\tau_p} \frac{d}{dV}(e^{qV/kT}) = \frac{q}{kT} I$ (5–67c)

Long p$^+$-n: $i(t) = \frac{Q_p(t)}{\tau_p} + \frac{dQ_p(t)}{dt}$ (5–47)

MOS-n CHANNEL

Oxide: $C_i = \frac{\epsilon_i}{d}$ Depletion: $C_d = \frac{\epsilon_s}{W}$ MOS: $C = \frac{C_i C_d}{C_i + C_d}$ (6–36)

Threshold: $V_T = \underbrace{\Phi_{ms} - \frac{Q_i}{C_i}}_{\text{Flat band}} - \frac{Q_d}{C_i} + 2\phi_F$ (6–38)

Inversion: $\phi_s\,(\text{inv.}) = 2\phi_F = 2\frac{kT}{q} \ln \frac{N_a}{n_i}$ (6–15) $W = \left[\frac{2\epsilon_s \phi_s}{qN_a} \right]^{1/2}$ (6–30)

$Q_d = -qN_a W_m = -2(\epsilon_s q N_a \phi_F)^{1/2}$ (6–32) At V_{FB}: $C_{FB} = \frac{C_i\, C_{\text{debye}}}{C_i + C_{\text{debye}}}$

Debye screening length: $L_D = \sqrt{\frac{\epsilon_s kT}{q^2 p_0}}$ (6–25) $C_{\text{debye}} = \frac{\epsilon_s}{L_D}$ (6–40)

Substrate bias: $\Delta V_T \simeq \frac{\sqrt{2\epsilon_s q N_a}}{C_i}(-V_B)^{1/2}$ (n channel) (6–63)

$$I_D = \frac{\bar{\mu}_n Z C_i}{L}\left\{(V_G - V_{FB} - 2\phi_F - \tfrac{1}{2}V_D)V_D - \frac{2}{3}\,\frac{\sqrt{2\epsilon_s q N_a}}{C_i}\left[(V_D + 2\phi_F)^{3/2} - (2\phi_F)^{3/2}\right]\right\} \quad (6\text{-}50)$$

$$I_D \simeq \frac{\bar{\mu}_n Z C_i}{L}\left[(V_G - V_T)V_D - \tfrac{1}{2}V_D^2\right] \quad (6\text{-}49)$$

Saturation: $I_D(\text{sat.}) \simeq \tfrac{1}{2}\bar{\mu}_n C_i \frac{Z}{L}(V_G - V_T)^2 = \frac{Z}{2L}\bar{\mu}_n C_i V_D^2(\text{sat.})$ (6–53)

$$g_m = \frac{\partial I_D}{\partial V_G} \quad ; \quad g_m(\text{sat.}) = \frac{\partial I_D(\text{sat.})}{\partial V_G} \simeq \frac{Z}{L}\bar{\mu}_n C_i (V_G - V_T) \quad (6\text{-}54)$$

For short L: $I_D \simeq Z C_i (V_G - V_T) v_s$ (6–60)

Subthreshold slope: $S = \frac{dV_G}{d(\log I_D)} = \frac{kT}{q}\ln 10\left[1 + \frac{C_d + C_{it}}{C_i}\right]$ (6–66)

BJT-p-n-p

$$I_{Ep} = qA\frac{D_p}{L_p}\left(\Delta p_E \,\text{ctnh}\,\frac{W_b}{L_p} - \Delta p_C \,\text{csch}\,\frac{W_b}{L_p}\right) \quad (7\text{-}18)$$

$$\Delta p_E = p_n(e^{qV_{EB}/kT} - 1), \quad \Delta p_C = p_n(e^{qV_{CB}/kT} - 1) \quad (7\text{-}8)$$

$$I_C = qA\frac{D_p}{L_p}\left(\Delta p_E \,\text{csch}\,\frac{W_b}{L_p} - \Delta p_C \,\text{ctnh}\,\frac{W_b}{L_p}\right)$$

$$I_B = qA\frac{D_p}{L_p}\left[(\Delta p_E + \Delta p_C)\tanh\frac{W_b}{2L_p}\right] \quad (7\text{-}19)$$

$$B = \frac{I_C}{I_{Ep}} = \frac{\text{csch}\,W_b/L_p}{\text{ctnh}\,W_b/L_p} = \text{sech}\,\frac{W_b}{L_p} \simeq 1 - \left(\frac{W_b^2}{2L_p^2}\right) \quad (7\text{-}26)$$

(Base transport factor)

$$\gamma = \frac{I_{Ep}}{I_{En} + I_{Ep}} = \left[1 + \frac{L_p^n n_n \mu_n^p}{L_n^p p_p \mu_p^n}\tanh\frac{W_b}{L_p^n}\right]^{-1} \simeq \left[1 + \frac{W_b n_n \mu_n^p}{L_n^p p_p \mu_p^n}\right]^{-1} \quad (7\text{-}25)$$

(Emitter injection efficiency)

$\frac{i_C}{i_E} = B\gamma \equiv \alpha$ (7–3)

(Common base gain)

$\frac{i_C}{i_B} = \frac{B\gamma}{1 - B\gamma} = \frac{\alpha}{1 - \alpha} \equiv \beta$ (7–6)

(Common emitter gain)

$\frac{i_C}{i_B} = \beta = \frac{\tau_p}{\tau_t}$ (7–7)

(For $\gamma = 1$)