ELEG 340 - Fall 08
Solid-State Electronics
Quiz 5

20 November 2008

NAME Solution

Time Limit: 30 minutes

Closed Books and Notes. You may use your own calculator, but may not loan or borrow one (ask proctor if you have questions). Put each expression in a final form as best you can.

Guidelines:

I. Full credit requires the dimensions/ units for all numerical quantities.

II. Show your work and calculations for full credit; accuracy to 2 significant figures is sufficient.

III. Assume that the semiconductor material is silicon at room temperature (300 K), unless otherwise stated.

IV. Data: at room temperature (300K): thermal energy $k_BT = 0.026$ eV; thermal voltage $k_BT/q = 0.026$ volts; silicon intrinsic concentration $n_i = 1$ (or 1.5) x 10^{10} cm^{-3}; recombination lifetimes: τ_n, $\tau_p = 1$ μsec; dielectric constant $\kappa_{Si} = 11.8$; permittivity of free space $\varepsilon_o = 8.85 \times 10^{-14}$ F/cm; electron charge $|q| = |e| = 1.6 \times 10^{-19}$ Coul; $\mu_n = 1200 \frac{cm^2}{V\text{-}s}$; $\mu_p = 400 \frac{cm^2}{V\text{-}s}$

V. Equations: see list at end of quiz

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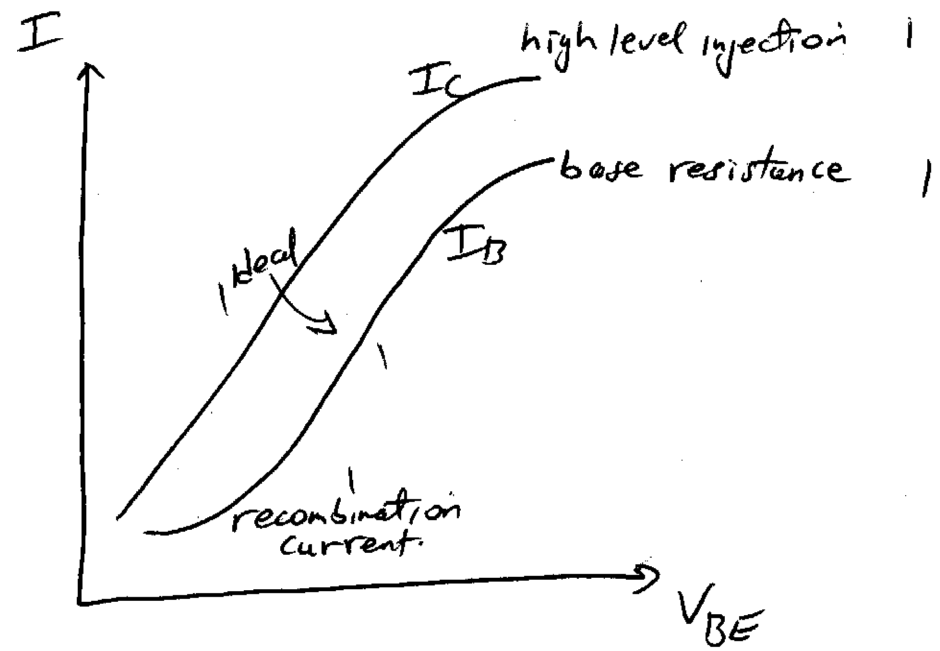
1. For a bipolar transistor (pnp or npn – your choice), draw the Gummel plot characteristics of the collector and base currents versus the emitter-base forward bias, V_{EB} (or V_{BE}), exhibiting real (non-ideal) behavior. Label the ideal current regions, and the regions at high and low bias where the currents depart from ideal behavior. Recall that at low bias, the collector current remains ideal because of transistor action.



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2. Calculate the pinch-off voltage, V_P, of the following n-channel junction field effect transistor (JFET). The transistor parameters are: p^+ gate doping $N_A = 1 \times 10^{19}$ cm^{-3}; n-channel doping $N_D = 1 \times 10^{16}$ cm^{-3}; channel half width $a = 1$ μm; depth $Z = 3$ μm, and channel length $L_{ch} = 5$ μm.

$$V_P = \frac{q a^2 N_D}{2 K_{Si} \epsilon_0}$$

$$= \frac{1.6 \times 10^{-19} C \; (10^{-4} cm)^2 \times 10^{16} cm^{-3}}{2 \times 11.8 \times 8.85 \times 10^{-14} F/cm}$$

$$= \frac{1.6}{2 \times 1.18 \times 0.885} \times \frac{10^{-19-8+16}}{10^{1-13}} \qquad \underbrace{\frac{C - cm^2 cm\, cm^{-3}}{F}}_{V}$$

$$\approx \frac{1.6}{2} \times 10^{-11+12}$$

$$= \frac{16}{2} \text{ Volts} = 8 \text{ Volts}$$

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3. For the n-JFET of problem 2 above, calculate the open channel conductance, G_O.

$$G_o = \frac{2aZ}{\rho_{elec} L} = \frac{2aZ\,(N_D q \mu_n)}{L}$$

$$G_o = \frac{2\times10^{-4}\,cm \times 3\,\mu m \times 10^{16}\,cm^{-3}}{5\,\mu m} \times 1.6\times10^{-19}\,C \times 1200\,\frac{cm^2}{V\text{-}s}$$

$$= \frac{2\times3\times1.6\times1.2}{5}\times10^{-4+16-19+3}\;cm^3\,cm^{-3}\,\frac{C}{V\text{-}s}$$

$$\frac{A}{V} = \frac{1}{\Omega}$$

$$= \frac{12}{5}\times10^{-4}\ \text{Siemen.}$$

$$= 2.4\times10^{-4}\,S = \frac{1}{4\,K\Omega}$$

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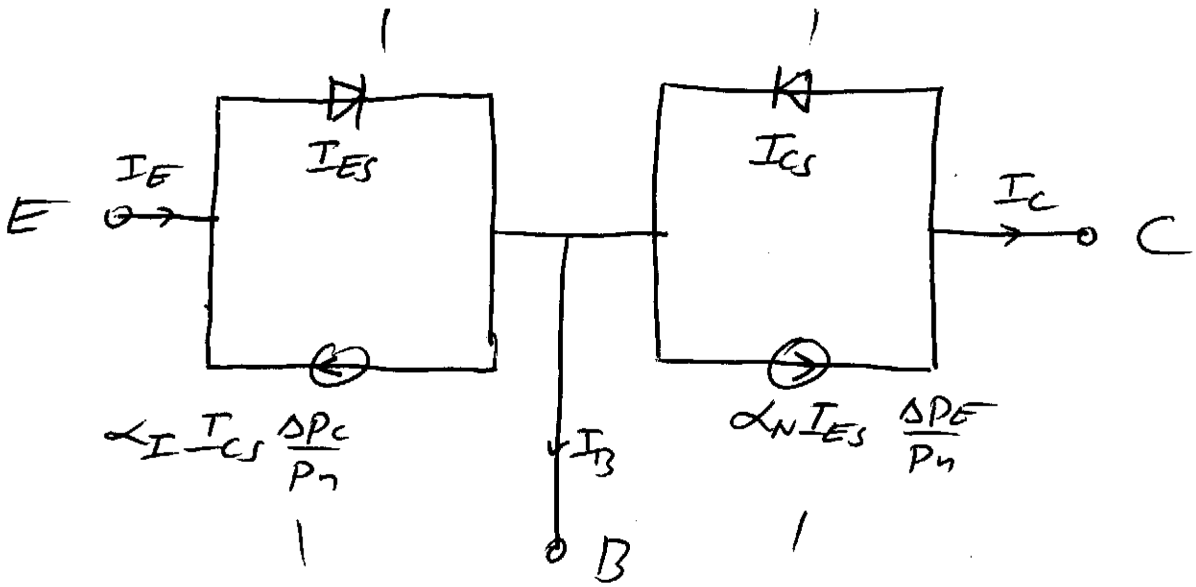
$$= 0.24\ mS$$

4. Based on the Ebers-Moll equations, draw the schematic of the coupled diode model. Indicate the values and directions of the current sources and the leakage currents of the diodes.



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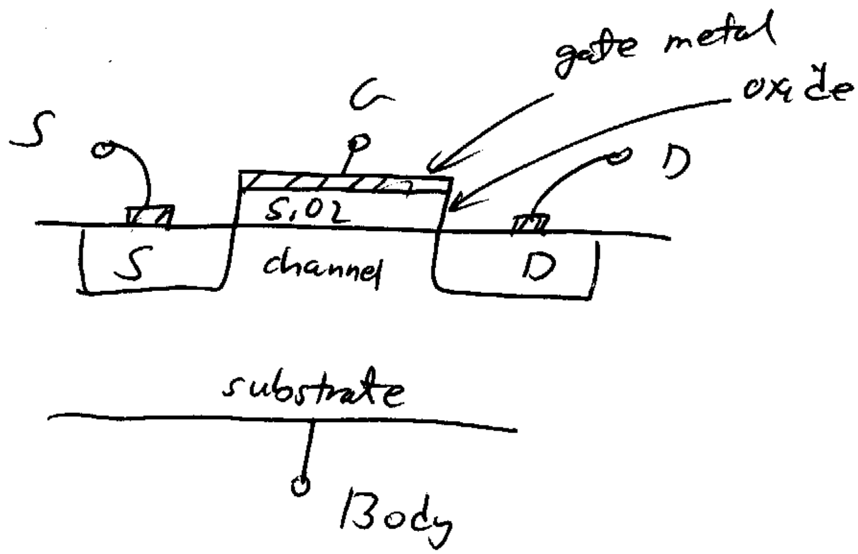
5. Draw a cross-section view of an MOSFET transistor, and label the gate, source and drain regions, including the gate insulator and substrate (body).



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V. Equations:

$p_{op} = i\hbar d/dx$ $f_{FD}(E) = 1/[1 + \exp(E-E_F)/k_BT]$ $E = Q\ V$

$n = n_i\exp[(E_F-E_i)/k_BT]$. $p = n_i\exp[(E_i-E_F)/k_BT]$ $n_op_o = n_i^2$ $np = n_i^2e^{(Fn-Fp)/kT}$

$J_n = q\mu_n n\mathcal{E} + qD_n dn/dx$ $J_p = q\mu_p p\mathcal{E} - qD_p dp/dx$ $\sigma_{elec} = q(n\mu_n + p\mu_p)$

$U_n = (n_p-n_{po})/\tau_n$ $U_p = (p_n-p_{no})/\tau_p$ $p' = p - p_o = g_{opt}\tau_p$ $n' = n - n_o = g_{opt}\tau_n$

$C_{dep} = \kappa_s\varepsilon_o\ A/W$ $C_{diff} = qI\tau/k_BT$ $D/\mu = k_BT/q$ $L = \sqrt{(D\tau)}$

$\partial p/\partial t = -1/q\ \partial J_p/\partial x - p'/\tau_p$ $\partial n/\partial t = -1/q\ \partial J_n/\partial x - n'/\tau_n$;

$\partial p/\partial t = D_p\ \partial^2 p/\partial x^2 - p'/\tau_p$ $\partial n/\partial t = D_n\ \partial^2 n/\partial x^2 - n'/\tau_n$

$\varphi_{bi} = k_BT/q\ \ln(N_AN_D/n_i^2)$ $W_{dep} = [2\kappa_s\varepsilon_o/q\ (1/N_A + 1/N_D)(\varphi_{bi} - V_F)]^{1/2}$

$p_n(x_{no}) = p_{no}(x_{no})e^{qVf/kT}$; $n_p(-x_{po}) = n_{po}(-x_{po})e^{qVf/kT}$

Diode current

$I = qA(D_pp_n/L_p + D_nn_p/L_n)[e^{qV/kT} - 1] = I_o[e^{qV/kT} - 1]$; $I_o = I_{th} = qA(D_pp_n/L_p + D_nn_p/L_n)$

Solar Cell and Photo detector diodes:

$I_{tot} = I_o\ [e^{qV/kT} - 1] - I_{opt}$; $I_{opt} = qAg_{opt}(L_n + L_p + W)$

BJTs:

$I_E = I_C + I_B$ $I_C = \alpha I_E + I_{CBO}$ $I_C = \beta I_B + I_{CEO}$

$I_E \approx qAD_p/L_p\ \Delta p_E\ \mathrm{ctnh} W_b/L_p$ $I_C \approx qAD_p/L_p\ \Delta p_E\ \mathrm{csch} W_b/L_p$

$I_B \approx qAD_p/L_p\ \Delta p_E\ \tanh W_b/2L_p \ldots$ $\alpha = \gamma\ B_T = (\beta/1+\beta)$ $\beta = \tau_p/\tau_{tr} = (\alpha/1-\alpha)$

$I_E = I_{Ep} + I_{En}$. $I_{Ep} = \gamma I_E$ $I_{Cp} = B_T\ I_{Ep}\ldots$ $I_C = I_{Cp} + I_{CBO}$

$\gamma = I_{Ep}/(I_{Ep} + I_{En})$ $B_T = \mathrm{sech} W_b/L_p \approx (1 - W_b^2/2L_p^2)$

Ebers-Moll:

$$I_E = I_{EN} + I_{EI} = I_{ES}(e^{qV_{EB}/kT} - 1) - \alpha_I I_{CS}(e^{qV_{CB}/kT} - 1)$$
$$I_C = I_{CN} + I_{CI} = \alpha_N I_{ES}(e^{qV_{EB}/kT} - 1) - I_{CS}(e^{qV_{CB}/kT} - 1)$$

$$I_C = \alpha_N I_{ES}\frac{\Delta p_E}{p_n} - I_{CS}\frac{\Delta p_C}{p_n} = \frac{I_{CS}}{p_n}(\alpha_I\Delta p_E - \Delta p_C)\ ,\quad I_E = I_{ES}\frac{\Delta p_E}{p_n} - \alpha_I I_{CS}\frac{\Delta p_C}{p_n} = \frac{I_{ES}}{p_n}(\Delta p_E - \alpha_N\Delta p_C)$$

Text cover page equations:

SEMICONDUCTOR PHYSICS

Electron Momentum: $\mathrm{p} = m^*\mathrm{v} = \hbar\mathbf{k} = \dfrac{h}{\lambda}$ Planck: $E = h\nu = \hbar\omega$

Kinetic: $E = \dfrac{1}{2}m^*\mathrm{v}^2 = \dfrac{1}{2}\dfrac{\mathrm{p}^2}{m^*} = \dfrac{\hbar^2}{2m^*}\mathbf{k}^2$ (3-4) Effective mass: $m^* = \dfrac{\hbar^2}{d^2E/d\mathbf{k}^2}$ (3-3)

Total electron energy = $P.E. + K.E. = E_c + E(\mathbf{k})$

Fermi-Dirac e^- distribution: $f(E) = \dfrac{1}{e^{(E-E_F)/kT} + 1} \cong e^{(E_F - E)/kT}$ for $E \gg E_F$ (3-10)

Equilibrium: $n_0 = \displaystyle\int_{E_c}^{\infty} f(E)N(E)dE = N_c f(E_c) = N_c e^{-(E_c - E_F)/kT}$ (3-15)

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \quad N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2} \quad (3\text{-}16), (3\text{-}20)$$

$$p_0 = N_v[1 - f(E_v)] = N_v e^{-(E_F - E_v)/kT} \quad (3\text{-}19)$$

$$n_i = N_c e^{-(E_c - E_i)/kT}, \quad p_i = N_v e^{-(E_i - E_v)/kT} \quad (3\text{-}21)$$

$$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT} = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}(m_n^* m_p^*)^{3/4} e^{-E_g/2kT} \quad (3\text{-}23), (3\text{-}26)$$

Equilibrium: $n_0 = n_i e^{(E_F - E_i)/kT}$, $p_0 = n_i e^{(E_i - E_F)/kT}$ (3-25) $n_0 p_0 = n_i^2$ (3-24)

Steady state: $n = N_c e^{-(E_c - F_n)/kT} = n_i e^{(F_n - E_i)/kT}$, $p = N_v e^{-(F_p - E_v)/kT} = n_i e^{(E_i - F_p)/kT}$ (4-15) $np = n_i^2 e^{(F_n - F_p)/kT}$ (5-38)

$$\mathscr{E}(x) = -\frac{d\mathcal{V}(x)}{dx} = \frac{1}{q}\frac{dE_i}{dx} \quad (4\text{-}26)$$

Poisson: $\dfrac{d\mathscr{E}(x)}{dx} = -\dfrac{d^2\mathcal{V}(x)}{dx^2} = \dfrac{\rho(x)}{\epsilon} = \dfrac{q}{\epsilon}(p - n + N_d^+ - N_a^-)$ (5-14)

$\mu \equiv \dfrac{q\bar{t}}{m^*}$ (3-40a) Drift: $\mathrm{v}_d \cong \dfrac{\mu\mathscr{E}}{1 + \mu\mathscr{E}/\mathrm{v}_s}$ $\begin{cases} = \mu\mathscr{E} \text{ (low fields, ohmic)} \\ = \mathrm{v}_s \text{ (high fields, saturated vel.)} \end{cases}$ (Fig. 6-9)

Drift current density: $\dfrac{I_x}{A} = J_x = q(n\mu_n + p\mu_p)\mathscr{E}_x = \sigma\mathscr{E}_x$ (3-43)

Conduction Current: (4–23)

$$J_n(x) = q\mu_n n(x)\mathscr{E}(x) + qD_n \frac{dn(x)}{dx}$$

drift diffusion

$$J_p(x) = q\mu_p p(x)\mathscr{E}(x) - qD_p \frac{dp(x)}{dx}$$

$$J_{\text{total}} = J_{\text{conduction}} + J_{\text{displacement}} = J_n + J_p + C\frac{dV}{dt}$$

Continuity: $\frac{\partial p(x,t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}$ $\quad \frac{\partial \delta n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$ (4–31)

For steady state diffusion: $\frac{d^2\delta n}{dx^2} = \frac{\delta n}{D_n\tau_n} \equiv \frac{\delta n}{L_n^2}$ $\quad \frac{d^2\delta p}{dx^2} = \frac{\delta p}{L_p^2}$ (4–34)

Diffusion length: $L \equiv \sqrt{D\tau}$ Einstein relation: $\frac{D}{\mu} = \frac{kT}{q}$ (4–29)

p-n JUNCTIONS

Equilibrium: $V_0 = \frac{kT}{q}\ln\frac{p_p}{p_n} = \frac{\mathrm{kT}}{\mathrm{q}}\ln\frac{N_a}{n_i^2/N_d} = \frac{\mathrm{kT}}{\mathrm{q}}\ln\frac{N_aN_d}{n_i^2}$ (5-8)

$\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{qV_0/kT}$ (5-10) $\quad W = \left[\frac{2\epsilon(V_0 - V)}{q}\left(\frac{N_a + N_d}{N_aN_d}\right)\right]^{1/2}$ (5-57)

One-sided abrupt p^+-n: $x_{n0} = \frac{WN_a}{N_a + N_d} \simeq W$ (5-23) $\quad V_0 = \frac{qN_dW^2}{2\epsilon}$

$\Delta p_n = p(x_{n0}) - p_n = p_n(e^{qV/kT} - 1)$ (5-29)

$\delta p(x_n) = \Delta p_n e^{-x_n/L_p} = p_n(e^{qV/kT} - 1)e^{-x_n/L_p}$ (5-31b)

Ideal diode: $I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)(e^{qV/kT} - 1) = I_0(e^{qV/kT} - 1)$ (5-36)

Non-ideal: $I = I_0'(e^{qV/\mathbf{n}kT} - 1)$ $\quad (\mathbf{n} = 1 \text{ to } 2)$ (5-74)

With light: $I_{\text{op}} = qAg_{\text{op}}(L_p + L_n + W)$ (8-1)

Capacitance: $C = \left|\frac{dQ}{dV}\right|$ (5-55)

Junction Depletion: $C_j = \epsilon A\left[\frac{q}{2\epsilon(V_0 - V)}\frac{N_dN_a}{N_d + N_a}\right]^{1/2} = \frac{\epsilon A}{W}$ (5-62)

Stored charge
exp. hole dist.: $Q_p = qA\int_0^{\infty}\delta p(x_n)dx_n = qA\Delta p_n\int_0^{\infty}e^{-x_n/L_p}dx_n = qAL_p\Delta p_n$ (5-39)

$I_p(x_n = 0) = \frac{Q_p}{\tau_p} = qA\frac{L_p}{\tau_p}\Delta p_n = qA\frac{D_p}{L_p}p_n(e^{qV/kT} - 1)$ (5-40)

$G_s = \frac{dI}{dV} = \frac{qAL_pp_n}{\tau_p}\frac{d}{dV}(e^{qV/kT}) = \frac{q}{kT}I$ (5-67c)

Long p^+-n: $i(t) = \frac{Q_p(t)}{\tau_p} + \frac{dQ_p(t)}{dt}$ (5-47)

MOS-n CHANNEL

Oxide: $C_i = \frac{\epsilon_i}{d}$ Depletion: $C_d = \frac{\epsilon_s}{W}$ MOS: $C = \frac{C_iC_d}{C_i + C_d}$ (6-36)

Threshold: $V_T = \underbrace{\Phi_{ms} - \frac{Q_i}{C_i}}_{\text{Flat band}} - \frac{Q_d}{C_i} + 2\phi_F$ (6-38)

Inversion: $\phi_s\,(\text{inv.}) = 2\phi_F = 2\frac{kT}{q}\ln\frac{N_a}{n_i}$ (6-15) $W = \left[\frac{2\epsilon_s\phi_s}{qN_a}\right]^{1/2}$ (6-30)

$Q_d = -qN_aW_m = -2(\epsilon_sqN_a\phi_F)^{1/2}$ (6-32) At V_{FB}: $C_{FB} = \frac{C_i\,C_{debye}}{C_i + C_{debye}}$

Debye screening length: $L_D = \sqrt{\frac{\epsilon_skT}{q^2p_0}}$ (6-25) $C_{debye} = \frac{\epsilon_s}{L_D}$ (6-40)

Substrate bias: $\Delta V_T \simeq \frac{\sqrt{2\epsilon_sqN_a}}{C_i}(-V_B)^{1/2}$ (n channel) (6-63)

$$I_D = \frac{\overline{\mu}_n Z C_i}{L}\left\{(V_G - V_{FB} - 2\phi_F - \tfrac{1}{2}V_D)V_D - \frac{2}{3}\,\frac{\sqrt{2\epsilon_s q N_a}}{C_i}\left[(V_D + 2\phi_F)^{3/2} - (2\phi_F)^{3/2}\right]\right\} \quad (6\text{-}50)$$

$$I_D \simeq \frac{\overline{\mu}_n Z C_i}{L}\left[(V_G - V_T)V_D - \tfrac{1}{2}V_D^2\right] \quad (6\text{-}49)$$

Saturation: $I_D(\text{sat.}) \simeq \frac{1}{2}\overline{\mu}_n C_i \frac{Z}{L}(V_G - V_T)^2 = \frac{Z}{2L}\overline{\mu}_n C_i V_D^2(\text{sat.}) \quad (6\text{-}53)$

$$g_m = \frac{\partial I_D}{\partial V_G} \;;\; g_m(\text{sat.}) = \frac{\partial I_D(\text{sat.})}{\partial V_G} \simeq \frac{Z}{L}\overline{\mu}_n C_i (V_G - V_T) \quad (6\text{-}54)$$

For short L: $I_D \simeq Z C_i (V_G - V_T) v_s \quad (6\text{-}60)$

Subthreshold slope: $S = \frac{dV_G}{d(\log I_D)} = \frac{kT}{q}\ln 10\left[1 + \frac{C_d + C_{it}}{C_i}\right] \quad (6\text{-}66)$

BJT-p-n-p

$$I_{Ep} = qA\frac{D_p}{L_p}\left(\Delta p_E \,\text{ctnh}\frac{W_b}{L_p} - \Delta p_C \,\text{csch}\frac{W_b}{L_p}\right) \quad (7\text{-}18) \qquad \begin{aligned}\Delta p_E &= p_n(e^{qV_{EB}/kT} - 1)\\ \Delta p_C &= p_n(e^{qV_{CB}/kT} - 1)\end{aligned} \quad (7\text{-}8)$$

$$I_C = qA\frac{D_p}{L_p}\left(\Delta p_E \,\text{csch}\frac{W_b}{L_p} - \Delta p_C \,\text{ctnh}\frac{W_b}{L_p}\right)$$

$$I_B = qA\frac{D_p}{L_p}\left[(\Delta p_E + \Delta p_C)\tanh\frac{W_b}{2L_p}\right] \quad (7\text{-}19)$$

$$B = \frac{I_C}{I_{Ep}} = \frac{\text{csch}\, W_b/L_p}{\text{ctnh}\, W_b/L_p} = \text{sech}\frac{W_b}{L_p} \simeq 1 - \left(\frac{W_b^2}{2L_p^2}\right) \quad (7\text{-}26)$$

(Base transport factor)

$$\gamma = \frac{I_{Ep}}{I_{En} + I_{Ep}} = \left[1 + \frac{L_p^n n_n \mu_n^p}{L_n^p p_p \mu_p^n}\tanh\frac{W_b}{L_p^n}\right]^{-1} \simeq \left[1 + \frac{W_b n_n \mu_n^p}{L_n^p p_p \mu_p^n}\right]^{-1} \quad (7\text{-}25)$$

(Emitter injection efficiency)

$\frac{i_C}{i_E} = B\gamma \equiv \alpha \quad (7\text{-}3)$ $\qquad \frac{i_C}{i_B} = \frac{B\gamma}{1 - B\gamma} = \frac{\alpha}{1-\alpha} \equiv \beta \quad (7\text{-}6)$ $\qquad \frac{i_C}{i_B} = \beta = \frac{\tau_p}{\tau_t} \quad (7\text{-}7)$

(Common base gain) (Common emitter gain) (For $\gamma = 1$)