6 November 2008

NAME ________________________

Solution

Time Limit: 30 minutes

Closed Books and Notes. You may use your own calculator, but may not loan or borrow one (ask proctor if you have questions). Put expression in a final form as best you can.

Guidelines:
I. Full credit requires the final dimensions/units for all numerical quantities that you calculate.

II. Show all work and calculations for full credit; accuracy to 2 significant figures is sufficient.

III. Assume that the material is silicon at room temperature (300 K), unless otherwise stated.

IV. Data: at room temperature (300K): thermal energy $k_B T = 0.026$ eV; thermal voltage $k_B T/q = 0.026$ volts; silicon intrinsic concentration $n_i = 1 \text{ or } 1.5 \times 10^{10} \text{ cm}^{-3}$; recombination lifetimes: $\tau_n, \tau_p = 1 \mu\text{sec}$; dielectric constant $\kappa_{Si} = 11.8$; permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-14} \text{ F/cm}$; electron charge $|q| = |e| = 1.6 \times 10^{-19} \text{ Coul}$;

V. Equations: see list at end of quiz

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1. Draw the current versus voltage (I-V) characteristic plot of a photovoltaic (pn junction) solar cell, and label the following parameters: (a) \( V_{dc} \); (b) \( I_{sc} \); and the maximum power point values: (c) \( V_{max} \); (d) \( I_{max} \).

2. For a solar cell, give the definition of either: (a) the fill-factor (FF); or (b) the power conversion efficiency, \( \eta \). Express your definition using conventional parameters including the ones in problem (1) above.

\[
FF = \frac{V_{max} I_{max}}{V_{oc} I_{sc}}
\]

\[
\eta = \frac{V_{max} I_{max} (= P_{max})}{P_{incident}}
\]
3. Derive an analytical expression for the open circuit voltage $V_{oc}$ of a solar cell in terms of $I_{th}$ and $I_{opt}$, starting with the current vs. voltage $(I - V)$ equation.

\[ I = I_{th} \left( e^{\frac{V}{n k T}} - 1 \right) - I_{opt} \]

\[ 0 = I_{th} \left( e^{\frac{V_{oc}}{n k T}} - 1 \right) - I_{opt} \]

\[ \frac{I_{opt}}{I_{th}} + 1 = e^{\frac{V_{oc}}{n k T}} \]

\[ V_{oc} = \left[ \ln \left( \frac{I_{opt} + I_{th}}{I_{th}} \right) \right] \cdot \frac{k T}{q} \]

4. Consider the following data plot for an npn bipolar transistor. Using standard transistor equations, from this data, estimate the numerical values of two parameters in the normal active mode, (that is with $V_{CE}$ above about 3 volts): (a) the forward current transfer ratio, $\alpha$, also known as the common base current gain; and (b) the common emitter current gain, $\beta$, also known as the collector to base current transfer ratio. Show your work.

\[ \alpha = \frac{\Delta I_C}{\Delta I_C} = \frac{(22 - 11) \text{mA}}{(22.2 - 11.1) \text{mA}} \]

\[ = \frac{11}{11.1} \approx 0.99 \]

\[ \beta = \frac{\Delta I_C}{\Delta I_B} = \frac{(22 - 11) \text{mA}}{(0.2 - 0.1) \text{mA}} = \frac{11}{0.1} = 110 \]
5. Sketch the minority carrier distribution in the neutral base region of a bipolar transistor in the normal active mode, that is, with the EB junction forward biased, and the BC junction reverse biased. Label the coordinate axes, and indicate the reference values of relevant carrier concentrations such as $n_{po}$ or $p_{no}$.
V. Equations:

\[ p_{op} = ihd/dx \]
\[ f_{FD}(E) = 1/[1 + \exp(E - E_F)/k_B T] \]
\[ E = Q V \]
\[ n = n_i \exp[(E_T - E_i)/k_B T] \]
\[ p = n_i \exp[(E - E_T)/k_B T] \]
\[ n_o p_o = n_i^2 \]
\[ n_p = n_i^2 e^{(E_n - F_p)/k_B T} \]
\[ J_n = q \mu_n n \mathcal{E} + q D_n d n/dx \]
\[ J_p = q \mu_p p \mathcal{E} - q D_p d p/dx \]
\[ \sigma_{elc} = q (n \mu_n + p \mu_p) \]
\[ U_n = (n_p - n_p_0)/\tau_n \]
\[ U_p = (p_n - p_n_0)/\tau_p \]
\[ p' = p - p_0 = g_{opt} \tau_p \]
\[ n' = n - n_0 = g_{opt} \tau_n \]
\[ C_{dep} = \kappa \varepsilon_0 A/W \]
\[ C_{diff} = q \tau/k_B T \]
\[ D/\mu = k_B T/q \]
\[ L = \sqrt{(D \tau)} \]
\[ \partial p/\partial t = -1/q \partial J_p/\partial x - p'/\tau_p \]
\[ \partial n/\partial t = -1/q \partial J_n/\partial x - n'/\tau_n \]
\[ \partial p/\partial t = D_p \partial^2 p /\partial x^2 - p'/\tau_p \]
\[ \partial n/\partial t = D_n \partial^2 n /\partial x^2 - n'/\tau_n \]
\[ \phi_{bi} = k_B T/q \ln(N_A N_D/n_i^2) \]
\[ W_{dep} = [2 \kappa \varepsilon_0 /q (1/N_A + 1/N_D) (\phi_{bi} - V_F)]^{1/2} \]
\[ p_n(x_n) = p_{n_0}(x_{n_0}) e^{qV_xt/k_B T} \]
\[ n_p(-x_{p_0}) = n_{p_0}(-x_{p_0}) e^{qV_xt/k_B T} \]

Diode current

\[ I = qA (D_p p_0/L_p + D_n n_0/L_n) [e^{qV_xt/k_B T} - 1] = I_0 [e^{qV_xt/k_B T} - 1] \]
\[ I_o = I_{th} = qA(D_p p_0/L_p + D_n n_0/L_n) \]

Solar Cell and Photo detector diodes:

\[ I_{tot} = I_o [e^{qV_xt/k_B T} - 1] - I_{opt} \]
\[ I_{opt} = qA_{opt} (I_n + L_o + W) \]

BJTs:

\[ I_E = I_C + I_B \]
\[ I_C = \alpha I_E + I_{CEO} \]
\[ I_C = \beta I_B + I_{CEO} \]
\[ I_E \approx qA D_p/L_p \Delta p E \tanh W_b/L_p \]
\[ I_B \approx qA D_p/L_p \Delta p E \cosh W_b/2L_p \]
\[ \alpha = \gamma B_T = (\beta/1 + \beta) \]
\[ \beta = \tau_p/\tau_n = (\alpha/1 - \alpha) \]
\[ I_E = I_{Ep} + I_{En} \]
\[ I_{Ep} = \gamma I_E \]
\[ I_{CP} = B_I E_{Ep} \ldots \]
\[ I_C = I_{CP} + I_{CEO} \]
\[ \gamma = I_{Ep}/(I_E + I_{En}) \]
\[ B_T = \text{sech}W_b/L_p \approx (1 - W_b^2/2L_p^2) \]