\textbf{ELEG 340 - Fall 08}\n\textit{Solid-State Electronics}\n\textbf{Quiz 3}\n
23 October 2008 \hfill NAME _Solution_

\textbf{Time Limit:} 30 minutes

Closed Books and Notes. You may use your own calculator, but may not loan or borrow one (ask proctor if you have questions). Put expression in a final form as best you can.

\textbf{Guidelines:}
I. Full credit requires the final dimensions/units for all numerical quantities that you calculate.

II. Show all work and calculations for full credit; accuracy to 2 significant figures is sufficient.

III. Assume that the material is silicon at room temperature (300 K), unless otherwise stated.

IV. At room temperature (300K), thermal energy $k_B T = 0.026$ eV, silicon has intrinsic concentration $n_i = 1 \times 10^{10}$ cm$^{-3}$, and recombination lifetimes: $\tau_n, \tau_p = 1 \mu$s; dielectric constant $\kappa_{Si} = 11.8$;.

Permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-14}$ F/cm; electron charge $|q| = 1.6 \times 10^{-19}$ Coul;

V. Equations:

\begin{align*}
    & p_{op} = ihd/dx & & f_{FD}(E) = 1/[1 + \exp(E-E_F)/k_B T] & & E = Q\ V \\
    & n = n_i \exp[(E_F-E_i)/k_B T] & & p = n_i \exp[(E_i-E_F)/k_B T] & & n_o p_o = n_i^2 & & n_p = n_i^2 \exp[(E_F-E_p)/k_B T] \\
    & J_n = q\mu_n n\varepsilon + qD_n dn/dx & & J_p = q\mu_p p\varepsilon - qD_p dp/dx & & \sigma_{elec} = q(n\mu_n + p\mu_p) \\
    & U_n = (n_0 - n_0)/\tau_n & & U_p = (p_0 - p_0)/\tau_p & & p' = p - p_o = g_{opt} \tau_p & & n' = n - n_o = g_{opt} \tau_n \\
    & C_{dep} = \kappa_{Si} \varepsilon_0 A/W & & C_{diff} = q\lambda T/k_B T & & D/\mu = k_B T/q & & L = \sqrt{(D/\tau)} \\
    & \partial p/\partial t = -1/q \partial J_p/\partial x - p'/\tau_p & & \partial n/\partial t = -1/q \partial J_n/\partial x - n'/\tau_n; \\
    & \partial p/\partial t = D_p \partial^2 p/\partial x^2 - p'/\tau_p & & \partial n/\partial t = D_n \partial^2 n/\partial x^2 - n'/\tau_n \\
    & \varphi_{bi} = k_B T/q \ln(N_A N_D/n_i^2) & & W_{dep} = [2k_\varepsilon_0 (1/N_A + 1/N_D)(\varphi_{bi} - V_F)]^{1/2} \\
    & l = qA(D_p p_0/L_p + D_n n_0/L_n)[e^{qV/kT} - 1] = I_0 [e^{qV/kT} - 1]; & & \text{Score} 23 \\
    & p_n(x_{no}) = p_{no}(x_{no})e^{qV/kT}; & & n_p(x_{po}) = n_{po}(x_{po})e^{qV/kT} \}
1. A sample of Ge at room temperature (300K) is p-type doped with $N_A = 1 \times 10^{16}$ cm$^{-3}$, which are fully ionized. For Ge at 300 K, $n_i = 2.5 \times 10^{13}$ cm$^{-3}$, hole mobility $\mu_p = 1900$ cm$^2$/V-s, and electron mobility $\mu_n = 3900$ cm$^2$/V-s. Calculate the electrical conductivity, $\sigma$.

\[
\sigma = \rho \mu_p = 10^{16} \text{ cm}^{-3} \times 1.6 \times 10^{-19} \text{ C} \times 1900 \text{ cm}^2/\text{V-s}
\]

\[
= 1.6 \times 10^{-9} \times 10^{16 - 19 + 3} \text{ cm}^{-3} + \text{Coul} \frac{1}{\text{V-s}}
\]

\[
= 1.6 \times 10^{-9} \frac{1}{\text{cm}}
\]

\[
= 3.15 \text{ S/cm}
\]

2. A one-sided step junction of silicon is uniformly doped with $N_A = 10^{18}$ cm$^{-3}$ on the p-side, and $N_D = 10^{15}$ cm$^{-3}$ on the n-side. Assume that the built-in (contact) voltage is $\Phi_0 = 0.7$ volts. What is the equilibrium depletion width, $W_{dep}$?

\[
W = \sqrt{\frac{2kT \ell_0}{q N_D}} \Phi_0
\]

\[
= \sqrt{2 \times 1.8 \times 8.85 \times 10^{-14} \text{F/cm} \times 0.7 \text{V}}
\]

\[
= \sqrt{2 \times 1.18 \times 8.85 \times 0.7 \times 10 \times 10^{-14} \text{F/cm} \times 0.7 \text{V}}
\]

\[
= \sqrt{1.18 \times 8.85 \times 0.7 \times 10 \times 10^{-14} \text{F/cm} \times 0.7 \text{V}}
\]

\[
= 0.9 \text{ mm}
\]
3. A junction of silicon has an equilibrium depletion width \( W = 0.5 \, \mu m \). Calculate the depletion capacitance per area \( C'_{\text{dep}} \), in thermal equilibrium. (Hint: the text calls \( C'_{\text{dep}} \) the "junction" capacitance)

\[
C' = \frac{K_0}{W} \frac{11.8 \times 8.85 \times 10^{-14} \text{F/cm}}{0.5 \times 10^{-4} \text{cm}}
\]

\[
= 2 \times 1.18 \times 0.885 \times 10^{1 - 14 + 4} \text{F/cm}^2
\]

\[
= 1 \times 10^{-8} \text{F/cm}^2
\]

4. Consider a one-sided step junction of silicon, uniformly doped with \( N_A = 10^{16} \, \text{cm}^{-3} \) on the p-side, and \( N_D = 10^{16} \, \text{cm}^{-3} \) on the n-side. (a) calculate the equilibrium concentration of minority holes at the depletion edge of the n-side, \( p_{n0}(x_{n0}) \). (b) using the law of the junction, calculate the new concentration of holes at the depletion edge \( p_n(x_{n0}) \) under a forward bias \( V_F = +0.5 \) volts.

\[a) \quad p_{n0} = \frac{n_e^2}{N_0} = \frac{10^{20} \, \text{cm}^{-6}}{10^{16} \, \text{cm}^{-3}} = 10^4 \, \text{cm}^{-3} \]

\[b) \quad p_{n}(x_{n0}) = p_{n0} e^\frac{V_F}{kT}
\]

\[
= 10^4 \, \text{cm}^{-3} e^{0.5 / 0.026}
\]

\[
= 10^4 \, \text{cm}^{-3} e^{0.5 / 0.026}
\]

\[
= 10^{12} \, \text{cm}^{-3}
\]
5. A junction of silicon under forward bias $V_F$ has a concentration of minority holes at the depletion edge on the n-side, $p_n(x=x_{n0})$ given by the law of the junction. The excess concentration of holes, $p_n' = p_n - p_{n0}$, decays exponentially as $e^{-x/L_p}$. With this $x$-dependence, calculate an analytical expression for the diffusion current (only) of holes that flows under this concentration gradient. Hint: ignore any other currents that may be flowing.

\[
J_p = -\varepsilon D_p \frac{dp_n}{dx}
\]

\[
J_p = \pm \varepsilon D_p p_{n0} (e^{\frac{\xi}{kT}} - 1) e^{-\frac{x}{L_p}} \quad (x_0' = 0, x = x_{n0})
\]
6. Sketch the I-V characteristics of a real (non-ideal) pn junction diode and label on your drawing the following regions: (a) forward bias; (b) reverse bias; (c) breakdown; (d) your choice of one other effect such as generation/recombination current, or Ohmic limit.

7. Sketch the band diagram of your choice of a metal-semiconductor junction, and indicate whether the semiconductor is n-type or p-type, and whether your band diagram shows a rectifying or Ohmic contact.