Geometric Modeling Using Focal Surfaces (Sketches 089)

Jingyi Yu
University of Delaware

Xiaotian Yin Xianfeng Gu
SUNY Stony Brook

Leonard McMillan
UNC Chapel Hill

Steve Gortler
Harvard

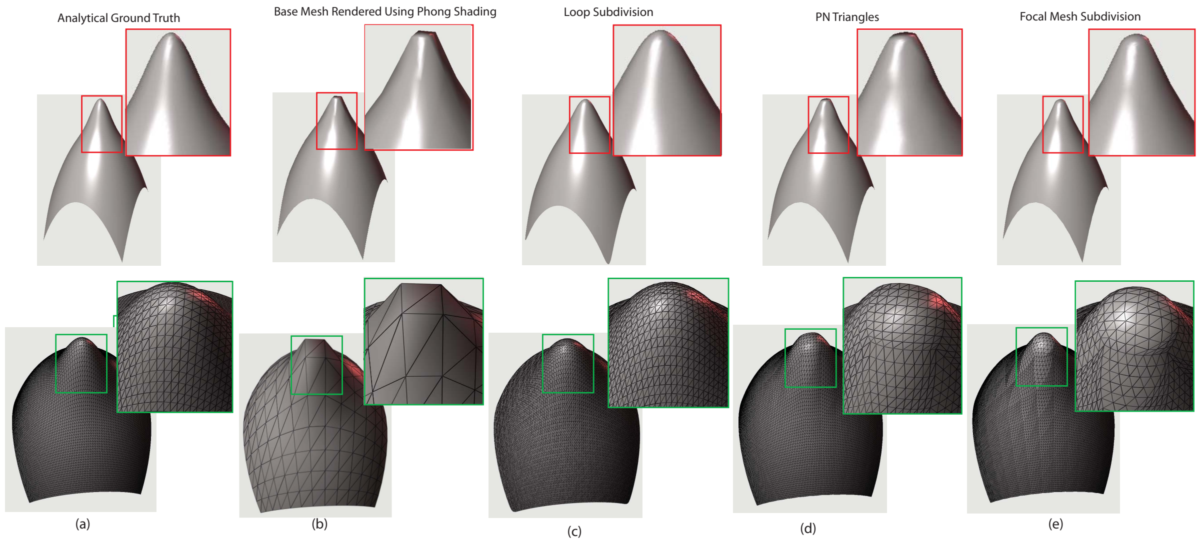

Figure 1: Top row shows the rendering of specular inflection on a pear-shaped surface (a). Bottom row shows the underlying triangulation. Phong shading the base mesh produces polygonal looking specular spots. Loop subdivision generates smooth specular spot yet loses specular inflection. PN triangles reproduces specular inflection yet generates polygonal highlights. Focal surface subdivision generates smooth specular inflection. Focal surface subdivision also generates anisotropic triangulation caused by the two parabolic curves.

Abstract

The differential geometry of smooth three-dimensional surfaces can be interpreted from one of two perspectives: in terms of oriented frames located on the surface, or in terms of a pair of associated focal surfaces. These focal surfaces are swept by the loci of the principal curvatures' radii. The normal of each focal surface indicates a principal direction at the corresponding point on the original surface [Pottmann and Wallner 2001]. In this article, we utilize piecewise linear focal surfaces, which we call focal meshes, to model smooth surfaces.

1 Introduction

Surface representations are crucial to computer graphics, numerical simulation, and computational geometry. Sampled representations, such as triangle meshes, have long served as simple, but effective, smooth surface approximations [van Overveld and Wyvill 1997]. The approximation of a smooth surfaces from a sampled geometric model, whether explicit or not, requires consistent notions of first-order and second-order differential geometric attributes.

We present a new framework for modeling discrete surfaces consistent with a differential geometry specified by an associated Piecewise Linear (PL) focal surface approximations. The focal meshes directly represent many fundamental geometric attributes such as the the surface's tangent plane, principal curvatures, and principal directions. We present a subdivision algorithm that uses focal meshes as a support structure. Focal mesh based subdivision generates smooth surfaces with consistent first and second order differential geometries. We demonstrate the importance of this shape consistency in rendering specular and reflective surfaces.

2 Exposition

Vertex and normal interpolation are paramount to rendering PL surfaces [Grimm and Zorin 2006]. The simplest and most widely used method is Phong interpolation. In Phong interpolation, the normalized barycentric coordinates of a triangle are used to blend its vertex normals. Another class of popular interpolation methods that generates smooth surfaces are subdivision schemes, such as the Loop subdivision. Closest to our approach is PN triangles, which interpolate the surface and normal separately while maintaining consistent normals. However, none of these interpolation approaches consider consistency with differential geometric features of the base surface.

We present a subdivision algorithm that utilizes the focal meshes as a control mesh. At each subdivision level, we split an edge on the base focal mesh by inserting a new focal vertex. We then find the normal direction and the principal radius of this vertex and trace back onto the original mesh surface. Finally, we apply regular subdivision to form four sub-triangles from one triangle.

Our results, shown in Figure 1, illustrate that the focal mesh subdivision generates a smooth surface with consistent first and second order differential geometries at each subdivision level. In contrast, classical subdivision schemes are either less smooth or their differential geometries vary at each level of subdivision. These artifacts are more noticeable in the animations (see the Supplemental Video).

Finally, we present a focal mesh approximation algorithm using a novel normal-ray surface representation that locally parameterizes the normals about a surface point as rays. We then show how to construct piecewise linear focal surfaces by computing the congruency of the sampled normal rays from a discrete mesh.

References

Grimm, C., and Zorin, D. 2006. Surface modeling and parameterization with manifolds: Siggraph 2006 course notes. In *SIGGRAPH '06: ACM SIGGRAPH 2006 Courses*, ACM Press, New York, NY, USA, 1–81.

Pottmann, H., and Wallner, J. 2001. *Computational Line Geometry*. Springer.

van Overveld, C. W. A. M., and Wyvill, B. 1997. Phong normal interpolation revisited. *ACM Trans. Graph. 16*, 4, 397–419.