How to compare different centrality concepts?

Normalization in one network

p-norm of the centrality vector for concept $\mathbf{X}$

$$||c_{\mathbf{X}}||_p = \begin{cases} \left(\sum_{i=1}^{n} |c_{\mathbf{X}i}|^p\right)^{1/p} & 1 \leq p \leq \infty \\ \max_i\{|c_{\mathbf{X}i}|\} & p = \infty \end{cases} \implies \frac{c_{\mathbf{X}}}{||c_{\mathbf{X}}||_p} \implies c_{\mathbf{X}i} \leq 1$$

separation of positive and negative values of $c_{\mathbf{X}}$

$$c'_{\mathbf{X}} = \begin{cases} c_{\mathbf{X}i} / \left(\sum_{j: c_{\mathbf{X}j} > 0} |c_{\mathbf{X}j}|^p\right)^{1/p} & c_{\mathbf{X}i} > 0 \\ 0 & c_{\mathbf{X}i} = 0 \\ c_{\mathbf{X}i} / \left(\sum_{j: c_{\mathbf{X}j} < 0} |c_{\mathbf{X}j}|^p\right)^{1/p} & c_{\mathbf{X}i} < 0 \end{cases}$$

Exercise (do not submit): Is $c'_{\mathbf{X}}$ a norm? Prove or disprove.

Freeman "Centrality in social networks: Conceptual clarification"

Normalization for different networks

Point-centrality

$$c''_{\mathbf{X}i} = c_{\mathbf{X}i} / \left(\max_{G \in \mathcal{G}_n} \max_{i \in V(G)} c_{\mathbf{X}i} \right)$$

set of all graphs with *n* vertices

Examples

- Degree centrality = normalization by factor *(n-1)*
- Shortest paths betweenness centrality $c_B(i) = \sum_{s \neq i} \sum_{t \neq i} \frac{\sigma_{st}(i)}{\sigma_{st}}$

  What is the upper bound (or normalization factor)?

  Star graph, $c_B(i) = (n-1)(n-2)/2$
- Closeness centrality

  $\forall i \in V \; C_i = 1/l_i$, where $l_i = \frac{1}{n} \sum_{j \in V} \delta_{ij}$, $\delta_{ij} =$ length of $i - j$ shortest path

  What is the upper bound (or normalization factor)? It is *1/(n-1)*

Summary and How Does It Work in Practice

Categories of centrality measures

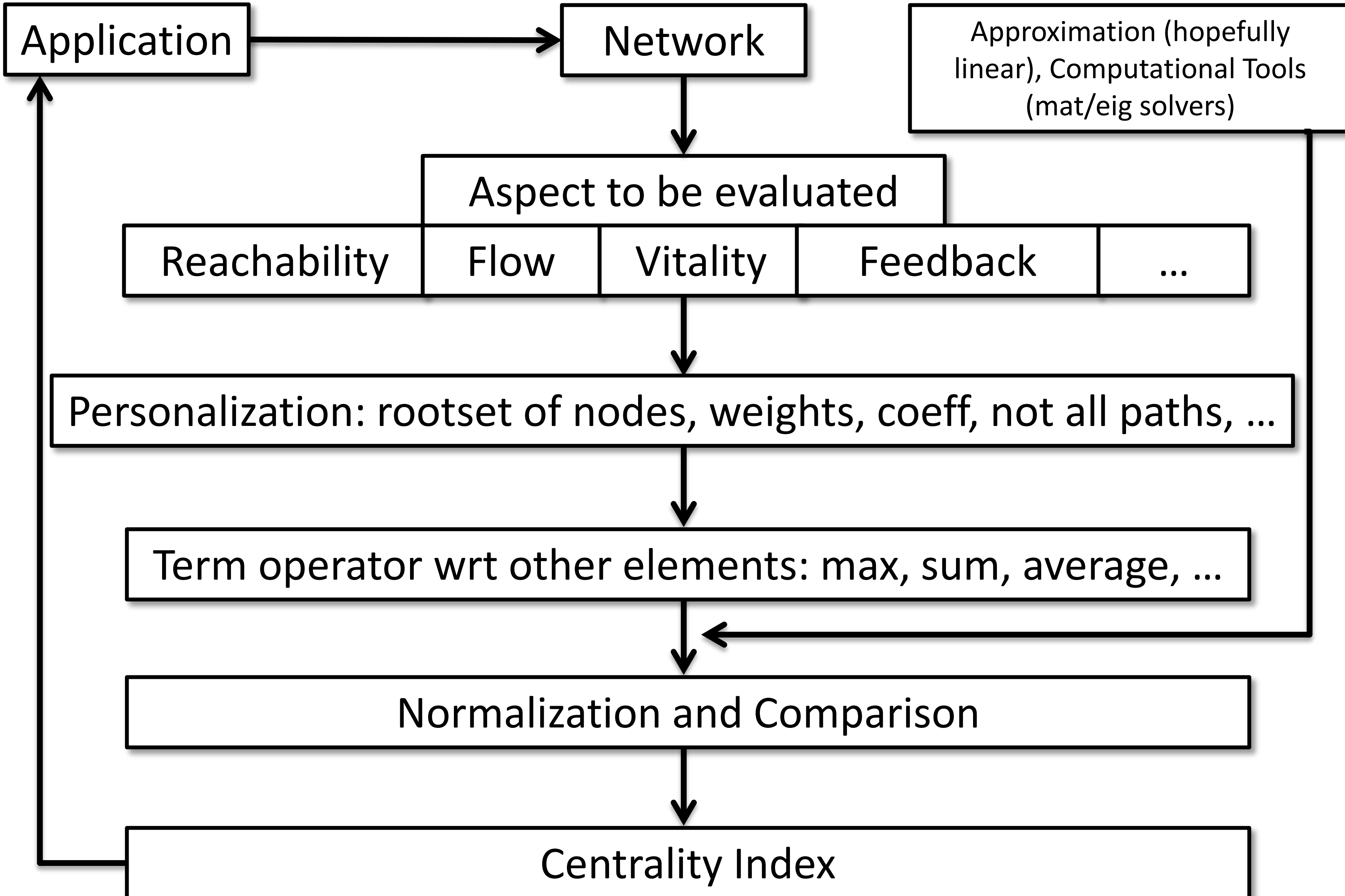
Reachability. A vertex is supposed to be central if it reaches many other vertices. Centrality measures of this category are the degree centrality, the centrality based on eccentricity and closeness, etc. All of these centralities rely on the distance concept between pairs of nodes.

Amount of flow. Based on the amount of flow $f_{st}(i)$ from a vertex s to a vertex t that goes through a vertex or an edge i. Can be based on current flow and random walks (will see how it works in Spectral Methods). Also measures that are based on the enumeration of shortest paths, stress centrality; betweenness centralities measure the expected fraction of times a unit flow goes through the element if every vertex s sends one unit flow consecutively to every other vertex t.

Vitality. Based on the vitality, i.e., the centrality value of an element x is defined as the difference of a real-valued function f on G with and without the element. Recall, a general vitality measure is identified $f(G) - f(G\setminus\{x\})$. Such as the max-flow betweenness vitality.

Feedback. Centrality measures that are based on implicit definitions of a centrality given by the abstract formula $c(i) = f(c(v1), . . . , c(vn))$, where the centrality value of i depends on the centrality values of all vertices. Includes Katz and some of eigenvector-based centralities.

[BE] "Network Analysis"


Application
Network
Approximation (hopefully linear), Computational Tools (mat/eig solvers)
Aspect to be evaluated
Reachability
Flow
Vitality
Feedback
…
Personalization: rootset of nodes, weights, coeff, not all paths, …
Term operator wrt other elements: max, sum, average, …
Normalization and Comparison
Centrality Index

Definitions and Axiomatization of Vertex Centrality

Sabidussi (1966) and Kishi (1980)
(on the blackboard)