ELEG 867  
RF and Microwave Technology  
Homework #1  
Comments & Corrections to Published Solutions

Problem 8-2 b
First, there is an error to be corrected. \( T_{12} = - \frac{S_{22}}{S_{21}} = - \frac{-1/3}{e^{j\theta}} = \frac{1}{3}e^{j\theta} \)

Then the question was raised, "The published solution has \( S_{21} = V_2/V_1 \), isn't this true only when \( S_{22} = 0 \) (matched condition)?" The answer is yes. However, even though the output of the line under test is not matched in 8-2 b, the values that have been plugged in for \( V_2 \) and \( V_1 \) to calculate \( S_{21} \) are those that would be true if the output were matched, i.e. \( V_2^- = V_1^- e^{j\theta} \) and \( V_1^+ = V_1^+ \).

Problem 8-6 b
Correction in the final line of solution:
\[
\text{GAMMA IN} = 0.2 - (-0.81)/(1 + 0.4) = 0.779
\]

Problem 9-3 d
Answer: \( Z_{IN} = 200 - j100 \) Ohms

Problem 9-5
The problem asks for the magnitude of the reflection coefficient in dB. This can be read from the "Return Loss" scale on the Smith Chart.

Problem 10-4
In calculating \( Z_A = Z_{IN} + Z_{\lambda/4} \) the author only went around the Smith Chart for \( \lambda/8 \) instead of \( \lambda/4 \). The correct answer should be \( Z_A = 0.24 - j0.28 \)

Problem 10-6
\( Z_{IN} = 120 - j50 \) Ohms
8.2) Find the scattering matrix and the transmission matrix of a loss-less transmission line of length "l" in a 50 Ω system when:
   a) Characteristic impedance of the line is (Z_o=50 Ω).
   b) Characteristic impedance of the line is (Z_o=100 Ω).

a) \( S_{11} = \Gamma_m = (50-50)/(50+50) = 0 \)
   symmetrical network: \( S_{22} = S_{11} = 0 \)
   \( S_{21} = V_2^*/V_1^* = V_1^* e^{j\beta l}/V_1^* = e^{j\beta l} \)
   symmetrical network: \( S_{12} = S_{21} \)
   \( T_{11} = 1/S_{21} = e^{j\beta l} \)
   \( T_{12} = -S_{22}/S_{21} = 0 \)
   \( T_{21} = S_{11}/S_{21} = 0 \)
   \( T_{22} = S_{12} = S_{21} \)
   \( S_{22}/S_{21} = e^{-j\beta l} \)

   symmetrical network: \( = S_{11} = 0 \)
   \( S_{21} = V_2^*/V_1^* = V_1^* e^{j\beta l}/V_1^* = e^{j\beta l} \)
   symmetrical network: \( S_{12} = S_{21} \)

b) \( Z_o=100 \) Ω

\( S_{11} = \Gamma_m = (50-100)/(50+100) = -1/3 \)
   symmetrical network: \( S_{22} = S_{11} = -1/3 \)
   \( S_{21} = V_2^*/V_1^* = V_1^* e^{j\beta l}/V_1^* = e^{j\beta l} \)
   symmetrical network: \( S_{12} = S_{21} \)
   \( T_{11} = 1/S_{21} = e^{j\beta l} \)

\( T_{12} = -S_{22}/S_{21} = e^{j\beta l} \)
\( T_{21} = S_{11}/S_{21} = -e^{j\beta l}/3 \)
\( T_{22} = S_{12} = S_{21} \)
\( S_{22}/S_{21} = e^{-j\beta l} - e^{j\beta l}/9 \)

8.6) A two-Port network has the scattering matrix as shown below. From this data:
   a) Determine whether the network is reciprocal or lossless,
   b) If the output terminals are shorted together, what will the input reflection coefficient be?

\[
[S] = \begin{bmatrix}
0.2 & j0.9 \\
-j0.9 & 0.4
\end{bmatrix}
\]

\( a) \) Not reciprocal because \( S_{11} \neq S_{22} \)

For lossless network we need: \( \Sigma S_{ij}S_{ij}^* = 1 \)
\( \Rightarrow S_{11}S_{11}^* + S_{21}S_{21}^* = 0.2(0.2) + j0.9(-j0.9) = 0.04 + .81 = 0.85 \neq 1 \)
Therefore the network is not lossless (i.e. is lossy).
\( b) \) Shorting the output gives: \( \Gamma_L = (0-50)/(0+50) = -1 \)
Using equation 8.70, we have:
\( \Gamma_n = S_{11} + S_{12}S_{21}(-1)(1-S_{22}(-1)) = S_{11} - S_{12}S_{21}/(1+S_{22}) \)
\( = 0.2 - (0.81)/(-0.81) = 0.875 \)
9.3 A lossless transmission line is connected to a load, \( Z_L = 100 + j100 \Omega \)

Using a Smith chart:

a) Determine the reflection coefficient at the load. Answer: \( 0.447 \angle 63^\circ \)
   (assume \( Z_o = 100 \Omega \))

b) Calculate the return loss. Answer: 7 dB

c) Find the VSWR on the line. Answer: 2.6 VSWR

d) Determine the reflection coefficient and the input impedance \( \lambda/8 \) away from the load. Answer: \( 0.447 \angle -27^\circ \)

9.5 VSWR on a lossless transmission line (\( Z_0 = 50 \Omega \)) is measured to be 3.0.

Using a Smith Chart determine:

a) The magnitude of the reflection coefficient in “ratio” and in “dB”.
   Answer: \( \Gamma = 0.5 \) and \( \Gamma_{\text{pwr}} = 0.25 \)

b) The return loss in dB. Answer: \( R_{\text{loss}} = 6.0 \text{ dB} \)

c) The mismatch loss in dB. Answer: \( M_{\text{loss}} = 1.25 \text{ dB} \)

d) If the load is located \( 3\lambda/8 \) away from the source, determine the load impedance value, the input impedance of the transmission line and the reflection coefficients at the load and at the source.

\[
Z_L = 3(50) = 150 \Omega
\]
\[
Z_{in} = (0.6+j0.8)50 = 30 + j40 \Omega
\]
\[
\Gamma_L = 0.5 \angle 0^\circ
\]
\[
\Gamma_S = 0.5 \angle 90^\circ
\]
10.4) A lossless transmission line \((Z_0=50 \Omega)\) is terminated in a load \((Z_L=100+j100 \Omega)\). A single shorted stub \((l=\lambda/8)\) is inserted \(\lambda/4\) away from the load as shown in figure P10.4. Using a Smith chart, determine the line's input impedance \((Z_i)\).

\[
\begin{align*}
\text{Radmanish} \quad & \quad Z_{LN}=(100+j100)/50=2+j2 \\
& Y_{LN}=j1.0 \\
& Z_A=Z_{LN}+Z_{AW}=0.8-j1.4 \Rightarrow Y_A=.31+j.54 \\
& Y_B=Y_A+j1.0=.31+j1.54 \\
& Z_0=.125-j.625 \\
& l=(.5-.41)+.375=.285\lambda \\
& Z_{AW}=1.6-j3.8 \\
& Z_{in}=50(1.6-j3.8)=80-j190\Omega \\
\end{align*}
\]

\[
\begin{align*}
\text{Hunsperger} \quad & \quad Z_{LN}=(100+j100)/50=2+j2 \\
& Y_{LN}=-j1.0 \\
& Z_{AV}=0.26-j0.24 \Rightarrow Y_A=2+j2 \\
& Y_B=(2+j2)-j1.0 = 2+j1.0 \\
& Z_0=0.4-j0.19 \\
& l=0.375\lambda \quad (318 \text{ \mu}m \text{ from Fig 10A}) \\
& Z_{in}^H=0.9-j0.94 \\
& Z_{in}^H=45-j47 \\
\end{align*}
\]

10.6) Determine the input impedance of a transmission line at a distance of 2 cm from the load impedance if the wavelength on the transmission line is found to be 16 cm. What is the VSWR on the line? Assume \(Z_L=40+j50 \Omega\) and \(Z_0=50\Omega\).

\(l=\lambda/8\)

\((ZL)N=(40+j50)/50=0.8+j1\)

We can plot \((ZL)N\) on the smith chart and read off the VSWR, or we can calculate as:

\[
\begin{align*}
\Gamma&=(40+j50-50)/(40+j50+50)=0.495 \angle 72^\circ \\
\text{VSWR}&=(1+0.495)/(1-0.495)=2.96 \\
Z_{inN} &= 2.4-j1 \\
Z_{in} &= 50(2.4-j1)=120-j50 \text{ ohms}
\end{align*}
\]