Problem 4-1a

\[ Z_1 = \frac{\frac{-2}{wC_j} R_P}{R_P - \frac{2}{wC_j}} \cdot \frac{R_P + \frac{1}{wC_j}}{R_P + \frac{1}{wC_j}} = \frac{-\frac{2}{wC_j} R_P + \frac{R_P}{wC_j}}{R_P - \frac{2}{wC_j}} \]

\[ Z_2 = R_s + \left( \frac{-\frac{1}{wC_j} R_P + \frac{R_P}{wC_j}}{R_P - \frac{1}{wC_j}} \right) \]

\[ = \frac{R_s R_P}{wC_j} + \frac{R_s}{wC_j} + \frac{R_P}{wC_j} - \frac{\frac{1}{wC_j}}{R_P - \frac{1}{wC_j}} \]

\[ Q = \frac{1}{R} = \frac{\frac{R_P}{wC_j}}{wC_j R_s R_P + \frac{R_s}{R_P} + 1} \]

\[ = \frac{R_P wC_j}{wC_j R_s R_P + \frac{R_s}{R_P} + 1} \]

\[ = \frac{wC_j R_P}{1 + wC_j R_s R_P} \quad \text{for} \quad \frac{R_s}{R_P} < 1 \]
Take \( \frac{dQ}{dw} \) and set equal to zero

\[
\frac{dQ}{dw} = \frac{WC_j R_p}{1 + W^2 C_j^2 R_p R_s}
\]

\[
= -WC_j R_p \frac{2WC_j^2 R_p R_s}{(1 + W^2 C_j^2 R_p R_s)^2} + \frac{C_j R_p}{(1 + W^2 C_j^2 R_p R_s)}
\]

\[
= -2 W^2 C_j^3 R_p R_s + C_j R_p (1 + W^2 C_j^2 R_p R_s)
\]

\[
= \frac{C_j R_p}{(1 + W^2 C_j^2 R_p R_s)^2} \left[ (1 + W^2 C_j^2 R_p R_s) - 2 W^2 C_j^2 R_p R_s \right]
\]

\[
= \frac{C_j R_p}{(1 + W^2 C_j^2 R_p R_s)^2} \left[ 1 - W^2 C_j^2 R_p R_s \right]
\]

Set \( Q = 0 \), get condition

\[
1 - W^2 C_j^2 R_p R_s = 0
\]

\[
W_0 = \frac{1}{C_j R_p R_s}
\]
4.2 a. The power absorbed by each diode when the diodes are d.c. biased on (switch off) is given by

\[ P_D = \frac{Z_0}{P_i \left(1 + \frac{n Z_0}{2 R}\right)^2} \approx \frac{4 R}{h^2 Z_0} \]

for \( R \ll Z_0 \).

Thus in this case

\[ P_D = 1 = P_i \frac{4 R}{h^2 Z_0} = 10^3 \times \frac{4.1}{h^2 50} = \frac{80}{h^2} \]

So 9 diodes would be needed to dissipate the power when the microwave transmission was switched off.

b. When the switch is off as above the power leaking through to the load is given by:

\[ \frac{P_{RL}}{P_i} = \frac{1}{\left(1 + \frac{n Z_0}{2 R}\right)^2} \approx \frac{4 R^2}{h^2 Z_0^2} \]

for \( R \ll Z_0 \).

\[ P_{RL} = P_i \frac{4 R^2}{h^2 Z_0^2} = \frac{10^3 \times 4}{2500} = 0.0164 \text{ watts} \]
4-3 (Prob. 5-12 Lia0)
Solution in textbook pp. 203 - (Parallel loading)
in class notes - (Series loading)

4-4 (Prob. 5-13 Lia0)
a. Assume parallel loading

Then \( A = \frac{R_n}{R_n - R_L} = 15 \text{db} = 31.62 \)

\( R_n = 69 + j\times 9.7 \) (Given)

\( R_e = R_n - \frac{R_n}{A} = 69 + j\times 9.7 - \frac{69 + j\times 9.7}{31.62} \)

\( = 69 + j\times 9.7 - (2.18 + j\times 0.31) \)

\( = [66.82 + j\times 9.39] \)

b. Assume series loading

Then \( A = \frac{R_e}{R_e - R_n} \) or \( R_L = \frac{-A \times R_n}{1 - A} \)

\( R_L = -31.62 \times \frac{(69 + j\times 9.7)}{1 - 31.62} = -(2182 + j\times 307) \)

\( = [71.26 + j\times 10.02] \)
a. The advantages of a parametric device are:
   1. very low noise figure—especially if cooled
   2. large gain-bandwidth product
   3. low bias power
   4. can be used as up-or down converters, as well as amplifiers at a given frequency

   The disadvantages are:
   1. requires a microwave "pump" power source
   2. complex cavity design—expensive
   3. nonlinear device characteristic can lead to unwanted frequency generation and instability
   4. generally larger and heavier than a FET amplifier

b. The applications of parametric amplifiers are:
   1. low noise front-end for radar and communications receivers
   2. down-converter to an intermediate frequency (IF)
   3. up-converter to shift to higher carrier frequency
   4. wide band amplifier at microwave frequencies
Parametric up-converter: $\delta Q = 8 \quad f_0 / f_3 - f_2 / f_3 = 8$

$T_4 = 300^\circ K \quad T_0 = 300^\circ K$

a. Maximum power gain:

$$\text{Gain} = \frac{f_0}{f_3} \frac{x}{(1 + 1 + x)^2}$$

where $x = \frac{f_3}{f_0} (\delta Q)^2 = \frac{1}{8} (8)^2 = 8$

$$G_{\text{max}} = 8 \times \frac{8}{(1 + 1 + 8)^2} = \frac{64}{16} = 4 - 6 \text{ dB}$$

b. Noise figure:

$$F = 1 + \frac{2 T_4}{T_0} \left[ \frac{1}{18} + \frac{1}{(8Q)^2} \right] = 1 + \frac{2 \times 300}{300} \left[ \frac{1}{8} + \frac{1}{8^2} \right]$$

$$= 1 + 2 (0.14) = 1.28 = 1.08 \text{ dB}$$

c. Bandwidth:

$$BW = 2\pi \sqrt{\frac{f_0}{f_3}} = 2 \times 0.2 \sqrt{8} = 1.13$$
Negative-resistance parametric amplifier:

- $f_s = 2$ GHz
- $R_i = 1$ kΩ
- $\varphi = 0.35$
- $f_p = 12$ GHz
- $R_g = 1$ kΩ
- $\gamma Q = 10$
- $f_i = 10$ GHz
- $R_{Ts} = 1$ kΩ
- $C = 0.01$ pF
- $R_{Ti} = 1$ kΩ
- $T_d = 300^\circ K$

a. Power gain:

$$\text{Gain} = \frac{4f_i R_g R_i}{T_d R_{Ts} R_{Ti} (\pi a)^2}$$

where $a = R/R_{Ts}$

$$R = \frac{\gamma^2}{\omega_s \omega_c C^2 R_{Ti}}$$

$$= \frac{(0.35)^2}{2\pi \times 2 \times 10^7 \times 2 \pi \times 10^{10} \times (10^{-14})^2 \times 10^3}$$

$$= \frac{0.12}{18.88 \times 10^{-6}} = 1.52 \times 10^3$$

$$a = \frac{R}{R_{Ts}} = \frac{1.52 \times 10^3}{10^3} = 1.52$$

$$\text{Gain} = \frac{4 \times 10^{10} \times 1 \times 1 \times 1.52}{2 \times 10^9 \times 1 \times 1 \times (1-1.52)^2} = 112.59 = 20.5 \text{ dB}$$

b. Noise figure:

$$F = 1 + 2 \frac{T_d}{T_0} \left[ \frac{1}{\delta Q} + \frac{1}{(\gamma Q)^2} \right]$$

$$= 1 + 2 \frac{300}{300} \left[ \frac{1}{10} + \frac{1}{(10)^2} \right] = 1.22 \approx 0.86 \text{ dB}$$

c. Bandwidth:

$$BW = \frac{\varphi}{2} \sqrt{\frac{f_i}{f_s \text{Gain}}} = \frac{0.35}{2} \sqrt{\frac{10^{10}}{2 \times 10^9 \times 112.59}} = 0.04$$