Chapter 14

14.1 Sketch the cross sectional view of a double-heterostructure GaAlAs laser diode. Choose the thickness and aluminum concentration in each layer required to produce light emission at \( \lambda_0 = 8500 \text{ Å} \), along with field confinement to the active layer for the lowest order TE and TM modes only.

Solution. The aluminium concentration required in the active region for light emission at \( \lambda_0 = 8500 \text{ Å} \) is determined from Fig. 12-7 to be \( X = 10\% \) (reading from the “p-n junction” curve).

The Al concentration difference \( (y - x) \) required for confinement of the lowest order mode is \( y - x = 3\% \) (interpolating from the \( m = 0 \) curve of Fig. 4.11 for \( X = 10\% \), and assuming an active region thickness of 0.8 \( \mu \text{m} \)). Note that the choice of active region thickness should be something less than 1 \( \mu \text{m} \) to take full advantage of the confinement effect because the diffusion length of the carriers is about this value.

The Al concentration difference required for confinement of the \( m = 1 \) mode is 14\%, from the same figure. Thus to waveguide light of only the \( m = 0 \) mode the Al concentration in the confining layer must be \( 13\% \leq y \leq 24\% \). The cross section would then look like

![Cross-sectional view of a double-heterostructure GaAlAs laser diode](image)

Note that a thickness of at least several times the wavelength must be chosen for the confining layers to insure adequate confinement.
14.3 Derive an expression for the threshold current density \( J_{th} \) in a heterojunction laser that has an active layer thickness \( d \), light emitting layer thickness \( D \), average loss coefficient \( \alpha \), and reflecting endfaces with different reflectivities \( R_1 \) and \( R_2 \). Use the symbols defined in the text for the other parameters, i.e., \( \eta_q, \lambda_0, \Delta \nu \), etc.

**Solution.**

\[ R_1 P_1 e^{(g \frac{d}{D} - \alpha L)} = P_2 \]

and

\[ R_2 P_2 e^{(g \frac{d}{D} - \alpha L)} = P_1 \]

\[ \therefore R_1 P_1 e^{(g \frac{d}{D} - \alpha L)} = P_2 \]

\[ e^{(g \frac{d}{D} - \alpha L)} = \frac{1}{R_1 R_2} \]

\[ \left( g \frac{d}{D} - \alpha \right) 2L = \ln \left( \frac{1}{R_1 R_2} \right) \]

\[ g = \frac{D}{d} \left[ \alpha + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \right] \]

From (11.1.8) \( g = \frac{n_q \lambda_0^2 J}{8 \pi e n^2 d \Delta \nu} \)

\[ \therefore J_{th} = \frac{8 \pi e n^2 \Delta \nu D}{n_q \lambda_0^2} \left[ \alpha + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \right] \]
For small active layer thickness $d$, the confinement factor $d/D$ may be written as

$$d/D \approx \gamma d.$$  

The compositional dependence of the refractive index of Ga$_{(1-x)}$Al$_x$As at $\lambda_0 = 0.90 \mu m$ can be represented by

$$n = 3.590 - 0.710x + 0.091x^2.$$  

a) Develop an expression for the confinement factor as a function of $x$, $d$ and $\lambda_0$ for a three-layer, symmetric GaAs-Ga$_{(1-x)}$Al$_x$As DH laser with $n_a = 3.590$.

b) For a DH laser with $d = 0.1 \mu m$, $x = 0.1$ and $\lambda_0 = 0.90 \mu m$, what is the magnitude of the confinement factor?

c) Compare your answer to Part b, with the data given in Fig. 14.4

Solution.

a) From Eqs. (12.2.7) and (12.2.11),

$$d/D \approx \gamma d = \frac{2\pi^2(n_a^2 - n_c^2)d^2}{\lambda_0^2}$$

For GaAs – Ga$_{(1-x)}$Al$_x$As at $\lambda_0 = 0.90 \mu m$

$$n_a^2 - n_c^2 = n_a^2 - (n_a^2 - 1.42 n_a \cdot a + 0.182 n_a \cdot a^2 + \ldots)$$

By neglecting the higher-order terms,

$$n_a^2 - n_c^2 = 1.42 n_a \cdot a$$

Since $n_a = 3.590$, $d/D$ becomes

$$d/D = \frac{2\pi^2(1.42 \times 3.59 a)d^2}{\lambda_0^2} = \frac{100 a d^2}{\lambda_0^2}$$

b) For a DH laser with $d = 0.1 \mu m$, $a = 0.1$ and $\lambda_0 = 0.90 \mu m$

$$d/D = \frac{100 \times 0.1 \times (0.1)^2}{(0.9)^2} = 0.12$$

c) The confinement factor is given by 0.11 in Fig. 12.4 and 0.12 by the approximate solution at small $d$. 
14.6 What are the advantages of a stripe-geometry laser as compared to a broad-area DH laser?

**Solution.**

i) lower threshold current because of the smaller cross-sectional area.

ii) suitable for long-distance optical fiber communications since stable single-mode oscillation can be obtained.

iii) longer lifetime of the laser because of retarded degradation, since most of the junction perimeter is removed from the surface.

14.7 The external *differential quantum* efficiency of a GaAs laser diode emitting at a wavelength of 0.9 μm is 30%. The applied voltage is 2.5 volts. Calculate the external *power* efficiency of the device.

**Solution.** The differential quantum efficiency is the number of photons emitted/s per electron (or hole) injected/s

\[ \eta_q = \frac{N_{ph}}{N_e} = 0.3 \]

The external power efficiency is given by

\[ \eta_{power} = \frac{\text{optical power emitted}}{\text{electrical power injected}} = \frac{N_{ph} \times h\nu}{VI} = \frac{N_{ph} \times h\nu}{V \cdot q \cdot N_e} \]

Since \( N_e = \frac{N_{ph}}{0.3} \)

\[ \eta_{power} = \frac{0.3 \times h\nu}{V \times q} = \frac{0.3 \times 1.38 \times 16 \times 10^{-19}}{2.5 \times 1.6 \times 10^{-19}} \]

\[ h\nu = \frac{1.24 \times 0.9}{0.9} = 1.38 \text{ eV} = 1.38 \times 16 \times 10^{-19} \text{ joules} \]

\[ \eta_{power} = \frac{0.3 \times 1.38 \times 16 \times 10^{-19}}{2.5 \times 1.6 \times 10^{-19}} = 0.166 = 16.6\% \]
Chapter 15

15.1 What is the grating spacing required for a DFB laser to operate at a wavelength $\lambda_0 = 8950 \text{ Å}$ in GaAs if a first-order grating is desired?

Solution.

$$\lambda_0 = \frac{2\Lambda n}{m}$$

or

$$\Lambda = \frac{\lambda_0 m}{2n}$$

for this case $\lambda_0 = 8950 \text{ Å}$, $m = 1$; from Fig. 4.9 for GaAs $n = 3.58$

$$\Lambda = \frac{8950 \times 1}{2 \times 3.58} = 1250 \text{ Å}$$

15.2 Repeat Problem 15.1 for the case of a laser formed in an active region of Ga$_{0.8}$Al$_{0.2}$As.

Solution.

From Fig. 4.9 for Ga$_{0.8}$Al$_{0.2}$As

$n = 3.4$

$$\Lambda = \frac{8950 \times 1}{2 \times 3.4} = 1316 \text{ Å}$$

Note the strong dependence of grating spacing on index of refraction.

15.3 If the grating of Problem 15.1 is fabricated by using the technique of Fig. 7.10, determine two combinations of prism material and angle ($\alpha$) that could be used with a He-Cd laser ($\lambda_0 = 3250 \text{ Å}$).

Solution.

$$\Lambda = \frac{\lambda_0}{2n_p \sin \alpha}$$

in this case $\lambda_0 = 3250 \text{ Å}$ and $\Lambda = 1250 \text{ Å}$ or

$$\sin = \frac{\lambda_0}{2n_p \Lambda} = \frac{3250}{n_p 2500}$$

If we use a quartz prism with $n_p = 1.6$ we need an angle $\alpha = 60.38^\circ$. If we use a rutile prism with $n_p = 2.4$ we need an angle $\tilde{\alpha} = 35.44^\circ$.

15.4 a) If a surface grating is formed on a waveguide, what is the grating spacing $\Lambda$ required for fourth-order Bragg diffraction, in terms of $\lambda$ and $n_g$?
b) Sketch the reflected wave directions, and indicate the angles between them.

c) In general, can a scattered wave perpendicular to the direction of the waveguide be observed for odd orders, such as the first and third order, of Bragg diffraction?

**Solution.**

a) From (13.1.10)

\[ \Lambda = \frac{\ell \cdot \lambda_0}{2n_g} \]

For 4th-order gratings, \( \ell = 4 \)

\[ \therefore \Lambda = 2 \cdot \frac{\lambda_0}{n_g} \]

b) Since \( \Lambda = 2\lambda_0/n_g \), (13.1.13) becomes

\[ \sin \theta = \frac{1}{2} \ell' - 1 \quad \text{with} \quad \frac{1}{2} \ell' - 1 \leq 1 \quad \text{for} \quad \ell' = 0, 1, 2, 3, 4 \]

- \( \ell' = 0 \): \( \theta = -90^\circ \), the ray angle \( \psi = 90^\circ + \theta = 0^\circ \) \( \rightarrow \) forward direction
- \( \ell' = 1 \): \( \theta = 30^\circ \), \( \psi = 90^\circ - 30^\circ = 60^\circ \)
- \( \ell' = 2 \): \( \theta = 0^\circ \), \( \psi = 90^\circ \rightarrow \) transverse direction
- \( \ell' = 3 \): \( \theta = 30^\circ \), \( \psi = 90^\circ + 30^\circ = 120^\circ \)
- \( \ell' = 4 \): \( \theta = 90^\circ \), \( \psi = 180^\circ \rightarrow \) backward scattering for DFB.

\[ \ell' = 3 \quad \ell' = 2 \quad \ell' = 1 \quad \ell' = 0 \]

\[ 60^\circ \quad 30^\circ \quad 60^\circ \]

\[ \ell = 4 \]

\[ 30^\circ \]

\[ 60^\circ \]

\[ c) \text{No. Only the scattered waves for even orders such as the second and fourth order can be observed perpendicular to the direction of the waveguide.} \]

**15.5** A buried heterostructure, multiple contact, distributed feedback (DFB) laser has a grating periodicity of 1200 Å and a waveguide index of refraction equal to 3.8. The interaction length of the grating is 500 μm. Assume that only a single mode oscillates and that feedback comes from a first-order Bragg reflection. In this
problem you may neglect the slight shift in wavelength which occurs in a DFB laser and assume that emission is at the Bragg wavelength.

a) What is the emitted wavelength of the laser?

b) If current is injected into the modulator section of the diode so as to change the waveguide index by one part in \(10^7\), what will be the resulting change in laser emitted wavelength (frequency)?

**Solution.** The oscillation frequency of a DFB laser is given by

\[
\omega_m = \omega_0 \left( m + \frac{1}{2} \right) \frac{\pi c}{n_g L} = \omega_0 - \frac{\pi C}{2n_g L} \quad \text{for} \quad m = 0
\]

where \(\omega_0\) is the Bragg frequency, calculated from

\[
\lambda_0 = \frac{2 \Lambda n_g}{\ell} = 2 \Lambda n_g = 2 \times 1200 \times 3.8 = 9120 \text{Å} \quad \text{for} \quad \ell = 1
\]

We will neglect the \(\pi C/(2n_g L)\) shift (\(\approx 5\) Å) in this problem.

b) \(\lambda_0 = 2 \Lambda n_g\)

\[
\frac{d\lambda_0}{dn_g} = 2\Lambda
\]

\[
d\lambda_0 = 2\Lambda dn_g
\]

but \(\frac{dn_g}{n_g} = 10^{-7}\)

\[
dn_g = 10^{-7}n_g = 10^{-7} \frac{\lambda_0}{2\Lambda}
\]

\[
\therefore d\lambda_0 = 2\Lambda \times 10^{-7} \frac{\lambda_0}{2\Lambda} = 10^{-7}\lambda_0
\]

\[
d\lambda_0 = 10^{-7} \times 9120 = 9.12 \times 10^{-4} \text{Å}
\]
Chapter 16

16.1 We wish to modulate the light output of a semiconductor laser having the optical power-current characteristic shown below by directly varying the input current. Sketch the intensity (or optical power density) waveform of the light output for a bias current of 300 mA and an applied current signal which is sinusoidal and has a peak to peak current variation of 200 mA.

Solution. Refer to Fig. 14.1 for graphical approach

16.2 Repeat problem 16.1 for a bias current of 150 mA, all other conditions remaining the same.

Solution. Refer to Fig. 14.1 for graphical approach

The key point of problems 14.1 and 14.2 is that because of the extreme non-linearity of the laser response to current it is essential that the laser be biased well above threshold to permit a linear response to an applied signal.

16.3 A cw double heterojunction GaAs-GaAlAs laser diode is to be directly current modulated by a sinusoidal ac signal current of 100 mA peak-to-peak. What must be
the minimum dc bias current to the laser to ensure that the laser output linearly follows the input signal, i.e., the output optical signal will be sinusoidal, following the input signal current waveform. Relevant characteristics of the laser diode are as follows:

1) Emitted wavelength: 9000 Å.
2) Half power points of the emission peak for spontaneous emission have been measured for this material at room temperature to be at 9200 and 8800 Å.
3) Index of refraction: 3.3.
4) Thickness of active layer: 1 µm.
5) Internal quantum efficiency: 0.8.
6) Average absorption coefficient: 10 cm$^{-1}$.
7) Length between cleaved end faces: 1 mm.
8) Reflectivity of end faces: 0.4.
9) Cross-sectional area normal to current flow: 10$^{-3}$ cm$^2$.

**Solution.** The D.C. bias current to the laser must be large enough that the laser current never drops below the threshold current level at any point in the cycle.

\[
J_{th} = \frac{8\pi e n^2 (\Delta \nu)d}{n q \lambda_0^2} \left( \frac{\alpha - \ln \frac{1}{R}}{L} \right)
\]

\[
(\Delta \nu) = \frac{c}{\lambda_1} - \frac{c}{\lambda_2} = 3 \times 10^8 \left( \frac{1}{8.8 \times 10^{-7}} - \frac{1}{9.2 \times 10^{-7}} \right) = 1.5 \times 10^{13} \text{ sec}^{-1}
\]

\[
J_{th} = \frac{8(1.6 \times 10^{-19})(3.3)^2(1.5 \times 10^{13})(1 \times 10^{-6})}{(0.8)(9 \times 10^{-7})^2(1 \times 10^3 + 1 \times 10^2(0.916))}
\]

\[
= 1.9 \times 10^6 \text{ A/m}^2
\]

\[
I_{th} = J_{th} \times A = 1.9 \times 10^6 \times 10^{-7} = 190 \text{ mA}
\]

Since the modulation signal varies from +50 mA to −50 mA about the bias point

\[
I_{DC \text{ bias}} = 190 + 50 = 240 \text{ mA}
\]

**16.4** Given a laser with threshold current $I_t = 100$ mA and peak current $I_p = 300$ mA, what is the maximum modulation depth achievable if the dc bias current is $I_{dc} = 130$ mA? Assume that the laser $L-I$ curve is linear above threshold and that, at threshold, the output power $P_t = 0.05 P_p$.

**Solution.**
With \( I_{DC} = 130 \text{ mA} \), \( \Delta I_{\text{max}} \) is \( \pm 30 \text{ mA} \) (i.e. \( 100 - 160 \text{ mA} \)).

At 100 mA, \( P_{\text{out}}/P_{\text{max}} = 0.05 \).

At 160 mA,

\[
\frac{P_{\text{out}}}{P_{\text{max}}} = \frac{95}{200} \left( \frac{160}{100} - 1 \right) + 0.05
\]

Slope of linear portion of curve

\[
\frac{P_{\text{out}}}{P_{\text{max}}} = 0.335
\]

Modulation depth = 0.335 – 0.5 = 28.5%

16.5 A SH laser diode is to be used for high-speed communications. It must be operated with no dc bias, and in a pulsed mode with no more than a 5% duty cycle, and a pulse width of 0.8 ns. The threshold and peak currents are 100 mA and 7 A, respectively. The spontaneous electron lifetime in the device is 60 ns.

a) What factor(s) limit the maximum modulation rate?

b) What is the maximum pulse repetition rate that can be used?

Solution.

a) With this device the turn-on delay and quiescent time needed between pulses are the two factors which should be considered.

b) Turn-on delay:

\[
\tau_d = \tau_{\text{SP}} \left( \ln \frac{I_p}{I_p - I_{\text{th}}} \right) \\
= 6 \times 10^{-8} \ln \frac{7}{7 - 0.1} \\
= 6 \times 10^{-8}(0.0144) \\
\tau_d = 8.84 \times 10^{-10} \text{ sec}
\]

\( \therefore f_{\text{max}} = 1.157 \text{ GHz} \)
Pulsing delay: 5% duty cycle, 0.8 ns pulse width

\[0.05 \times 0.8 \text{ nsec} = 0.8 \text{ nsec}
\]
\[x = 1.6 \times 10^{-8} \text{ sec}
\]
\[\therefore f_{\text{max}} = 62.5 \text{ MHz}
\]

So the delay due to pulsing will be the limiting factor.