Given that

\[ B_{12} N_1 \rho(v_{12}) = A_{21} N_2 + B_{21} N_2 \rho(v_{12}) \]

show that for a very large photon density \( \rho(v_{12}) \to \infty \) it follows that

\[ B_{12} = B_{21} \]

**Solution.**

\[ B_{12} n_1 \rho(v_{12}) = A_{21} n_2 + B_{21} n_2 \rho(v_{12}) \quad (12-2) \]

\[ B_{12} = \left[ \frac{A_{12}}{\rho(v_{12})} + B_{21} \right] e^{-(E_2 - E_1)/kT} \quad (\leftarrow \text{This is } n_2/n_1) \]

As \( T \to \infty \), \( e^{-(E_2 - E_1)/kT} \to 1 \)

Thus as \( e(v_{12}) \to \infty \)

\[ B_{12} = B_{21} \]

11.3 If the absorption of a photon causes an electron to make a direct transition (i.e., no phonon involved) from a state with wavevector \( |k| = 1.00 \times 10^7 \text{ cm}^{-1} \) in a given direction to a state with wavevector \( |k| = 1.01 \times 10^7 \text{ cm}^{-1} \) in the same direction, what was the wavelength of the photon?

**Solution.** from (10.1.12)

\[ \bar{k}_t + \frac{2\pi}{\lambda_{\text{phot}}} \bar{\mu} = \bar{k}_f \]

\[ \lambda_{\text{phot}} = \frac{2\pi}{|k_f| - |k_i|} = \frac{2\pi}{0.01 \times 10^7} = 2\pi \times 10^{-5} \text{ cm} \]

\[ = 6280 \text{ Å} \]

11.5 Sketch three different plots of typical diode laser emission spectra which show output light intensity vs. wavelength a) below threshold, b) at threshold, c) above threshold.

**Solution.**

[Sketches of intensity vs. wavelength plots for spontaneous emission below threshold, super-radiance emission at threshold, and laser emission above threshold.]
a) Draw the \((E \text{ vs. } x)\) energy band diagram of a \(p - n\) junction laser under forward bias and indicate the inverted population region, as well as \(E_{Fn}, E_{Fp}, E_c, E_v\) and \(E_g\).

b) What energies establish the long and short wavelength limits of the laser emission spectrum?

**Solution.**

\[ E_{Fn} - E_{Fp} \quad \text{and} \quad E_c - E_v \quad (= E_g) \]
Chapter 12

12.1 We would like to design a p-n junction laser for use as the transmitter in a range finder. The output is to be pulsed with a peak power out of each end of 10 W (only the output from one end will be used) and a pulse duration of 100 ns. Wavelength is to be 9000 Å. Room temperature operation is desired and some of the pertinent parameters have been either measured or established so that:

1) Half-power points of the emission peak for spontaneous emission have been measured for this material at room temperature to be at 9200 and 8800 Å.
2) Index of refraction: 3.
3) Thickness of light emitting layer: 10 µm.
4) Thickness of active (inverted pop.) layer: 1 µm.
5) Internal quantum efficiency: 0.7.
6) Average absorption coefficient: 30 cm⁻¹.
7) W = 300 µm.
8) Reflectivity of the Fabry-Perot surfaces: 0.4.

a) What must be the separation between Fabry-Perot surfaces if we wish to have a peak pulse current density of $3 \times 10^4$ A/cm²?

b) What is the threshold current density?

Solution.

a) $P_{out} = \frac{1}{L} \ln \frac{1}{R} \frac{J_{nq}}{e} (L \times W) h\nu$

$P_{out} + \frac{1}{L} \ln \frac{1}{R} = \ln \frac{J_{nq} W h\nu}{e P_{out}}$

$\frac{1}{L} \ln \frac{1}{R} = \ln \frac{J_{nq} W h\nu}{e P_{out}} - \alpha$

$\frac{1}{L} = \frac{J_{nq} W h\nu}{e P_{out}} - \alpha$

$\frac{1}{L} = \frac{3 \times 10^4 \times 0.7 \times 3 \times 10^{-2} \times 1.38}{0.916} = 43.5 \approx 10.7$

$L = 1/10.7 = 0.0935$ cm $= 935$ µm

b) $J_{th} = \frac{8 \pi e n^2 \Delta \nu D}{n_q \lambda^2} \left( \alpha + \frac{1}{L} \ln \frac{1}{R} \right)$

$J_{th} = 8 \times 1.6 \times 10^{-19} (3.3)^2 10^{-3} 1.5 \times 10^{-13} \frac{0.7(9 \times 10^{-5})^2}{0.916 + 0.0935}$

$J_{th} = 4620$ A/cm²
12.2 In the case of the laser of Problem 12.1, if the heat sink can dissipate 1 W at room temperature, what is the maximum pulse repetition rate that can be used without causing the laser crystal to heat above room temperature (neglect $I^2R$ loss)?

Solution.

$$\eta_{as} = \frac{1}{\alpha} \ln \frac{1}{\eta} - \eta_{n} = \frac{9.8}{39.8} \times 0.7 = 0.172$$

$$P_{in} \times 0.172 = 20 \text{ W} ; \quad P_{in} = \frac{20}{0.172} = 116 \text{ W}$$

Avg. power lost = 1 W = \((20/0.172)(1 - 0.172)(1 \times 10^{-7})\) (Repetition Rate)

Maximum rep. rate = \(X = 0.104 \times 10^6\) pps.

12.3 For the laser of Problem 12.2, what is the minimum range for which transmitted and target reflected pulses will not overlap in time at the detector, assuming that transmitter and detector are located at essentially the same point and transmission is through air?

Solution.

$$\text{dist} = \frac{3 \times 10^8 \text{ m/sec} \times 10^{-7} \text{ sec}}{2} = 15 \text{ m}$$

12.5 A semiconductor laser formed in a direct bandgap material is found to have an emission wavelength of 1.2 \(\mu\text{m}\). The external quantum efficiency is 15%.

a) What is the approximate bandgap energy of the material?

b) If the output power is 20 mW, give an approximate estimate of the input current.

Solution.

a) The energy of the emitted photons is approximately equal to the bandgap. Hence:

$$E = \frac{hc}{\lambda} = \frac{1.24}{1.2} = 1.03 \text{ eV}$$

b) The external quantum efficiency is defined as the number of photons emitted for each hole–electron pair injected. Hence the input current is given by

$$I_{in} = \frac{0.02 \text{ Joules/sec}}{1.03 \text{ eV} \times 1.6 \times 10^{-19} \text{ Joules/eV}} \times \frac{1}{0.15} \times 1.6 \times 10^{-19} \text{ coul/carrier}$$

$$= 129 \text{ mA}$$

If you don't remember the definition of external quantum efficiency you could approximate the solution by assuming a conversion efficiency of 15% and an applied voltage equal to the bandgap energy. Then

$$I_{in} = \frac{P_{out}}{\eta_{ext} \times V_{in} \times 0.15} = \frac{0.02}{1.03 \times 0.15} = 130 \text{ mA}$$

The problem was stated in terms of ±50% to allow reasonable approximation of the input voltage about the bandgap value.
12.8 The following parameters are known for a semiconductor laser:
Emission wavelength $\lambda_0 = 0.850 \mu m$.
Lasing threshold current $I_{th} = 12 \text{ A}$

**External quantum efficiency = 1%** below threshold

**External quantum efficiency = 10%** above threshold

a) Sketch the light output power [W] vs. input current [A] curve for this device for currents ranging from 0 to 20 A; label the value of the output power at threshold.

b) What is the output power [W] for an input current = 18 A?

**Solution.**

a) The energy of 1 photon is given by

$$ E = \frac{1.24}{\lambda_0} = \frac{1.24}{0.85} = 1.46 \text{ eV} $$

$1.46 \text{ eV} \times 1.6 \times 10^{-19} \text{ Joules/eV} = 2.34 \times 10^{-19} \text{ Joules/photon.}$

$$ \frac{\text{# hole-electron pairs injected per second}}{\text{amp}} = \frac{\text{Current (in amps)}}{\text{Charge per carrier (coulombs)}} $$

$$ \frac{\text{# of pairs/sec}}{\text{amp}} = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18} \text{ pairs/sec/amp} $$

Therefore the ratio of power out to input current is below threshold

$$ \frac{P_{out}}{I_{in}} = 2.34 \times 10^{-19} \text{ Joules/photon} \times 0.01 \frac{\text{Photons}}{\text{pair}} \times 6.25 \times 10^{18} \frac{\text{pairs/sec}}{\text{amp}} $$

$$ = 1.46 \times 10^{-2} \text{ Watts/amp} $$

**above threshold**

$$ \frac{P_{out}}{I_{in}} = 2.34 \times 10^{-19} \text{ Joules/photon} \times 0.1 \frac{\text{Photons}}{\text{pair}} \times 6.25 \times 10^{18} \frac{\text{pairs/sec}}{\text{amp}} $$

$$ = 1.46 \times 10^{-1} \text{ Watts/amp} $$

**at threshold**

$$ P_{out} = 1.46 \times 10^{-2} \text{ Watts/amp} \times 12 \text{ amp} = 0.208 \text{ Watts} $$

b) **At $I_{in} = 18 \text{ amps}$**

$$ P_{out} = 1.46 \times 10^{-2} \times 12 + 1.46 \times 10^{-1} \times 6 $$

$$ = 0.175 + 0.876 $$

$$ = 1.051 \text{ Watts} $$
12.10 The external differential quantum efficiency of a GaAs laser diode emitting at a wavelength of 0.9 \( \mu m \) is 30%. The applied voltage is 2.5 volts. Calculate the external power efficiency of the device.

Solution. The differential quantum efficiency is the number of photons emitted/s per electron (or hole) injected/s.

\[
\eta_q = \frac{N_{ph}}{N_e} = 0.3
\]

The external power efficiency is given by

\[
\eta_{power} = \frac{\text{optical power emitted}}{\text{optical power injected}} = \frac{N_{ph} \times h\nu}{VI} = \frac{N_{ph} \times h\nu}{V \cdot q \cdot N_e}
\]

Since \( N_e = N_{ph}/0.3 \)

\[
\eta_{power} = \frac{0.3 \times h\nu}{V \times q} = \frac{0.3 \times h\nu}{2.5 \times 1.6 \times 10^{-19}}
\]

\[
h\nu = \frac{1.24}{0.9} = 1.38 \text{ eV} = 1.38 \times 1.6 \times 10^{-19} \text{ joules}
\]

\[
\eta_{power} = \frac{0.3 \times 1.38 \times 1.6 \times 10^{-19}}{2.5 \times 1.6 \times 10^{-10}} = 0.166 = 16.6\%
\]
13.2 Which of the following wavelengths of light can be used to pump an erbium-doped fiber amplifier (EDFA)? 980 nm, 982 nm, 855 nm, 1350 nm, 1480 nm.

Solution. 980 nm, 982 nm, 1480 nm

13.4 A fiber laser has a cavity length of 2 m and an effective index of refraction of 1.75. It emits at a vacuum wavelength of 2.4 µm. If it is to be harmonically modelocked to the third harmonic of its fundamental frequency, what must be the modulation frequency of the locking signal?

Solution.

\[ f_m = n f_c = 3 f_c \]

in general \( f v = v = c/n = 3 \times 10^8 / 1.75 \)

\( f_c \) = inverse of the cavity round trip time
\[ \frac{1}{L/v} = \frac{1}{2 \times 2} = \frac{1}{4 \times 1.75 / 3 \times 10^8} = \frac{3 \times 10^8}{7.00} = 4.29 \times 10^7 \]

\[ f_m = 3 \times 4.29 \times 10^7 = 1.29 \times 10^8 = 129 \text{ MHz} \]

13.5 What are the ranges of wavelength over which each of the following types of optical amplifier can be used?

- **a.** Semiconductor optical amplifier (SOA)
- **b.** Erbium doped fiber amplifier (EDFA)
- **c.** Raman amplifier
- **d.** Thulium doped fiber amplifier (TDFA)

**Solution.**

- **a.** Semiconductor 800–900 nm, 1300–1360 nm, 1480–1520 nm, 1525–1565 nm, 1570–1620 nm
- **b.** (EDFA) 1525 to 1600 nm (from text)
- **c.** Raman 1525 to 1989 nm (from Fig. 13.5)
- **d.** TDFA 1430 to 1500; also \( \approx 1.3 \mu \text{m} \) (get all data from Table 13.1)