4.4 Solution

The first step is to determine the index of refraction in the two layers using the Sellmeir Equation

\[ n^2 = A + \frac{B}{\lambda_2^2 - C} - D \lambda_0^2. \quad (\lambda_0 \text{ in } \mu \text{m}) \]

for the waveguiding layer \( X = 0.1 \)

\[ n_2^2 = (10.906 - 2.92 \times 0.1) + \frac{0.97501}{(0.9)^2 - (0.52886 - 0.735 \times 0.1)^2}
- 0.002467(1.41 \times 0.1 + 1)(0.9)^2 \]
\[ = 10.614 + 1.618 - 0.00228 \]
\[ = 12.23 \]
\[ n_2 = 3.49 \]

for the confining layer \( X = 0.83 \)

\[ n_3^2 = (10.906 - 2.92 \times 0.83) + \frac{0.97501}{(0.9)^2 - (0.00306 - 0.105 \times 0.03)^2}
- 0.002467(1.41 \times 0.83 + 1)(0.9)^2 \]
\[ = 8.482 + 1.2778 - 0.00434 \]
\[ = 9.76 \]
\[ n_3 = 3.12 \]

Thus \( n = n_2 - n_3 = 0.37 \)
(Solutions continued)

4.4 Solution (continued)

The number of modes is determined from

\[ \Delta n > \frac{(2M+1)^2 \lambda_o^2}{32 n_3 t^2} \quad \text{or} \quad \sqrt{\frac{\Delta n \cdot 32 \cdot n_3 \cdot t^2}{\lambda_o^2}} - 1 \geq M \]

\[ \sqrt{0.37 \times 32 \times 3.12 \times \frac{3}{0.9}} - 1 \]

\[ \frac{2}{y.61} \geq M \]

Therefore \( M = 0 \) \( \ldots \ldots \) 9

or 10 modes can be guided
4.5 Solution

\[ \lambda_o = 1.15\mu m \]

\[ n_2^2 = 10.614 + \frac{0.97501}{(1.15)^2 - (0.52886 - 0.735 \times 0.1)^2} \times 0.002467(1.41 \times 0.1 + 1)(1.15) \]

\[ = 11.48 \]

\[ n_2 = 3.39 \]

\[ n_3^2 = 8.482 + \frac{0.97501}{(1.15)^2 - (0.30386 - 0.105 \times 0.83)^2} - 0.002467(1.41 \times 0.83 + 1)(1.15) \]

\[ = 9.24 \]

\[ n_3 = 3.04 \]

\[ n = 0.35 \]

\[ \sqrt{0.35 \times 32 \times 3.04} \times \frac{3}{1.15} - 1 \geq M \]

7.1 > M \rightarrow M = 0 \ldots 7

or 8 modes can be guided.
4.7 Since the upper confining layer has both an Al concentration difference and a carrier concentration difference to produce optical confinement, while the lower confining layer has only a carrier concentration difference, the number of modes that can propagate will be determined by conditions at the lower interface.

There we have a required $\Delta n$

$$\Delta n = n_2 - n_3 > \frac{(2m+1)^2 \frac{\lambda_o^2}{t_o}}{32 n_3 t_q^2} = \frac{(2m+1)^2 (1.06 \times 10^{-6})^2}{32 \times 3.6 \times (5 \times 10^{-6})^2}$$

$$= (2m+1)^2 \times 3.9 \times 10^{-4}$$

The carrier concentration difference provides a $\Delta n$

$$\Delta n = n_2 - n_3 - \frac{(N_3-N_2) e^2}{2n_3 \varepsilon_o m^* \omega^2} = \frac{10^{24} \times (1.6 \times 10^{-19})^2 (1.06 \times 10^{-6})^2}{2 \times 3.6 \times 8.85 \times 10^{-12} \times 0.08 \times 9.1 \times 10^{-3} (2 \pi \times 3 \times 10^8)^2}$$

where $\omega = \frac{2 \pi c}{\lambda_o} = 1.75 \times 10^{-3}$

has been used

$$3.9 \times 10^{-4} \times 9 = 3.5 \times 10^{-3}$$

Thus only the $m=0$ mode can be guided
SOLUTIONS

4.9 If the waveguide in 4.4 were to be modified so as to support one addition mode, would it be practical to add the needed change in \( n \) using the electro-optic effect?

\[ r_{41} = 1.4 \times 10^{-12} \]

This waveguide currently supports 10 modes. The index of refraction needed to support 11 modes can be found from:

\[
\Delta n \geq \frac{(2m+1)^2 \lambda_0^2}{16(n_2+n_3) t_g^2} \quad \text{eq. (3.1.29)}
\]

\[
\Delta n \geq \frac{(411)(8.1 \times 10^{-12})}{16(6.61)(9 \times 10^{-12})}
\]

\[
\Delta n \geq \frac{3.57 \times 10^{-10}}{9.51 \times 10^{-10}}
\]

\[ \Delta n \geq 0.3753 \text{ needed to guide 11th mode. Without E-O effect } \]

\[ \Delta n = 0.37 \]

\[ \text{E-O effect must produce } 0.0053 = \Delta n \]

\[ \text{\( \uparrow \)seems small enough, but ...} \]

\[ \Delta n = n^3 r_{41} \frac{v}{2t_g} \quad \text{eq. (4.5.1)} \]

\[ v = \frac{2 \Delta n t_g}{n^3 r_{41}} = \frac{2(0.0053)(3 \times 10^{-6})}{(3.49)^3(1.4 \times 10^{-12})} \]

\[ = \frac{3.18 \times 10^{-8}}{5.95 \times 10^{-11}} \]

\[ v = 534 \text{v} \]

\[ \varepsilon = 1.78 \times 10^8 \text{ V/m} \]

This is much too large (it will cause breakdown). Therefore the E-O effect can't be used.
Chapter 5

5.1 What are the factors that determine the thickness of a polystyrene waveguide formed by spinning a solution of dissolved polystyrene onto a substrate?

Solution.
- density of polymer
- concentration of polymer in solution
- spin speed

5.2 An array waveguide multiplexer like that shown in Fig. 5.9 is designed to operate around a center wavelength of 1.55 µm and to have 20 channels spaced 1 nm apart in wavelength. The radius of curvature in the focusing slabs is 8 mm. The input and output waveguide spacing is 125 µm and the array waveguide separation is 7 µm. The index of refraction in the slab regions is 1.67. What is the required path length difference between adjacent waveguides in the array?

Solution.
\[
\Delta L = \frac{(n_s d D \lambda_0)}{(N_{ch} f \Delta \lambda)} = \frac{(1.67 \times 7 \times 10^{-6} \times 1.25 \times 10^{-4} \times 1.55 \times 10^{-6})}{20 \times 8 \times 10^{-3} \times 1 \times 10^{-9}} = 14.156 \times 10^{-6} = 14.156 \mu m
\]

5.3 What can be done in order to double the number of channels in the AWG multiplexer of Prob. 5.2 without changing the path length difference between the array waveguides? Describe all of the potential ways that it can be done. Assume that the center wavelength and the wavelength spacing between channels can not be changed.

Solution.
- \(\lambda_0 = 1.55 \mu m\) is fixed
- \(\Delta \lambda = 1 \text{ nm}\) is fixed

To increase \(N_{ch}\) to 40 you can do any one of the following:
- a. double array waveguide separation \(d\)
- b. double input and output waveguide separation \(D\)
- c. double index of refraction in slab \(n_s\)
- d. halve the slab radius of curvature
5.4 How is it that light emitting diodes of different wavelengths can be produced in the same polymer?

**Solution.**
The polymer can be doped with different substances that change its effective “bandgap and hence its emission wavelength.

5.5 What is a Fiber Bragg Grating (FBG)? Describe three different ways of producing Fiber Bragg Gratings.

**Solution.**
A Fiber Bragg Grating is a periodic multilayered structure formed in an optical fiber waveguide that has the wavelength-selective transmission properties of a grating. An FBG can be formed by exposing a step-indexed, germanosilica fiber to intense ultraviolet (UV) light. Three different methods for producing the required periodicity are interferometrically combining two coherent laser beams, using a phase mask, or using masked projection of a laser beam.
Chapter 6

6.1 If \( P = P_0 \exp(-\alpha z) \), where \( P_0 \) is the power at the input end of a waveguide and \( P \) is power as a function of distance traveled in the propagation direction (\( z \)), show that

\[
\alpha = \frac{\text{power lost per unit length}}{\text{power transmitted}}.
\]

Solution.

\[ P(z) = P_0 e^{-\alpha z} \]

The power lost per unit length is

\[
\frac{dP}{dz} = -\alpha P_0 e^{-\alpha z} = -\alpha P(z)
\]

\( \alpha = -\frac{dP/\,dz}{P(z)} \)

(\( dP/\,dz \) is negative for a loss, hence \( \alpha \) is positive).

6.2  Show that the relationship between attenuation coefficient \( \alpha \) (in cm\(^{-1}\)) and loss \( L \) (in dB/cm) is given by

\[ L = 4.3\alpha. \]

Fig. 6.9. Optical integrated circuit. The curved waveguides are each exactly 1/4 of a circle long for the radii given

Solution.
\[ P(z) = P_0 e^{-\alpha z} \]

\[ \therefore \ln \frac{P}{P_0} = -\alpha z \]

but loss \((\mathcal{L})\) in dB/cm is given by

\[ \mathcal{L} = 10 \log_{10} \frac{P}{P_0} = \frac{10 \log_{10}(e)}{z} \ln \frac{P}{P_0} = \frac{4.3}{z} \ln \frac{P}{P_0} \]

\[ \therefore \mathcal{L} = -\frac{4.3}{z} \cdot \alpha z = -4.3\alpha \]

6.3 In the optical integrated circuit shown above, all of the waveguides have the same cross-sectional dimensions and loss per unit length due to scattering and absorption. However, the curved waveguides have an additional loss per unit length due to radiation.

If the total loss between the following elements is:

- Between \(D\) and \(E\) \(L_T = 1.01\) dB
- Between \(C\) and \(D\) \(L_T = 1.22\) dB
- Between \(B\) and \(C\) \(L_T = 1.00\) dB.

What is the total loss \(L_T\) between elements \(A\) and \(B\)? (Neglect coupling losses – consider only waveguide loss as above.)

**Solution.** Loss \(B\) to \(C\) = 1.00 dB

Since this is a straight waveguide the only loss is that due to absorption and scattering.

\[ \mathcal{L}_{a,s} = \frac{1.00\ dB}{0.5\ cm} = 2\ dB/cm \]

**Loss C to D = 1.22 dB**

The loss due to radiation is

\[ \mathcal{L}_R = 1.22 \pi \times 0.6/4 - 2 = 0.589\ dB/cm \]

**Loss D to E = 1.01 dB**

\[ \mathcal{L}_R = \frac{1.01}{\pi \times 0.2/4} - 2 = 4.4298\ dB/cm \]

Thus we have two equations with unknowns \(C_1\) and \(C_2\)

\[ \mathcal{L}_R = 0.589 = C_1 e^{-C_2 \times 0.3} \quad (C \ to \ D) \]

\[ \mathcal{L}_R = 4.4298 = C_1 e^{-C_2 \times 0.1} \quad (D \ to \ E) \]

dividing the two equations
\[ 7.52088 = e^{C_2(0.2)} \]
\[ C_2 = 10.088 \]
\[ \therefore C_1 = \frac{0.589}{e^{-10.088 \times 0.3}} = 12.147 \]

**Loss A to B**

\[ L_{\text{Total}} = (L_{a,s} + L_R) \ell \]
\[ = (2 + 12.147 e^{-10.088 \times 0.2}) \times \frac{\pi \times 0.4}{4} \]
\[ = 0.507 + 0.628 = 1.135 \text{ dB} \]

**6.4** Describe the physical reason why radiation loss from a guided optical mode in a curved waveguide increases as the radius of curvature is reduced.

**Solution.** See Sect. 5.3.2 of text for explanation.

**6.5** A certain ribbed channel waveguide, 1-μm deep, is used in an OIC for guiding light of vacuum wavelength \( \lambda_0 = 6328 \text{ Å} \). Loss measurements made on test sections of the guide have shown that the loss coefficient in a straight sample is \( \alpha = 0.3 \text{ cm}^{-1} \), while in a curved section with radius of curvature \( R = 0.5 \text{ mm} \) it is \( \alpha = 1.4 \text{ cm}^{-1} \), and in a curved section with \( R = 0.3 \text{ mm} \) it is a \( \alpha = 26.3 \text{ cm}^{-1} \).

What is the minimum radius of curvature that can be used if \( \alpha \) must be less than 3 cm\(^{-1} \) at all points in the circuit?

**Solution.**

\[ \alpha_R = C_1 \exp(-C_2 R) \text{ cm}^{-1} \]

straight \( \alpha = 0.3 \text{ cm}^{-1} \)

(a) \( R = 0.5 \text{ mm} = 0.05 \text{ cm} \quad \alpha = 1.4 \text{ cm}^{-1} \quad \alpha_R = 1.4 - 0.3 = 1.1 \text{ cm}^{-1} \)

(b) \( R = 0.3 \text{ mm} = 0.03 \text{ cm} \quad \alpha = 26.3 \text{ cm}^{-1} \quad \alpha_R = 26.3 - 0.3 = 26 \text{ cm}^{-1} \)

(a) \( 1.1 = C_1 \exp(-C_2 \times 0.05) \)

Solving eq. (a)

\[ C_1 = \frac{1.1}{\exp(-C_2 \times 0.05)} \]

(b) \( 26 = C_1 \exp(-C_2 \times 0.03) \)

Substituting for \( C_1 \) in eq. (b)
\[ 26 = \frac{1.1 \exp(-0.03C_2)}{\exp(-0.05C_2)} \]
\[ \ln 26 - \ln 1.1 = -0.03C_2 + 0.05C_2 = 0.02C_2 \]
\[ C_2 = \frac{3.16}{0.02} = 158.14 \]
\[ C_1 = \frac{1.1}{\exp(-158.14 \times 0.05)} = 2787.75 \]
\[ \alpha_R = 2787.75 \exp(-158.14R) \]

\[ \alpha_R \text{ must be less than } (3 - 0.3) = 2.7 \text{ cm}^{-1} \text{ for total } \alpha \text{ less that } 3 \text{ cm}^{-1} \]
\[ 2.7 = 2787.75 \exp(-158.14R) \]
\[ R = \frac{\ln(2.7/2787.75)}{-158.14} = 0.044 \text{ cm} = 0.44 \text{ mm} \]

6.6  a) Sketch the index of refraction profile in a strip-loaded waveguide, indicating the relative magnitudes of the index in different regions.

   b) III-V semiconductor waveguide research has progressed to different materials in the order shown in the list below. What reason (i.e., advantage) motivated each step in this progression?

LPE GaAs waveguides on GaAs substrates
LPE Ga\(_{1-x}\)Al\(_x\)As waveguides on GaAs substrates
LPE Ga\(_{1-x}\)In\(_x\)As waveguides on GaAs substrates
MBE Ga\(_{1-x}\)In\(_x\)As\(_y\)P\(_{1-y}\) waveguides on GaAs substrates
(LPE – Liquid phase epitaxial; MBE – Molecular beam epitaxial)

Solution.
\[ n_1 > n_2 \text{ (bottom)} \quad \quad n_{\text{eff}} > n_2 \]
\[ \text{or } n_1 < n_2 \text{ (top)} \]
but need \( n_1 > n_0 \)

(b) LPE GaAs waveguides on GaAs substrates – optical waveguiding
LPE GaAlAs waveguides on GaAs substrates – emission and transmission of shorter wavelength
LPE GaInAs waveguides on GaAs substrates – emission and detection of longer wavelength
MBE GaInAsP waveguides on GaAs substrates – better lattice match for long wavelength emission and detection.

6.7 A certain uniform waveguide 2-cm long is transmitting an optical signal, the power of which is measured at the output of the waveguide to be 1 Watt. If the waveguide is cut so that its length is reduced by 10\%, the optical power at the output is found to be 1.2 Watts. What is the attenuation coefficient (in cm\(^{-1}\)) in the waveguide?

**Solution.**

\[
\alpha = \frac{\ln \frac{P_1}{P_2}}{Z_2 - Z_1} = \frac{\ln \frac{1.2}{1.0}}{Z_2 - (Z_2 - 0.1Z_2)} = \frac{\ln 1.2}{Z_2 - 0.9Z_2} = \frac{\ln 1.2}{Z_2 - 0.9Z_2} = \ln 1.2 \approx 0.91 \text{ cm}^{-1}
\]
6.8 Describe two different methods for separating the coupling losses from the propagation losses when experimentally measuring the losses of an optical waveguide.

**Solution.**
One method is to measure the relative transmission versus length in cleaved waveguide segments and then plot \( \ln \text{[relative transmission]} \) vs. length as in Fig. 6.8. The slope of the resulting curve in a straight line region is the propagation loss. The same procedure also can be performed with prism couplers as shown in Fig. 6.9.

6.9 A planar optical waveguide of length = 3 cm is observed to have a transmitted optical power of 5 mW. When the waveguide is cut in half, the optical power at the output of the remaining half is measured to be 5.5 mW.

(a) What is the loss coefficient of the waveguide in cm\(^{-1}\)?

(b) What is the loss of the waveguide in dB/cm?

**Solution.**

a) \[ \alpha = \frac{\ln(P_1/P_2)}{(Z_1 - Z_2)} = \frac{\ln(5/5.5)}{(3-1.5)} = -0.0635 \]

b) Loss\[\text{dB/cm}\] = 4.3 \( \alpha [\text{cm}^{-1}] \) = 4.3(-0.0635) = -0.273 dB/cm.