Exploring Fitts' Law in Web Browsing

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ABSTRACT

As one of the most influential laws in human-computer interaction for decades, Fitts' law quantitatively models human pointing behaviors. However, the majority of Fitts' law experiments have been conducted in a laboratory environment, which lends doubt to its validity in a real-world GUI (Graphical User Interface) context. This paper attempts to stress-test Fitts' law in the "wild", namely, under natural web browsing environments, instead of restricted laboratory settings. In our study, more than 1,000 users' mouse movement data are collected passively, during their online sessions of web browsing. Our analysis shows that, the *averaged* pointing times follow Fitts' law very well, with linear correlation above 98%. However, in *unaveraged* raw data, we observe considerable deviations from Fitts' law, with a 46.40% mean absolute deviation from predicted values. Based on the predicted pointing time from Fitts' model, we further classify the pointing actions into *fast*, *medium*, and *slow* movements. We observe that, in natural browsing, a fast movement has a different error model from the other two movements. This is different from previous findings in restricted environments. A complete profiling on user pointing performance should be done in more details, for example, constructing different error models for slow and fast movements.

1. INTRODUCTION

Web browsing is mainly driven by *target acquisition* – the movement of a pointing device, such as a mouse, touchpad, and stylus, to an on-screen target. The ability to quantify the way these pointing actions are performed has been studied heavily before. The previous research works have helped us better understand the effect human motor control has on pointing actions, with applications ranging from the design of more efficient graphical user interfaces (GUIs) [3], to the creation of accurate pointing devices [16], and human biometrics security [33, 47].

Fitts' law [14] is a classic and well-studied law, which quantifies pointing actions in terms of the size, distance, and time to reach the target. The most common formulation of Fitts' law comes from [30] and is referred to as the "Shannon formulation":

$$MT = a + b \cdot log_2 \left(\frac{A}{W} + 1 \right). \quad (1)$$

In Eq. 1, the log term is known as the index of difficulty (ID). Fitts' law describes a linear relationship between the mean time to complete the pointing action (MT) and the index of difficulty (ID) of the pointing task, which includes the distance to the target (A) and the width of the target in the direction of movement (W). The parameters a and b are environment- and user-specific.

This law has been one of the most widely used and most well-respected formulas in human-computer interaction, but it represents an idealized view of pointing actions. Nearly all of the experimental results currently in the literature regarding Fitts' law have been performed in extremely clinical settings – on a blank background, a single target square or circle appears, and a user must move the cursor from a starting position to the target as quickly as possible. This style of pointing, however, is not typical of real-world GUI applications. Typically, there are many potential targets on the screen at once. The environment is full of distractions, instead of a blank white screen, and the user spends time considering his or her next move, instead of pointing as quickly as possible. All of these environmental factors might affect the amount of time it takes for a user to complete a pointing action. There is very little research detailing how well Fitts' law applies, if at all, in such scenarios.

Moreover, Fitts' law is not merely a single equation in a vacuum. The law describes a linear model of human pointing actions, and a model cannot be defined by its mean alone. Other side metrics such as the standard deviation or variance of the model should also be considered when discussing Fitts' law, but the majority of the current literature seems to focus exclusively on the mean as presented by the common formulation.

This paper attempts to answer the question: how well does Fitts' law truly model real human pointing tasks in web browsing? We examine Fitts' law in a natural web browsing environment to determine its validity outside of a structured experimental setting. This is accomplished through a data set collected from 1,047 users' natural mouse traces on a real-world website. The major contributions of this work are summarized as follows:

- an application of the Fitts' law formula to pointing actions in a natural web browsing environment, involving a large-scale data collection from 1,047 real-world users on an Internet forum (Section 3.1), to assess Fitts' law's applicability to typical GUIs outside of an experimental setting (Section 4.1);

- an observation that in web browsing, *fast* movements have a different error model from *slow* movements, which deviates from previous laboratory studies. We speculate that this is partially due to the *open-loop* nature of fast movements (Section 4.2);

- a comparison of Fitts' law results for natural browsing using two different pointing devices − physical mouse and laptop touchpad − to determine whether the choice of pointing device has an effect on the linear relationship described by Fitts' law (Section 4.3);
- an analysis of the standard deviation of Fitts' law calculations of mean pointing time, to better understand the variance present in the Fitts model (Section 4.4).

The remainder of the paper is organized as follows. Section 2 describes the background of Fitts' law and web browsing behavior, and surveys related work. Section 3 details our data collection and processing. Section 4 evaluates the validity of Fitts law under natural web browsing, along with its proposed error model. Section 5 discusses several Fitts' law related issues. Section 6 concludes the paper.

2. BACKGROUND AND RELATED WORK

A large number of research works have been conducted to learn web browsing behaviors, with the goal of measuring user interests [6, 28, 36], web page quality [39], search quality [1, 21], predicting user demographic [20], and providing personalization [40]. The existing studies are heavily based on information of pageview activities, including pageview paths, time spent on webpages, frequencies of webpage visiting, etc. By contrast, in this paper, we explore human browsing behavior from a different perspective: the kinetics of user point-and-click actions in web browsing. Our work is useful in complementing previous works to better model user browsing behavior in a more comprehensive manner.

We focus on studying Fitts' law, one of the most influential laws in human-computer interaction research for decades. Essentially, Fitts' law reveals the length of time it takes to perform a task with a pointing device such as a mouse. For instance, how long does it take to move the mouse cursor to a particular position on the screen. It is expressed as in Eq. 1. The significance of Fitts' law is that it provides quantitative information regarding the accumulated time of multiple perceptual-motor feedback cycles for users to interact with a system using a pointing device.

Fitts' law is closely related with several theories on submovement analysis, and the major ones, in chronological order, consist of, *the iterative corrections model* [8, 24], *the impulse variability model* [35] (also known as the Schmidt's law), and *the optimized initial impulse model* [32] (also known as the Meyer's law). A comprehensive review of the three models can be found in [34]. The first two models emphasize on either solely feedback control or solely initial impulse, while the third model (the Meyer's law) combines these two views, and gave a satisfactory explanation supported by empirical evidence. Therefore, the underlying message from Fitts' law is an optimal planning of human motor-control bounded by speed-accuracy tradeoff[1]. In another word, even in a task as simple as reaching for a target, human motor skill automatically balances the speed and accuracy in an optimal way, with an outcome of target-reaching both accurately and rapidly.

Note that Schmidt's and Meyer's laws can serve as independent models for pointing actions, and they are closely related to Fitts' law. The former is for rapid pointing, and the latter constitutes a more generalized law (combining both Fitts' and Schmidt's laws). However, in terms of application, Schmidt's and Meyer's laws require submovement analysis at low level, while Fitts' law only involves measurements (i.e., *total* movement time, *total* distance, and target size) without any submovement-level variables. Therefore, due to its simplicity and ease-of-use, along with its success in many aiming-related experiments [2, 23, 25, 26, 29, 34], we have chosen Fitts' law as the focus of this study, instead of the other laws. And our objective is to specifically verify if Fitts' law alone is applicable for modeling daily pointing actions with computer mice in a natural web browsing environment.

[1]In presence of speed-accuracy tradeoff, one cannot accurately aim for a target with no error while moving extremely fast.

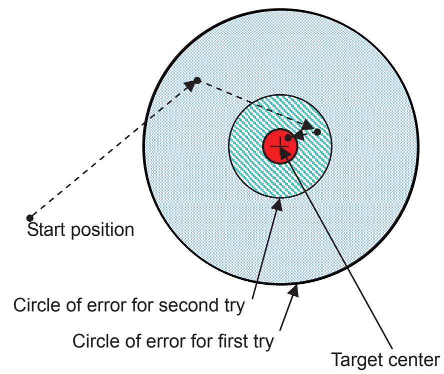

Figure 1: Step-wise movement towards target [11]

Here for simplicity, we present the first model, i.e., the iterative corrections model, proposed by Crossman and Goodeve [8] in 1963. This is not a perfect model in itself, but it especially reveals how the logarithmic term in Fitts' law came from perceptual-motor feedback loops. Figure 1 shows how the movements in each step get gradually smaller as the target gets closer, involving discrete cycles of sensing and movement [11].

It is assumed that each of submovement reduces the distance to the target geometrically, that is, it moves a constant fraction $1 - r$ of the remaining distance. Each of them takes the same time t. When remaining distance is such that the error circle of the remaining movement is less than the size of the target then we get inside the target. When the target has been reached:

$$r^N A = \frac{W}{2}.$$

Solving for N:

$$N = \frac{1}{\log_2 1/r} \log_2 \frac{2A}{W}.$$

The time required for completing all submovements are:

$$T = Nt = \frac{t}{\log_2 1/r} + \frac{t}{\log_2 1/r} \log_2 \frac{A}{W}.$$

This especially explains the logarithmic term in Fitts' law.

A large body of prior works have been dedicated to Fitts' law, since it was first proposed in 1954 [14]. Seminal works in the HCI (human-computer interaction) community include [4], [38], [43], [45], etc. However, only a few of existing literatures concern with real-life pointing behaviors, based on unobtrusively collected data.

Chapuis *et al.* [7] are among the first to notice the need to stress-test Fitts' law in natural GUI settings. They questioned if one can apply the Fitts' law obtained from controlled laboratory experiments to characterize the pointing activities "in the wild". Their underlying motivation is the same as ours, pointing "in the wild" involves far more cognitive processes than in a controlled laboratory setting, such as deciding what is the target, coping with possible interference from the field environment or planning for higher-level tasks. In their field study of 24 users, the results indeed deviate from those in controlled laboratory environments.

Slijper *et al.* [37] applied Bayesian statistics to model hand movements, drawn from a large-scale collection of users' daily mouse movements. Human arm movements are found to be strongly correlated to prior experience, making them predictable via Bayesian statistics analysis. Thus, Slijper *et al.* achieved their primary goal, which is to predict hand moving directions by utilizing the directional distribution.

Hust *et al.* [22] conducted another field study on the evaluation of real-life pointing performance for motor-impaired people. High variance is found within each participant, implying that it is insufficient to measure performance based on a single laboratory session. However, as target width and distance are not captured in the data collection, the theme of their study is not focused on the evaluation of Fitts' law. Moreover, since the experimented subjects are with motor impairments, it is unclear if a healthy person will still display considerable variance.

More recently, the differences between the natural and laboratory controlled mouse movements are further acknowledged in Gajos *et al.*'s work [17]. They found that, many of those mouse pointing movements "in the wild" are affected by extraneous factors, which include, for example, deciding what task to perform next and searching for the right user interface element. Motivated by this observation, the authors unobtrusively collected mouse pointer trajectories from 18 participants. A classifier is then trained to discriminate between deliberate, targeted mouse pointer movements and those movements that are distracted.

Meanwhile, Evans and Wobbrock [13] developed the *Input Observer*, a novel tool to passively collect user input data, from which they measured both text entry and mouse pointing performance "in the wild". With regard to pointing performance, the authors carefully measured target size and pointing errors by utilizing crowdsourcing. In the process of raw pointing data, a novel segmentation technique is employed to identify each trial, and further, outlier removal is used to damp the noise. As a result, the pointing performances "in the wild" are measured to be very close to that from laboratory studies, in terms of average pointing error, movement time, and throughput. However, in their work, only a small subset of collected data ($\sim$11% of pointing data) are considered in order to obtain laboratory-quality results; by contrast, in this paper, as our goal is to see if Fitts' law works well in regular web browsing activities, there is minimal data filtering done.

Overall, our paper significantly differs from these previous works in terms of scale. Whereas the largest data set of the previous studies includes 24 volunteer users, our study involves two orders of magnitude more human users. There are more than 1,000 participants who are real-world Internet users, and their pointing actions are recorded while using a web browser. Moreover, we solely focus on pointing behaviors in web browsing, while the previous works study pointing data from many different applications at the client side.

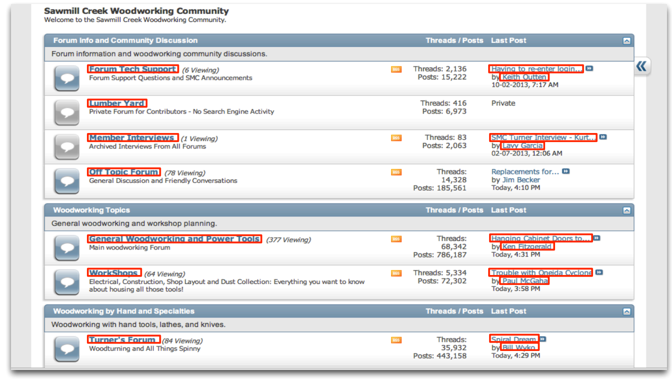

Figure 2: Layout of the webpage for data collection. Text links are highlighted by red boxes.

3. DATA COLLECTION

This paper includes two different data sets, obtained under two different circumstances, for the purposes of two different types of measurements. The data sets are described below, followed by an explanation of how the data is processed into a meaningful form.

3.1 Data Sets

In order to assess Fitts' law in a real-world setting, we first collect data outside of controlled laboratory conditions. More than 1,000 unique Internet users' mouse movements are recorded by JavaScript code embedded on a web forum [2], and submitted passively via AJAX requests to the web server. Figure 2 shows the layout of the website homepage. Based on the vBulletin template, the webpage has a simple outlook comprised of text links stacked vertically. Icons and images are present too, but not visually prominent as text links. It is impossible to know about online users' biographical information (gender, age, education background), and there is no guarantee on the amount of data collected for a certain user (a forum user could be logged in for a long time with frequent mouse activities, or could perform just one click and then leave). On the other hand, the breadth of this corpus represents a large sample of real Internet users browsing naturally, as they were free to browse webpages without being interrupted. Thus, it is ideal for studying how well Fitts' law models natural pointing behavior when using a web browsing application.

The second data set is collected to study the effect on Fitts' law of pointing actions using different pointing devices. This data collection is conducted in a controllable environment. Ten people are invited personally to participate in the second set of data collection. They use two different pointing devices, mouse and touchpad, to interact with

[2] The website is generated from vBulletin forum software, and has a similar layout as `https://www.vbulletin.com/forum/forum.php`. In addition, we do not argue that this is a representative website, but it suffices as a case study for exploring the Fitts' law in the wild.

GUIs. During the data collection, the users' activities are performed naturally without any interference. Their pointing actions are recorded using the RUI tool [27].

3.2 Data Processing

The raw mouse movements are represented as the tuples of timestamp and Cartesian coordinate pairs. Each tuple is in the form of $\langle \text{ACTION-TYPE}, t, x, y, \text{TARGET-TYPE} \rangle$, where ACTION-TYPE is the mouse action type (a *mouse-move* or *mouse-click*), t is the timestamp of the mouse action, x is the x-coordinate, y is the y-coordinate, and TARGET-TYPE is the type of clicked target (a text link, text, image, or form element). All timestamps are collected in milliseconds, and coordinates in pixels.

These raw data points then are further processed to extract the pointing actions. Here we choose to consider only *point-and-click* actions with a text link target, because the overwhelmingly dominant majority of pointing targets in the forum website are text links. A point-and-click is defined as a continuous movement followed by a click. A continuous movement is defined as a sequence of movement with little to no pause between the beginning and end of the movement. Here "little to no pause" means that the time lapse is less than 100 milliseconds between any two adjacent mouse records. Here 100-millisecond is an empirical threshold, which is roughly the shortest time scale of human perception [5]. The time for a point-and-click action is measured from the first mouse-move event to the last mouse-move event. In our data collection algorithm, raw mouse records are only generated with either mouse-move or mouse-click events. Therefore, with respect to a pointing action with multiple pauses, as long as the period of zero-velocity is less than 100ms (which is true in most cases), multiple submovements are still preserved as one point-and-click. We only choose point-and-click actions because a movement that ends in a pause (rather than a click) could just be the user idly "shaking" the mouse cursor or moving it out of the way on the screen. These types of actions do not have a definite target, so they are not covered by Fitts' law. Conversely, if a user clicks at the end of the movement within a web browser, highly likely it is clicking on a text link at the forum website, and thus we can assume that the pointing action has an aiming target—the text link. Note that our collected pointing data are associated with clicking *on* text-links, as missing the target would fail to send a server request. The only exception here is when a user clicks on a text-link accidentally while aiming for a different GUI target, but this is very unusual. Therefore, we assure that each collected point-and-click action within a web browser has a definite target and is error-free.

For each of these point-and-click actions, we calculate the time to complete the pointing action as the difference in timestamp from the first mouse-move event to the last one before the click. The ID is then calculated from the distance, the difference in Cartesian coordinates between the beginning of the movement and the click, and the target size. It is important to measure the target size as accurately as possible. Several models have been proposed to extend Fitts' law to two dimensional targets. It has been evaluated by [30, 44] that one of the best model is this formula:

$$T = a + b \cdot log_2(\frac{D}{min(W,H)} + 1),$$

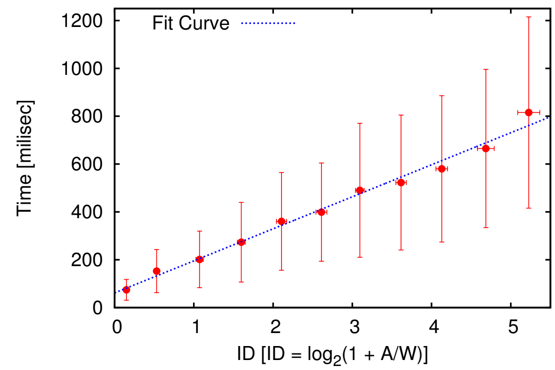

Figure 3: ID vs. Mean Time plotted for the forum users' data set. The linear correlation is over 98%. Every data point in the figure is averaged over 180 raw data points.

where $min(W,H)$ is the the smaller dimension of the height (H) and width (W) of a rectangle target. This model reflects the wide founded intuition that the smaller of H or W should dominant overall performance. For this experiment, we filter all point-and-clicks so that only those with a text-link as target are considered. Of all clicks, 17 pixels is the font height of the text hyperlink, which is the smaller dimension of the text link as a clicking target. For this very reason, we choose 17 pixels as a universal target width for all clicks on a text link.

To get the mean movement time MT in Fitts' formula Eq. 1, raw data must be averaged. We group the time to point values by ID into buckets of width 0.5, then each bucket is averaged to produce a single mean time (MT) point on the graph.

4. EVALUATION

Fitts' law describes a linear relationship between the index of difficulty (ID) of a pointing task and the mean time (MT) to complete that task. We may not know the values of parameters a and b ahead of time, but we know the fit should be linear. Thus, to evaluate how well Fitts' law applies on a given data set, we plot the MT vs. ID of the pointing actions extracted from the data set. We calculate the correlation coefficient (r) for each plot to measure how closely it fits a linear function. An r value of close to 1.0 means perfectly linear data (100% linear correlation), while an r value of close to 0.0 means completely random data (0% linear correlation).[3]

4.1 Fitts' Law in Natural Browsing

How well does Fitts' law apply in a natural web browsing environment? To answer this question, we evaluate the linearity of the first data set, containing 1,047 real-world forum users' data. Figure 3 shows the mean movement time (MT) as a function of ID, where error bars show the standard de-

[3]The correlation coefficient can actually range from -1 to 1; an r value close to -1.0 would mean the data perfectly fits a line with negative slope (i.e., an inverse relationship). All data discussed in this paper has a positive slope (i.e., a direct relationship).

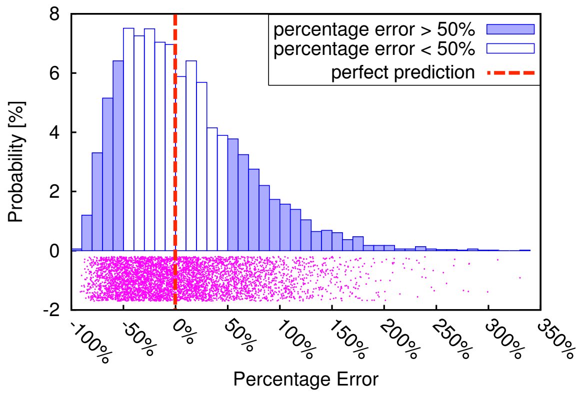

Figure 4: Probability distribution of $E_{\%}$, the relative deviation from Fitts' prediction, from raw movement times of natural web browsing. Histogram bars denote the portion of $E_{\%}$ with values in the corresponding block. Below the histogram is the scattering of raw data, where the location of dense area (between -50% and +50%) indicates the most probable values of $E_{\%}$.

viation of movement time. The data fit a linear regression with r showing a 98.28% correlation, which strongly suggests that the forum data follows Fitts' law. We can safely conclude from this that Fitts' law is robust enough to have real-world applications, not just under contrived laboratory situations. The a and b coefficients are 48.00 ms and 145.84 ms/bits, respectively. Here the non-zero value of parameter a is partially due to fact that Fitts' law does not apply to movements with very low IDs [8].

However, without averaging, Fitts' law has a poor fit. Given an actual raw movement time T_e, we define *percentage error* to measure the relative deviation to the Fitts' prediction MT:

$$E_{\%} = 100\% \times \frac{T_e - MT}{MT}. \tag{2}$$

A perfect prediction from Fitts' law corresponds to $E_{\%} = 0$. The larger the absolute value of $E_{\%}$ is, the more T_e deviates from Fitts' prediction. Figure 4 shows the probability density of percentage error $E_{\%}$ (defined in Eq. 2), with the scattering of raw data in the lower panel. The vertical line at $E_{\%} = 0\%$ (means zero error) indicates a perfect prediction. The percentage errors from Fitts' law to raw data span from -100% to 350%, and are highly concentrated between -50% and +50%. This wide range of scattering in terms of percentage error demonstrate, when applied to natural browsing, Fitts' prediction is not very accurate and a proper error model should be taken into account.

To measure the average deviation from Fitts' model, we define the *mean absolute percentage error* (MAPE) as:

$$M = \frac{100\%}{n} \sum_{i=1}^{n} \left| \frac{T_{e,i} - MT_i}{MT_i} \right|, \tag{3}$$

where n is the number of data points, $T_{e,i}$ is the ith actual movement time, and MT_i is the Fitts' prediction corresponding to the ID of ith data point. By averaging all 5,270 raw records, we have an MAPE of 46.40%. This is a significant deviation and cannot be ignored. In other words, when estimating user movement time on webpages using Fitts' model, one should take into account a relative error of around ±46.40%.

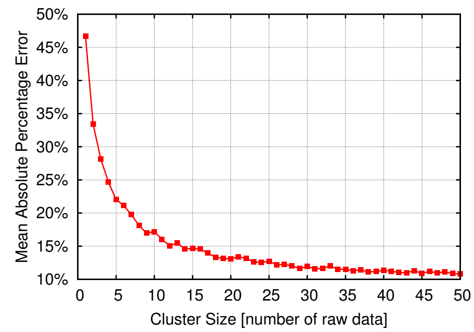

Figure 5: Mean absolute percentage error (MAPE) drops exponentially with the increase of cluster size. A cluster size S means every S raw data are clustered and averaged. Note that above 40 actions per cluster, MAPE stabilizes at around 10%.

Of course, the above deviation from Fitts' model are all based on *raw* data points, not *averaged* ones as in Fitts' original definition. And from a practical point of view, if one intends to utilize Fitts' law as a model on raw data, it is crucial to ask: how should raw data be clustered and averaged? How would the size of clusters affect the accuracy of Fitts' law?

As in our case ID is continuous, we have to partition the observations along the ID axis before averaging raw movement time T_es. The raw data (represented as pairs $\langle ID, T_e \rangle$) are clustered and averaged by groups of size S, ranging from 1 to 50. For instance, a group size of 10 means every 10 adjacent records in the assorted data are averaged. (In Evans and Wobbrock's work [13], a more sophisticated clustering technique is employed; by contrast, as our goal is not to produce laboratory-quality results, but to verify if Fitts' law works well in web browsing, we choose a relatively straightforward and simple clustering method.) Intuitively, when more data are included for averaging in each group, the closer the averaged data should be to the Fitts' predicted values, which leads to a smaller mean absolute percentage error (MAPE). Figure 5 plots MAPE as a function of cluster size, with its value from 1 to 50. It confirms our expectation, and furthermore, MAPE drops with an exponential rate with the increasing number of actions in a cluster. Note that with a cluster size of above 40, MAPE stabilizes at around 10%.

4.2 Error model

In fact, the large deviation of raw movement time from Fitts' prediction is mentioned in many previous studies [7, 11] under laboratory settings, but few of them elaborate in details on this issue. And it is learned that the deviation from Fitts' model is due to endpoint variability in human movement. More specifically, the spread of hits in aimed movement forms a Gaussian distribution about the target center [8, 14, 18, 30, 41, 46].

An *error* in aimed pointings occurs when a user misses a target. Errors are the outcome of endpoint variability. An error model aims to predict error rates – the probability

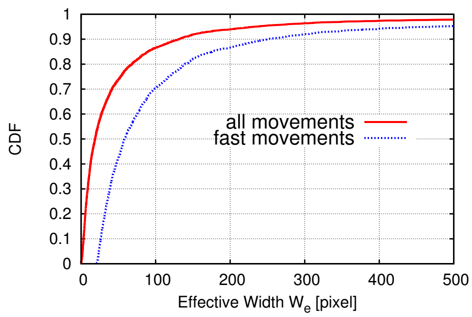

Figure 6: CDF for the effective width W_e for all movements and for the fast movements. Note that both distributions are not Gaussian, as were assumed in prior works.

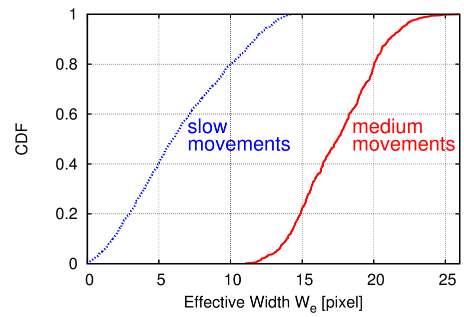

Figure 7: CDF for the effective width W_e for the medium and slow movements.

of missing a target – given the index of difficulty ID and the movement time. Intuitively, a higher ID (meaning a more difficult pointing task) and a shorter movement time (meaning a faster pointing action), lead to a higher error rate.

In particular, Wobbrock *et al.* [42] derived an error model from Fitts' law itself, based on the assumption that distance from endpoint to target center forms a Gaussian distribution. They evaluated their error model through a series of controlled experiments, and it is proved to be a very well fit. By employing the "effective movement time" from [41, 42], Fitts' law (Eq. 1) can be rewritten as

$$T_e = a + b \cdot \log_2\left(\frac{A}{W_e} + 1\right), \tag{4}$$

where W_e, is the *effective target width* coincident with the *actual* (unaveraged) movement time T_e. Solving for W_e, we have

$$W_e = \frac{A}{2^{\frac{T_e - a}{b}} - 1}. \tag{5}$$

Introducing W_e allows for mathematically growing or shrinking the effective target, to correspond to a *actual* movement time. Under the heuristic that W_e follows a normal distribution about the real target width, Wobbrock *et al.* are able to derive an accurate error model for laboratory controlled movements.

In this paper, we intentionally follow Wobbrock's error model as a guideline to examine if pointing actions "in the wild" can be interpreted with the same error model in laboratory studies. And we find that, different from laboratory results, in a natural browsing environment, fast movements have a different error model from slower movements. To further understand the high uncertainty of the actual movement time T_e with respect to Fitts' prediction (shown in Figure 4), we define three categories of movements: one is with better pointing performance (i.e., shorter movement time) than Fitts' prediction (*fast movements*), one is with nominal performance comparative to Fitts' prediction (*medium movements*), and the third one with worse performance than Fitts' prediction (*slow movements*).[4] They are defined as follows:

$$\begin{cases} T_{\%} < 0.9 & \rightarrow \text{fast movements;} \\ 0.9 \le T_{\%} \le 1.1 & \rightarrow \text{medium movements;} \\ T_{\%} > 1.1 & \rightarrow \text{slow movements.} \end{cases}$$

Our choice of the boundary values above is based on the human users' perception of fast, medium, and slow movements. Given the near-perfect fit on the average movement time (refer to Fig. 3), the Fitts' predicted value is set as the nominal case. Thus, fast movement corresponds to moving towards a target faster than Fitts' prediction (better pointing performance); slow movement is with a slower average speed than Fitts' prediction (worse pointing performance); and medium movement is the average case in accordance with Fitts' prediction (typical pointing performance). And our intention of dividing into the three categories is to explore how the range of deviation from Fitts' law affects their error models. Of all 5,084 raw data records, there are 45.44% fast movements, 12.84% medium movements, and 41.72% slow movements. Faster and slower movements have distinct nature, because of the difference in *closed-loop* and *open-loop* movements [18, 30, 42]. An aimed movement by human consists of multiple loops of feedback from neural system (to determine the next submovment) and constant motor fine-tuning. Open-loop movements are without fine-tuning, while close-loop movements are in contrast with careful fine-tuning. Setting Fitts' prediction as a standard bar, we speculate that faster movements are more prone to open-looped.

For each of the categories, we calculate the effective width W_e by Eq. 5 and examine its statistical properties. We apply the values of parameters a and b we get in Section 4.1. Note that in Eq. 5, for those records with $T_e < a$, which implies a very quick point-and-click, $W_e < 0$. We first filter out those records with a negative W_e, which represent 3.53% of all data. Figure 6 plots the distribution of effective width W_e. As we can see from the CDF curve for *all* movements, it does *not* readily follow a normal distribution as prior works assumed. The same applies to fast movements as one category. Figure 7 shows the distributions for slow and medium movements, which are Gaussian-shaped, with medium movements more Gaussian-like. Overall, it is clear that fast movements follow a completely different statistical model from other two categories, which implies a different error model.

The derivation of the Fitts' law error model from Wob-

[4]This is in the same spirit with Card, Moran & Newell's division of three imaginary humans according their HCI performance: Fastman, Slowman and Middleman [5].

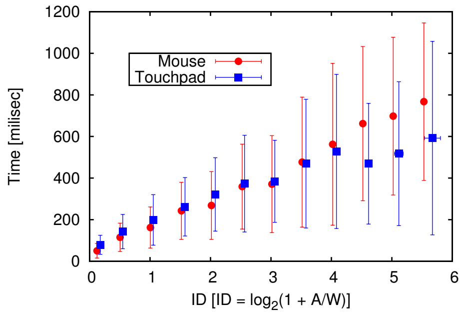

Figure 8: ID vs. Mean Time plotted for all collected physical mouse traces and all touchpad traces. Each data point in the figure is a result of averaging 180 raw data records. Note that the linear correlation is similar for both devices (over 98% in both cases).

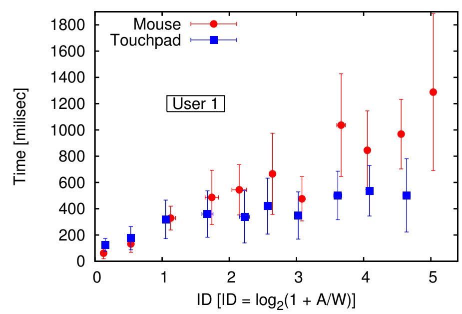

Figure 9: ID vs. Mean Time plotted for User 1's physical mouse trace and touchpad mouse trace. Each data point in the figure is a result of averaging 10 raw data records.

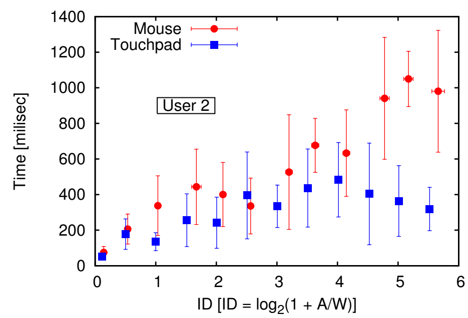

Figure 10: ID vs. Mean Time plotted for User 2's physical mouse trace and touchpad trace. Each data point in the figure is a result of averaging 10 raw data records.

brock *et al.* [42] then becomes:

$$P(E) = 1 - \int_{-z}^{+z} f(x)dx,$$

where

$$z = 2.066\frac{W}{A}\left(2^{\frac{T_e - a}{b}} - 1\right),$$

and $f(x)$ is the probability density function of the distance from target center, which is not necessarily a normal distribution.

Regarding the observed distinction between faster and slower movements, it is evident that Fitts' law does not model fast movements well. Rather, faster movements tend to be dominated by initial impulse, which is more fit into Schmidt's law [35].

4.3 Effect of Pointing Device

How does the choice of pointing device affect Fitts' law? While many early Fitts' law studies were performed using a stylus [15], recent studies have used many other pointing devices, including physical mice [4]. Does this choice affect the accuracy of the Fitts model? What about the use of a laptop touchpad? To answer this question, we evaluate the linearity of the second data set, containing both physical mouse and touchpad traces from across 10 people, for a total of 8 mouse traces and 7 touchpad traces. Figure 8 shows the combined physical mouse data plotted alongside the combined touchpad data. The physical mouse data show a linear correlation of 99.71%, a nearly perfect linear fit. The touchpad data matches the physical mouse data for low ID pointing tasks, but diverges somewhat for high ID pointing tasks. However, its r value still shows a 98.04% linear correlation, strongly suggesting that Fitts' law accurately models pointing tasks with either type of pointing device. The close similarity between their correlation coefficients indicates that the choice of pointing device does not impact the law's applicability – again, showing that Fitts' law is robust to different real-world environments.

The divergence of touchpad data at high ID values implies that, in reaching for a distant hyperlink (which leads to a high ID), touchpad is different from a traditional mouse. In the case of using touchpad, a user tend to make multiple swipes before reaching a distant target. While using a traditional mouse would usually be completed in a single swipe. In the touchpad data, what we have recorded as "continuous movement" is actually the last swipe, in which the user is likely to speed up, and that makes MT smaller than Fitts predicted.

One can also compare the traces recorded from one individual on two different pointing devices, for those users who recorded both physical mouse and touchpad data. There are three such users. Figure 9 shows the results of plotting User 1's mouse and touchpad traces. The relationship shows a similar trend as in the combined case – the data matches at low ID values but diverges at high ID values. However, the data still appears to be mostly linear.

Figure 10 shows the results of plotting User 2's single mouse trace and touchpad trace. Here, the mouse trace is linear as expected, but touchpad trace diverges greatly, even showing a downward nonlinear curve at high ID values. This shows that even for a single user, environmental factors can cause a difference in pointing actions from trace to trace, but when averaged the pointing actions all fit the Fitts model.

Figures 11, 12 and 13 show the results of plotting User 3's single mouse trace and two touchpad traces. All of this user's data follows a tightly correlated strong linear relationship. This shows that it is possible for a user's pointing actions

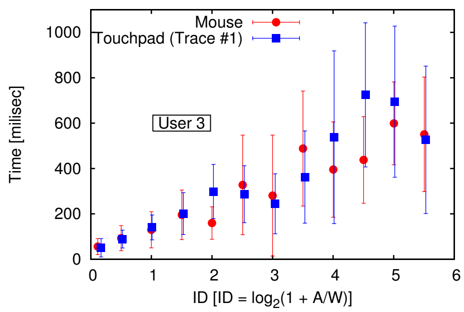

Figure 11: ID vs. Mean Time plotted for User 3's physical mouse trace and touchpad trace #1. Each data point in the figure is a result of averaging 10 raw data records.

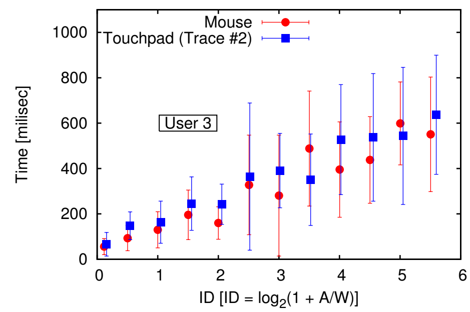

Figure 12: ID vs. Mean Time plotted for User 3's physical mouse trace and touchpad trace #2. Each data point in the figure is a result of averaging 10 raw data records.

using a touchpad to very closely match his or her pointing actions using a physical mouse. Thus, the use of different pointing devices does not break the Fitts model.

Note that the two users in our measurement above point faster with touchpad than mouse (refer to Figs. 9 and 10). This somewhat deviates from the observation of previous works [12, 31], where users point slightly faster with mouse than touchpad. Two reasons might apply here. Firstly, in our measurement, a user's movements are confined in the browser window. As touchpad is a friendly environment for short-range movement (i.e., one finger stroke), it is no surprise to see some faster pointings with touchpad than mouse within the browser window. Secondly, both touchpad and mouse devices have been greatly improved than two decades ago (when previous experiments [12, 31] are done), and thus it is quite possible that the observation made two decades ago does not hold nowadays in some scenarios.

4.4 Standard Deviation of Movement Time

The Fitts' law formula describes the mean time to complete a pointing action, and thus Fitts' law research in general focuses mainly on metrics involving the mean. However, a model is not defined solely by its mean value; there are also other considerations to take into account. For the purposes of this paper, we focus specifically on the standard deviation (and, by extension, the variance) of the time to complete a pointing action given by the Fitts' model under a natural web browsing environment.

To analyze the variance present in the Fitts model, we perform the same Fitts' law calculations on the first data set containing 1,047 forum users' traces. This time, however, we plot the ID of the pointing task versus the *standard deviation* of the time to complete the pointing action, rather than the *mean* time to point. Figure 14 shows the results. Clearly, Fitts' law describes not only a linear relationship between ID and the mean pointing time, but also a linear relationship between ID and the standard deviation of pointing time. In other words, the higher the ID of a pointing task, the more variance there is in the time it takes different users to complete such a task. This can be explained by signal-dependent noise in human neuromotor systems [19], which increases with the growth of control signal strength.

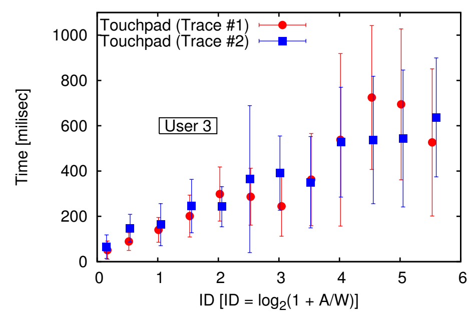

Figure 13: ID vs. Mean Time plotted for User 3's touchpad traces #1 and #2. Each data point in the figure is a result of averaging 10 raw data records.

5. DISCUSSION

In this section, we discuss four related issues of Fitts' law in the context of web browsing: firstly, we derive an analytical estimate on the inaccuracy in our distance measurement, and further show that the caused inaccuracy is minor overall; secondly, in acknowledge of the difference between web browsing and traditional Fitts' law experiments, we provide a guideline on how to apply Fitts' law to web browsing; thirdly, as touchscreen (on mobile phones and tablets) has become increasingly popular for web browsing, we discuss if it is possible to extend Fitts' law to touchscreens; lastly, we present the Fitts related parameters for certain users, and preliminarily explore the possibility of user profiling.

5.1 Inaccuracy on Distance Measure

In our experiment, the distance to the target (A in Eq. 1) is measured as the distance from beginning of the continuous movement to the clicked endpoint. However, ideally A should be the distance from starting position to the *center* of the target. Due to human movement variance, the clicked endpoint could not be exactly at the target center. In fact, it has been found in previous works [8, 14, 18, 30, 41, 46] that distance of the endpoint to the target center forms a normal distribution. Therefore, in our measurement, inaccuracy on the true distance A occurs especially when clicking on long hyperlinks. Figure 15 shows that, when clicking on a very long link, a user could either take path #1 to the left of the

User	Gender	a [ms]	b [ms/bits]	Correlation R^2	MAPE	Movement Categories Slow	Medium	Fast	Device
# 1	Male	6.28	117.14	0.933	41.73%	41.33%	13.68%	44.99%	Mouse
# 2	Male	103.08	138.05	0.919	28.18%	40.13%	19.11%	40.76%	Mouse
# 3	Male	-21.68	242.52	0.936	33.24%	43.80%	14.60%	41.61%	Mouse
# 4	Female	7.09	194.50	0.971	34.80%	32.65%	16.41%	50.94%	Mouse
# 5	Female	50.52	118.74	0.963	42.59%	41.54%	13.03%	45.42%	Mouse

Table 1: Fitts Law Related Parameters for Users of Mouse

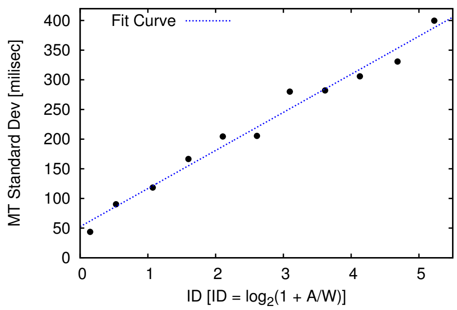

Figure 14: ID vs. Standard Deviation of Fitts' law calculations for the forum users' data set. Note the linear relationship.

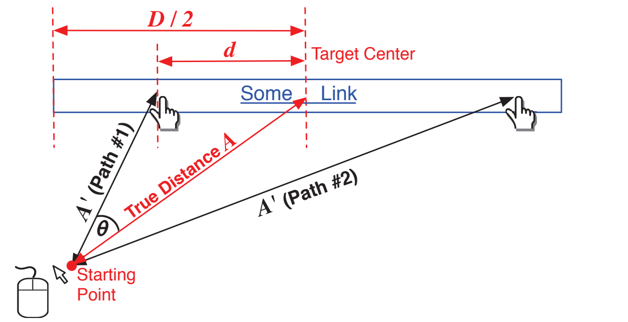

Figure 15: Click inaccuracy.

target center, thus rendering an underestimated distance; or he/she could take path #2, resulting in an overestimated distance. Overall, we believe that the inaccuracy on distance in our experiment is minor, since a typical webpage should be dominated by short links which only contain one or two words.

In particular, when link length is much less than the true distance to target center, the deviation of our measurement to the true distance is negligible. We denote D as the horizontal width of the target link, A as the true distance to target center, they are illustrated in Figure 15. For a certain *actual* click action, we denote the distance from the clicked endpoint to target center as d. Angle θ is angle between the line of true distance and the line of actual path taken. The relative error of measured distance A' to the true distance A is

$$\delta A = \frac{\|A - A'\|}{A}. \tag{6}$$

From the law of cosines, we have

$$A' = \sqrt{A^2 + d^2 - 2A \cdot d \cdot \cos\theta}. \tag{7}$$

Therefore,

$$\begin{aligned}(\delta A)^2 &= \frac{(A - A')^2}{A^2} \\ &= \frac{A^2 + (A^2 + d^2 - 2A \cdot d \cdot \cos\theta)}{A^2} \\ &\quad - \frac{2A\sqrt{A^2 + d^2 - 2A \cdot d \cdot \cos\theta}}{A^2} \\ &= 2 + (\frac{d}{A})^2 - 2 \cdot \frac{d}{A} \cdot \cos\theta \\ &\quad - 2\sqrt{1 + (\frac{d}{A})^2 - 2 \cdot \frac{d}{A} \cdot \cos\theta}. \end{aligned} \tag{8}$$

In case of clicking on short links with a horizontal length D, we assume $D \ll A$. For a successful click, we have $d < D/2$. Thus $D \ll A$ means $d \ll A$, or $d/A \to 0$. Define $\varepsilon = d/A$, Eq. 8 becomes

$$(\delta A)^2 = 2 - 2\sqrt{1 + \varepsilon^2 - 2\varepsilon \cdot \cos\theta} + \varepsilon^2 - 2\varepsilon \cdot \cos\theta. \tag{9}$$

As we can see, since $\|\cos\theta\| \leq 1$, when $\varepsilon \to 0$, which corresponds to short links, $\varepsilon \cdot \cos\theta \to 0$ as well. Therefore, the relative error of our distance measure $\delta A \to 0$.

5.2 Guidelines on Applying Fitts' Model to Web Browsing

From our experiments, we learn that the way one can apply Fitts' law to web browsing is different from what previous works describe for restricted laboratory settings. Therefore, we summarize a suggested guideline on how to apply Fitts' law model in web browsing as follows:

1. **Data Collection**: Besides x-y coordinates and timestamps, target types must be recorded as well, as it is needed to measure the target width.

2. **Clustering and Averaging**: Sort all records with increasing IDs, choose a proper cluster size S (our results show that $S > 40$ yields optimal results), then average every S raw data.

3. **Linear Regression**: Plot the averaged ID and MT pairs, fit them in a straight line, and calculate parameters a and b. Note that they are user- and environment-specific.

5.3 Possible Extension to Touchscreens?

Touch screen in tablets and mobile phones has become an increasingly popular device for web browsing. However, with a touch screen, to reach for a link on a webpage, users usually slide and tap. This is very different from using mice and

User	Gender	a [ms]	b [ms/bits]	Correlation R^2	MAPE	Movement Categories Slow	Medium	Fast	Device
# 6	Male	27.70	226.34	0.925	30.80%	32.91%	16.08%	51.01%	Mouse
# 6	Male	121.04	97.32	0.851	41.65%	41.64%	12.79%	45.57%	Touchpad
# 7	Male	26.84	101.51	0.933	46.22%	37.52%	13.12%	49.36%	Mouse
# 7	Male	14.88	118.45	0.874	44.63%	41.52%	13.35%	45.13%	Touchpad
# 8	Female	61.57	167.51	0.931	39.98%	40.56%	17.46%	41.97%	Mouse
# 8	Female	108.64	65.83	0.678	49.01%	38.59%	10.68%	50.73%	Touchpad

Table 2: Fitts' Law Related Parameters for Users of Both Mouse and Touchpad

touchpad, which always involves continuous cursor movements before clicking. Although one can do a deliberately structured Fitts' law test as in [9], we believe that there is no regular pointing behavior under natural browsing in a mobile device or tablet. And even in [9]'s Fitts' law results [10], we can see that the data points with an iPhone are very noisy, and thus lack a linear pattern, especially compared to those with mice and touchpad.

5.4 User Profiling by Fitts' Parameters

As we mentioned in Section 1, the two constants a and b in Fitts' law formula Eq. 1 are affected by both the user and the environment factors. Those two constants, along with other Fitts law related parameters, such as correlation coefficient, mean absolute percentage error (MAPE), percentage of slow, medium, and fast movements, how do they differ from user to user under the same environment? And how are they affected by different environments given a same user? If the parameters are more affected by users than by environmental factors, then it would be possible to predict which user group an online user belongs to. For example, it may be possible to make a good guess about if the user online is male or female, right-handed or left-handed, age ranges, or even education backgrounds. This could be very useful for online authentication because for a certain website, such as online banking or student account, the registered users are relatively stable. Here we present preliminary results from our controlled data set, on how the parameters are affected by different users and environments.

Table 1 shows Fitts' law related parameters for all 5 users using mice. There are 3 male users and 2 female users. We can see that, with respect to dividing user groups, male vs. female as an example, there is no strong indication in any of the parameters for gender differences. For example, female user #5 is very similar to male user #1 in all parameters, except for parameter a; but a alone cannot reflect gender differences as female user #4's a is very close to that of male user #1.

Which of the two factors, user or environment, plays a major role in affecting Fitts' law related parameters? To answer this question, we look at the mouse and touchpad data of the same user. Table 2 shows the parameters for 3 users with both mouse and touchpad data. In the table, we can see the diverging effects of changing environments for different users. For example, both user #6 and #8's parameters are greatly affected by changing from a mouse to touchpad, while user #7' parameters are relatively stable regardless of using mouse or touchpad.

Overall, from the preliminary results, there is no definite line in dividing different user groups; and the changing of environments has different effects on different users. However, the size of our controlled data set, 10, may be too small to draw a definite conclusion. Moreover, the parameters we explored cannot capture all characteristics of human mouse movements in web browsing. It is possible that other kinetic-related metrics are able to differentiate user groups. We leave this for future works.

6. CONCLUSION AND FUTURE WORK

This paper examined the Fitts' model in the context of natural web browsing. Mouse movement data from over 1,000 real-world Internet users was collected via Javascript embedded on a web forum, and the analysis showed a linear relationship between the ID and MT of the task with over 98% correlation, suggesting strong evidence that Fitts' law extends well to web browsing behavior.

In addition, we evaluated the deviation in raw movement time from Fitts' predicted MT, especially the error model proposed by previous works. From the raw data, there exists a large deviation from Fitts' predicted values, with a 46.40% mean absolute deviation. We further divided all movements into three categories by the Fitts' predicted MT: slow, medium, and fast movements. And fast movements were shown to have an error model different from the other two categories, which indicates their open-looped nature.

Moreover, this paper examined the effect of differing pointing devices on the Fitts model. Pointing data was collected from 10 people variously using physical mice and laptop touch pads. The analysis showed that both devices had a strong linear relationship between ID and MT (over 98% correlation in both cases), and that the results were nearly identical at low ID values, yet diverged slightly at high ID values.

Finally, this paper discussed other Fitts' Law considerations, namely the standard deviation in Fitts' Law calculations. The forum data set was analyzed and the standard deviation of MT plotted against ID. The result showed that Fitts' Law also describes a linear relationship between ID and standard deviation, implying that variance in time to point increases as ID increases.

There are a number of possible directions for our future work. We plan to conduct a large-scale user data collection, with their demographic information known. Then, we plan to further verify the possibility of classifying users into different groups (gender, age, handedness, etc) based on Fitts' law parameters. We also plan to explore the possibility to detect web bots by checking if the mouse movement pattern follows Fitts' law.

7. REFERENCES

[1] E. Agichtein, E. Brill, S. Dumais, and R. Ragno. Learning user interaction models for predicting web

search result preferences. In *Proceedings of the 29th annual international ACM SIGIR conference on research and development in information retrieval*, SIGIR '06, pages 3–10, 2006.

[2] J. Annet, C. W. Golby, and H. Kay. The measurement of elements in an assembly task: The information output of the human motor system. *Quarterly Journal of Experimental Psychology*, 10:1–11, 1958.

[3] R. Blanch, Y. Guiard, and M. Beaudouin-Lafon. Semantic pointing: improving target acquisition with control-display ratio adaptation. In *ACM CHI'04*, pages 519–526. ACM, 2004.

[4] S. K. Card, W. K. English, and B. J. Burr. Evaluation of mouse, rate-controlled isometric joystick, step keys, and text keys for text selection on a CRT. *Ergonomics*, 21(8):601–613, 1978.

[5] S. K. Card, T. P. Moran, and A. Newell. *The Psychology of Human-Computer Interaction*. Lawrence Erlbaum Associates, Publishers, 1983.

[6] P. K. Chan. A non-invasive learning approach to building web user profiles. In *ACM SIGKDD International Confreence*, 1999.

[7] O. Chapuis, R. Blanch, and M. Beaudouin-Lefon. Fitts' law in the wild: A field study of aimed movements. Technical Report Rapport de Recherche No 1480, LRI, December 2007.

[8] E. R. F. W. Crossman and P. J. Goodeve. Feedback control of hand-movement and Fitts' law. *Quarterly J. of Exp. Psychology*, 35A(2):251–178, 1963/1983.

[9] J. Donovan. Fitts's Law Test. `http://jareddonovan.com/programming/fitts_law/`. [Accessed: Nov. 2012].

[10] J. Donovan. Fitts's Law Test Result. `http://jareddonovan.com/programming/fitts_law/graph.png`. [Accessed: Nov. 2012].

[11] H. Drewes. Only one Fitts' law formula please! In *ACM CHI'10*, pages 2813–2822, 2010.

[12] B. W. Epps. Comparison of six cursor control devices based on Fitts' law models. In *Proceedings of the Human Factors Society 30th Annual Meeting*, pages 327–331, 1986.

[13] A. C. Evans and J. O. Wobbrock. Taming wild behavior: the input observer for obtaining text entry and mouse pointing measures from everyday computer use. In *ACM CHI'12*, pages 1947–1956, 2012.

[14] P. M. Fitts. The information capacity of the human motor system in controlling the amplitude of movement. *Journal of Experimental Psychology*, 47(6):381–391, 1954.

[15] P. M. Fitts and J. R. Peterson. Information capacity of discrete motor responses. *Journal of Experimental Psychology*, 67(2):103–112, 1964.

[16] C. Forlines, D. Vogel, and R. Balakrishnan. Hybridpointing: fluid switching between absolute and relative pointing with a direct input device. In *ACM UIST'06*, pages 211–220. ACM, 2006.

[17] K. Z. Gajos, K. Reinecke, and C. Herrmann. Accurate measurements of pointing performance from in situ observations. In *ACM CHI'12*, 2012.

[18] T. Grossman and R. Balakrishnan. A probalilistic approach to modeling two-dimensional pointing. *ACM Transactions on Computer-Human Interaction*, 12(3):435–459, 2005.

[19] C. M. Harris and D. M. Wolpert. Signal-dependent noise determines motor planning. *Nature*, 394:780–784, 1998.

[20] J. Hu, H.-J. Zeng, H. Li, C. Niu, and Z. Chen. Demographic prediction based on user's browsing behavior. In *Proceedings of the 16th international conference on World Wide Web*, WWW '07, pages 151–160, 2007.

[21] J. Huang, R. W. White, and S. Dumais. No clicks, no problem: using cursor movements to understand and improve search. In *ACM CHI'11*, pages 1225–1234, 2011.

[22] A. Hurst, J. Mankoff, and S. E. Hudson. Understanding pointing problems in real world computing environments. In *Proceedings of the 10th international ACM SIGACCESS conference on Computers and accessibility*, Assets '08, pages 43–50, 2008.

[23] R. J. Jagacinski, D. W. Repperger, M. S. Moran, S. L. Ward, and B. Glass. Fitts' law and the microstructure of rapid discrete movements. *Journal of Experimental Psychology: Human Perception and Performance*, 6:309–320, 1980.

[24] S. W. Keele and M. I. Posner. Processing visual feedback in rapid movement. *Journal of Experimental Psychology*, 77:155–158, 1968.

[25] B. A. Kerr and G. D. Langolf. Speed of aiming movements. *Quarterly Journal of Experimental Psychology*, 29:475–481, 1977.

[26] R. Kerr. Movement time in an underwater environment. *Journal of Motor Behavior*, 5:175–178, 1973.

[27] U. Kukreja, W. E. Stevenson, and F. E. Ritter. RUI: Recording user input from interfaces under Windows and Mac OS X. *Behavior Research Methods*, 38(4):656–659, 2006.

[28] R. Kumar and A. Tomkins. A characterization of online browsing behavior. In *Proceedings of the 19th international conference on World Wide Web*, WWW '10, pages 561–570, 2010.

[29] G. D. Langolf, D. B. Chaffin, and J. A. Foulke. An investigation of Fitts' law using a wide range of movement amplitudes. *Journal of Motor Behavior*, 8:113–128, 1976.

[30] I. S. MacKenzie. Fitts' law as a research and design tool in human-computer interaction. *Human-Computer Interaction*, 7:91–139, 1992.

[31] I. S. MacKenzie, A. Sellen, and W. Buxton. A comparison of input devices in elemental pointing and dragging tasks. In *ACM CHI'91*, pages 161–166, 1991.

[32] D. E. Meyer, R. A. Abrams, S. Kornblum, C. E. Wright, and J. E. K. Smith. Optimality and human motor performance: Ideal control of rapid aimed movements. *Psychological Review*, 95:340–370, 1988.

[33] M. Pusara and C. E. Brodley. User re-authentication via mouse movements. In *Proceedings of the 2004 ACM workshop on visualization and data mining for computer security*, pages 1–8, 2004.

[34] D. A. Rosenbaum. *Human Motor Control*. Academic

Press, Inc., 1991.

[35] R. A. Schmidt, H. N. Zelaznik, B. Hawkins, J. S. Frank, and J. T. J. Quinn. Motor output variability: A theory for the accuracy of rapid motor acts. *Psychological Review*, 86:415–451, 1979.

[36] Y. Skadberg and J. R. Kimmel. Visitors' flow experience while browsing a web site: its measurement, contributing factors and consequences. *Computers in Human Behavior*, 20(3):403–422, 2004.

[37] H. Slijper, J. Richter, E. Over, J. Smeets, and M. Frens. Statistics predict kinematics of hand movements during everyday activity. *Journal of Motor Behavior*, 41:3–9, 2009.

[38] R. W. Soukoreff and I. S. MacKenzie. Towards a standard for pointing device evaluation, perspectives on 27 years of Fitts' law research in hci. *Int. J. Hum.-Comput. Stud.*, 61:751–789, December 2004.

[39] M. Spiliopoulou. Web usage mining for web site evaluation. *Commun. ACM*, 43:127–134, August 2000.

[40] K. Sugiyama, K. Hatano, and M. Yoshikawa. Adaptive web search based on user profile constructed without any effort from users. In *Proceedings of the 13th international conference on World Wide Web*, WWW '04, pages 675–684, 2004.

[41] A. T. Welford. *Fundamentals of Skill.* Methuen, London, 1968.

[42] J. O. Wobbrock, E. Cutrell, S. Harada, and I. S. MacKenzie. An error model for pointing based on Fitts' law. In *ACM CHI'08*, pages 1613–1622, 2008.

[43] J. O. Wobbrock, K. Shinohara, and A. Jansen. The effects of task dimensionality, endpoint deviation, throughput calculation, and experiment design on pointing measures and models. In *ACM CHI'11*, pages 1639–1648, 2011.

[44] H. Yang and X. Xu. Bias towards regular configuration in 2D pointing. In *ACM CHI'10*, pages 1391–1400, 2010.

[45] S. Zhai. Characterizing computer input with Fitts' law parameters: the information and non-information aspects of pointing. *Int. J. Hum.-Comput. Stud.*, 61:791–809, December 2004.

[46] S. Zhai, J. Kong, and X. Ren. Speed-accuracy tradeoff in Fitts' law tasks–on the equivalency of actual and nominal pointing precision. *Int'l J. of Human-Computer Studies*, 61(6):823–856, 2004.

[47] N. Zheng, A. Paloski, and H. Wang. An efficient user verification system via mouse movements. In *ACM CCS'11*, pages 139–150, 2011.