ELEG240- Spring, 2005
Homework 8
Due 4/26, noon

1. Using the one-dimensional crystal model, and also the approximate formula 8.C.17, calculate the lowest energy state in joules and in eV for a barrier height of 5 eV, a crystal period of 5 Å, and a barrier width of 1 Å. Note: you cannot solve this analytically, but must do it by plugging in values until the equation is solved. Helpful hint: when plugging values into sines and cosines, use radians.
ANSWER:
U_0=5 eV, a=5 Å, and w=1 Å, so in equation 8.C.17, P=3.27. To find the lowest energy state, we must find the lowest value of βa for which the right side of equation 8.C.17 is 1. Plugging numbers into the right side of the equation,

βa	$\cos(ka) = P\frac{\sin(\beta a)}{\beta a} + \cos(\beta a)$
2	1.07
2.1	0.84
2.03	1.00

So, when βa=2.03, that is the lowest energy state. Using

$$\frac{\hbar^2 \beta^2}{2m} = E,$$

we have that E_0=0.63 eV.

2. For the same crystal as problem 1, calculate the effective mass of the lowest energy band. Again, you will need to do it numerically but can plug in your variables with intelligent values.
ANSWER:
The effective mass is defined in equation 8.C.20 as

$$E = E_0 + \frac{\hbar^2 k^2}{2m^*}, \text{ or}$$

$$Ea^2 = E_0 a^2 + \frac{\hbar^2 (ka)^2}{2m^*}$$

Starting with βa=2.03, we use equation 8.C.17 to plot k vs. E (plotting E on the x-axis and k on the y-axis), and then fit the effective mass equation by varying the effective mass until it fit, and so determine the effective mass:

βa	Ea^2(joule-m^2)	ka
2.03	2.52E-38	0
2.1	2.69E-38	0.575
2.17	2.88E-38	0.822

Thus

Ea^2 =2.52E-38+5.13E-39 $(ka)^2$,

so

$$\frac{\hbar^2}{2m*} = 5.13x10^{-39}$$ in MKS units,

solving for $m*$,

$m* = 1.1x10^{-30}$ kilograms, or slightly greater than the mass of a free electron.