ELEG240- Spring, 2005
Homework 6
Due 4/12, noon

1. Light with wavelength λ is incident upon a diffraction grating nearly parallel, that is, along the x-axis in figure 6.B.1. What does the period of the grating need to be to reflect a beam back in the opposite direction to the incident light?
ANSWER:
$\sin\phi_{out} = \sin\phi_{in} + m\lambda / a$.

For this problem, $\phi_{in} = 90^o$. A beam in the opposite direction will have $\phi_{out} = -90^o$. Thus,

$-1 = 1 + m\lambda / a$,

$a = -m\lambda / 2$,

or $a = \lambda / 2$, λ , $3\lambda / 2$ …

2. For light with a wavelength of 10 microns in free space, what is the photon energy in both joules and electron volts? For light with a wavelength of 1 micron in free space, what is the photon energy in both joules and electron volts?
ANSWER:
$$E = hf = \frac{hc}{\lambda} .$$
For 10 microns, $E = 2x10^{-20}$ joules = 0.12 eV. For 1 micron, multiply times ten to get $E = 2x10^{-19}$ joules = 1.2 eV.

3. Apply Schrodinger's Equation to the problem of an electron wavefunction in a one-dimensional box with infinite sides, that is, treat only one dimension where the potential energy is zero from $-\mathrm{L}/2 < z < \mathrm{L}/2$, and it is infinite otherwise. Think what an infinite potential energy implies about the probability of finding and electron outside the box and what this means the wavefunction must be outside the box.
ANSWER:
If the bottom of the well is taken to be the zero of potential energy (we can set the zero anywhere), then in the well the electrons must obey Schrodinger's Equation (one-dimensional), setting $U = 0$:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dz^2} = E\psi \text{ , inside metal.}$$

A solution to this differential equation is

$\psi = A\cos kz$,

which can be seen by plugging in and seeing that ψ and x cancel out of the equation, leaving

$$\frac{\hbar^2 k^2}{2m} = E .$$

Note that $\psi = A\sin kx$ is also a solution, but we may focus on only one solution here. Outside the metal, $U = \infty$, which means that $\psi = 0$, since the probability of finding an electron in an infinite potential is zero.

$\psi = 0$, outside metal.

Since ψ must be continuous,

$\psi(x = \pm \mathrm{L}/2) = 0 = A\cos(k\mathrm{L}/2)$,

Which means that

$k\mathrm{L}/2 = \pi/2 + m\pi$, where m is any integer.

Since E is proportional to k, the *ground state* or lowest energy state of the wavefunction has

$k\mathrm{L}/2 = \pi/2$, lowest energy state.

Focusing on that,

$\psi_0 = A\cos(\pi z/\mathrm{L})$.

To find A, realize that the probability of finding an electron somewhere in the well is 1. This means that the sum of the probabilities of finding the electron in any differential of length dz is 1. That is,

$$\int_{-L/2}^{L/2} |\psi|^2 dz = 1 = \int_{-L/2}^{L/2} A^2 \cos^2(\pi z/\mathrm{L}) dz = A^2\mathrm{L}/2 .$$

Thus,

$A = \sqrt{2/\mathrm{L}}$,

and,

$$\psi_0 = \sqrt{\frac{2}{L}} \cos(\pi z / L) .$$