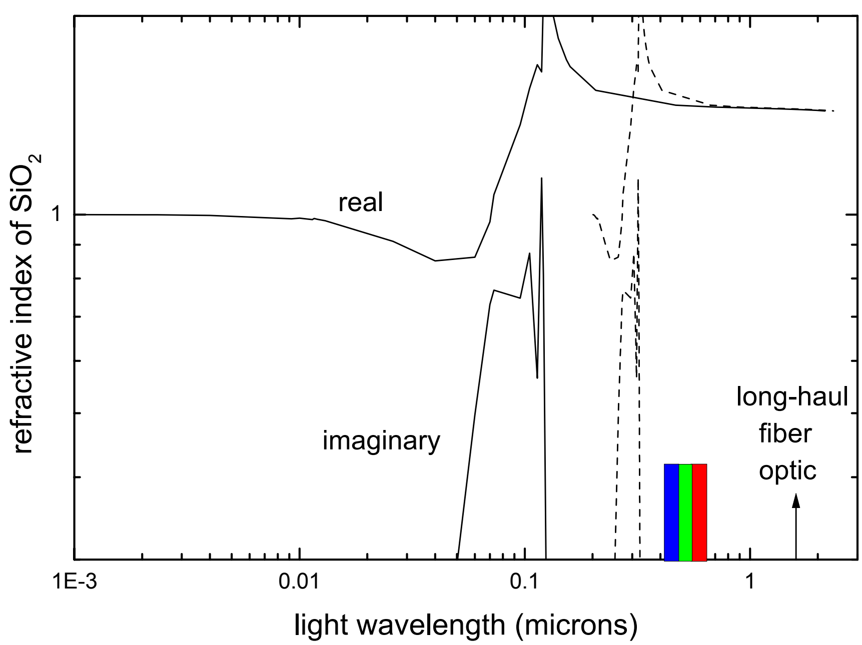
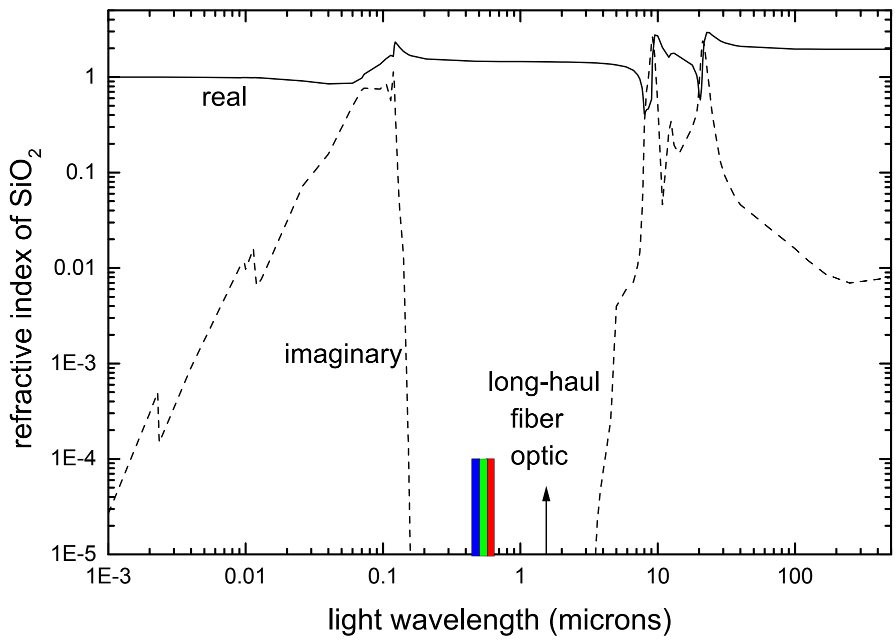
ELEG240- Spring, 2005
Homework 4
Due 3/16, 1 PM

1. A prism made of lithium niobate will separate the colors of light much more than a glass prism. What does this indicate about the difference in the wavelengths where lithium niobate and glass begin absorbing light as you move to shorter wavelengths from the visible to the ultraviolet?
ANSWER:
Referring to figure 4.B.3 of the text (magnifying the scale), one sees that the dispersion or variation in the refractive index, that causes the colors to separate, is greater closer to the absorption line.



Thus if the absorption line is moved closer to the visible, the dispersion will be greater (dashed lines). Thus lithium niobate begins absorbing closer to the visible.

2. Is the refractive index of silicon roughly the same in the radio wave and fiber optic portions of the spectrum? Why?
ANSWER:
Referring to figure 4.B.6 of the text for *silicon dioxide* (SiO2), the refractive index at fiber optic wavelengths is different than that at radio wavelengths:

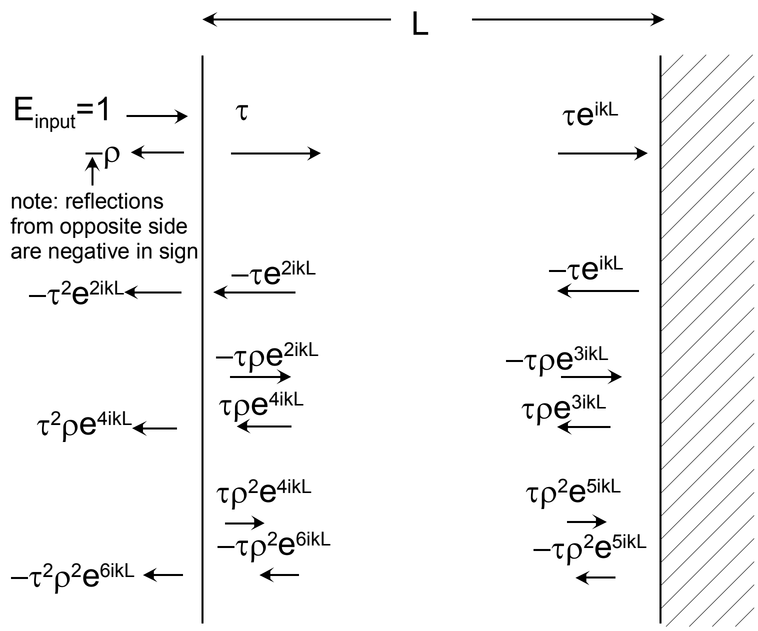


This is because there is absorption occurring in the infrared (around 10-20 microns) that causes the real part to shift upward going to longer wavelengths. That infrared absorption is due to the charged atoms of SiO2 oscillating in response to light. But <u>silicon</u> does not have absorption in the infrared due to oscillating charged atoms since its atoms are not charged (because they are all the same). Thus the refractive index of silicon is the same at fiber optic and radio wavelengths.

3. When light reflects off of a perfect conductor, the electric field of the reflected light is exactly equal but opposite in sign to the incident light. It has been suggested that another way to make a laser than stated in the text is to place a perfect conductor on one side. Using the same type of analysis (I start it for you in the figure) as in the text for the laser of figure 4.D.1, determine what L should be in terms of wavelength λ (as measured in the gain medium) to get lasing. note: Use $k = k_{real} - ik_{imag}$ to denote gain. Also use $k_{real} = \frac{2\pi}{\lambda}$. Hint: $\frac{1}{1+x^2} = 1 - x^2 + x^4 \ldots$, and the denominator can still be zero! Think complex!

ANSWER:

Apply the same reasoning, using the above principle for light reflected off a perfect conductor, to draw the bouncing waves:



Thus,

$$
\begin{aligned}
\mathrm{E}_{refl} &= -\rho - \tau^2 e^{2ikL} + \tau^2 \rho e^{4ikL} - \tau^2 \rho^2 e^{6ikL} ..... \\
&= -\rho - \tau^2 e^{2ikL}(1 - \rho e^{2ikL} + \rho^2 e^{4ikL} .....)
\end{aligned}
.
$$

Since

$$
\frac{1}{1+x^2} = 1 - x^2 + x^4 ...
$$

this equals

$$
\begin{aligned}
\mathrm{E}_{refl} &= -\rho - \tau^2 e^{2ikL} \frac{1}{1+(\sqrt{\rho} e^{ikL})^2} = -\rho - \frac{\tau^2 e^{2ikL}}{1+\rho e^{2ikL}} \\
&= \frac{-\rho - \rho^2 e^{2ikL} - \tau^2 e^{2ikL}}{1+\rho e^{2ikL}} = \frac{-\rho - e^{2ikL}}{1+\rho e^{2ikL}} = -\frac{\rho + e^{2ikL}}{1+\rho e^{2ikL}}
\end{aligned}
.
$$

For lasing to occur, the denominator must be zero, or

$$
-1 = \rho e^{2ikL} = \rho e^{2i(k_{real} - ik_{imag})L} = \rho e^{2k_{imag}L} e^{2ik_{real}L}
$$

Thus two conditions must occur,

$$
1 = \rho e^{2k_{imag}L},
$$

and

$$-1 = e^{2ik_{real}L}.$$

From the latter, since $e^{i(\pi+2\pi m)} = -1$, where m=0, 1, 2, …

$$\pi + 2\pi m = 2k_{real}L = \frac{4\pi L}{\lambda}.$$

$$L = \frac{\lambda}{4} + m\frac{\lambda}{2}, \qquad m=0, 1, 2 \ldots$$

We see that there is a quarter wave shift in the required thickness, which is because the conductor causes a phase shift on reflection, and hence waves are in phase with a half-wavelength total round trip.