ELEG240- Spring, 2005
Homework 3
Due 3/9, 1 PM

1. Light has a velocity of $2x10^8$ meters/second in a material. A light ray is incident on the material from air at an angle of 50 degrees from surface-normal. What will be the angle of the ray from normal after it enters the material?
ANSWER:
Since $\mathrm{v} = c/n$, and $c = 3x10^8$ meters/second, this implies that n=1.5 for the material. Applying Snell's Law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2,$$

where $\theta_1 = 50^o$ and for air $n_1 = 1$, and for the material from above $n_2 = 1.5$, we have that $\theta_2 = 30.7^o$.

2. After light passes through 1 centimeter of a material only 50 % of it remains. What is the absorption coefficient in the material?
ANSWER:
Since light propagates in absorbing material as $e^{-\alpha z}$, where α is the absorption coefficient, we have that

$$e^{-\alpha(1cm)} = 0.5.$$

Solving, $\alpha = 0.69$ 1/cm = 69 1/m.

3. For the material of problem 2, the wavelength of the light was 1 micron if the light was in free space. What is the imaginary part of the refractive index?
ANSWER:
Since the electric and magnetic fields of light propagate as $e^{-i(kz-\omega t)}$, where $k = \frac{\omega}{c}(n_{real} + in_{imag})$, and the power goes as the field squared, this implies that $\alpha = 2\frac{\omega}{c}n_{imag}$. (I am simply repeating the derivation of equation 4.12.) Noting that

$$c = \lambda f = \frac{\lambda\omega}{2\pi}$$ (where λ is the wavelength of the light when it is in free space)

we have that

$$\alpha = \frac{4\pi}{\lambda}n_{imag},$$ or

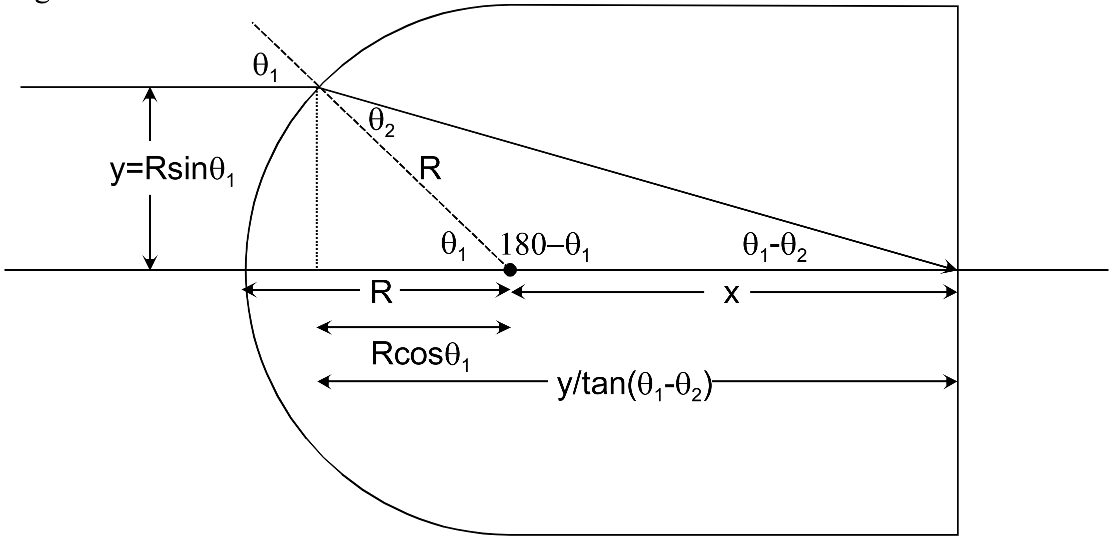
$$n_{imag} = \frac{\alpha\lambda}{4\pi} = \frac{(69)(10^{-6})}{4\pi} = 5.5x10^{-6}.$$

Note that for most materials, n_{real} ~1-10, so for a material with such a transparency listed, the real part of the refractive index dominates.

4. The focal point of a semi-spherical lens is the point at which it collects all light rays incident parallel to its axis, as shown in this figure. For the spherical lens of the drawing, calculate x in terms of R, the refractive index of the lens n, and θ_1. That's all I require, but for the students with a real interest, use a computer to plot x/R vs. θ_1 from 0 to 90°. You will not be able to plug in $\theta_1 = 0$, as that will give a 0/0 in the equation, so just get as close as you can. What does this imply about the focal "point" using a spherical lens?

ANSWER:

Label the Snell's Law angle and use the principles of geometry to find the other angles as shown:



Thus,

$$x = y/\tan(\theta_1 - \theta_2) - R\cos\theta_1 = \frac{R\sin\theta_1}{\tan(\theta_1 - \theta_2)} - R\cos\theta_1.$$

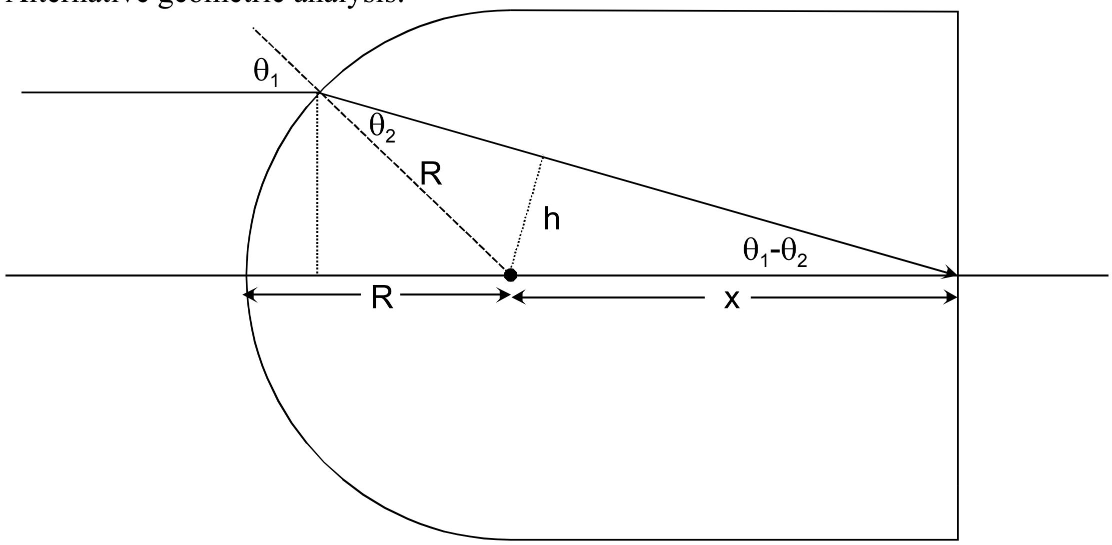
Applying Snell's Law,

$$\sin\theta_1 = n\sin\theta_2.$$

Plugging into the first equation,

$$x = \frac{R\sin\theta_1}{\tan[\theta_1 - \sin^{-1}(\frac{\sin\theta_1}{n})]} - R\cos\theta_1 .$$

Alternative geometric analysis:



$$\sin\theta_2 = \frac{h}{R},$$

$$\sin(\theta_1 - \theta_2) = \frac{h}{x},$$

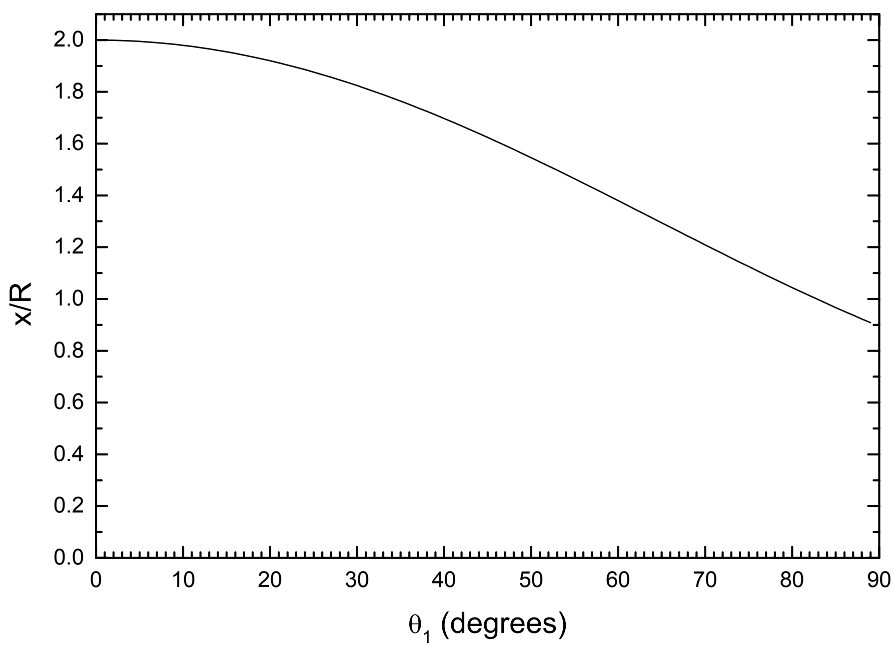
$$R\sin\theta_2 = x\sin(\theta_1 - \theta_2),$$

$$x = \frac{R\sin\theta_2}{\sin(\theta_1 - \theta_2)} = \frac{R(\frac{1}{n}\sin\theta_1)}{\sin[\theta_1 - \sin^{-1}(\frac{\sin\theta_1}{n})]} .$$

One can show this is equal to the above solution by noting that

$$x = R[\frac{\sin\theta_1}{\tan(\theta_1-\theta_2)} - \cos\theta_1] = R[\frac{\sin\theta_1\cos(\theta_1-\theta_2)}{\sin(\theta_1-\theta_2)} - \cos\theta_1]$$
$$= R[\frac{\sin\theta_1\cos(\theta_1-\theta_2) - \cos\theta_1\sin(\theta_1-\theta_2)}{\sin(\theta_1-\theta_2)}]$$
$$= R[\frac{\sin\theta_1(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) - \cos\theta_1(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)}{\sin(\theta_1-\theta_2)}].$$
$$= R[\frac{\sin\theta_2(\sin^2\theta_1 + \cos^2\theta_1)}{\sin(\theta_1-\theta_2)}] = R[\frac{\sin\theta_2}{\sin(\theta_1-\theta_2)}]$$

Plotting this for n=1.5,



We see that for rays near the axis of the lens, x/R=2, but for rays incident away from the axis the focal point moves closer to the lens. This is called spherical aberration, and for this reason sophisticated lens have a profile other than spherical that causes nearly all the rays to come to a single point.