1. Two point charges exist on the z-axis, a positive charge at $z=+z_q$ and negative charge of equal magnitude at $z=-z_q$:

On the y-axis, what direction does the electric field point in? If the charges are oscillating, that is, $z_q = \pm z_q \cos(\omega t)$, what direction does the magnetic field point in?

**ANSWER:**
To find the direction of the electric field, use Coulomb’s Law and take the superposition:

Thus, the field points in the –z direction. If the charges are oscillating in time, the electric field is also oscillating in time. Thus, Maxwell’s 3rd equation applies,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$
One could just as easily use the 4\textsuperscript{th} equation to discern the direction of $\mathbf{B}$. One needs to find the vector direction of $\nabla$, and apply the cross product. Noting that there is symmetry around the z-axis, cylindrical coordinates are appropriate. There is variation in z, but as on the y-axis the field will be a maximum, the derivative with respect to z goes to zero on the y-axis. Thus, the vector direction of $\nabla$ is in the $-r$ direction, or on the y-axis in the $-y$ direction. $\nabla$ is in the $-y$ direction since the fields decay as $y$ increases. Thus, $\nabla \times \mathbf{E}$ and hence $\mathbf{B}$ point in the $-x$ direction, or generally along the $\phi$ direction.

Note, this analysis implies that there is an electromagnetic wave flowing in the y direction. Thus, oscillating charges (as drawn) result in the production of an electromagnetic wave flowing away from them.

2. Show that the units of $\varepsilon \frac{\partial \mathbf{E}}{\partial t}$ are current/area.
   \begin{align*}
   \text{ANSWER:} \\
   & \text{Applying } Q=CV, \text{ and current}=\text{charge/time}, \\
   & \frac{\text{farads}}{\text{meter}} x \frac{\text{volts/meter}}{\text{second}} = \frac{\text{coulombs}}{\text{meter}^2 \text{second}} = \frac{\text{amps}}{\text{meter}^2}.
   \end{align*}

3. A conducting material, such as a metal, can be represented as having a conductivity $\sigma$ and permittivity $\varepsilon_0$. Assuming that the solution for the electric field of a plane electromagnetic wave is still $Ae^{(kt-\omega x)}$ (equation 3.15, forward going wave only), and the solution for the magnetic field of an electromagnetic wave is $\frac{k}{\omega} Ae^{(kt-\omega x)}$ (equation 3.7, phasor notation), and letting $k$ be complex, what is $k$? Note: you will need to redo equation 3.2 with a term including conductivity. Also, you may assume that $\sigma \gg \omega \varepsilon_0$. What does this imply about light propagation though a conductor?
   \begin{align*}
   \text{ANSWER:} \\
   & \text{Leaving the current term in Maxwell’s 4\textsuperscript{th} equation, and for the moment retaining the unit vectors in the } \nabla \times \mathbf{B} \text{ and } \frac{\partial \mathbf{E}}{\partial t} \text{ terms, equation 3.2 becomes} \\
   & -\frac{\partial B_z}{\partial z} \mathbf{a}_z = \mu_0 (\mathbf{j} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \mathbf{a}_z). \\
   & \text{At this point, one may infer that for the equation to hold, the current must be in the x direction, but also, one may deduce this by noting from Chapter 1 that} \\
   & \mathbf{j} = \sigma \mathbf{E},
   \end{align*}
and hence for this problem $\mathbf{j} = \sigma\mathbf{E}_x a_x$. Plugging in, and dropping the unit vectors,

$$-\frac{\partial B_y}{\partial z} = \mu_0 (\sigma \mathbf{E}_x + \epsilon \frac{\partial \mathbf{E}_x}{\partial t}).$$

Plugging in $E_x = A e^{i(kz - \omega t)}$ and $B_y = \frac{k}{\omega} A e^{i(kz - \omega t)}$, we obtain

$$-\frac{i k^2}{\omega} A e^{i(kz - \omega t)} = \mu_0 (\sigma A e^{i(kz - \omega t)} - i \omega \epsilon A e^{i(kz - \omega t)}),$$

or

$$-\frac{i k^2}{\omega} = \mu_0 (\sigma - i \omega \epsilon).$$

Using $\sigma \gg \omega \epsilon$, this becomes

$$-\frac{i k^2}{\omega} \approx \mu_0 \sigma,$$

or

$$k^2 \approx i \mu_0 \sigma \omega.$$  

Noting that $\sqrt{i} = \frac{1 + i}{\sqrt{2}}$ (square the right side to prove it to yourself),

$$k \approx (1 + i) \sqrt{\frac{\mu_0 \sigma \omega}{2}}.$$

Since $k$ has an imaginary component,

$$E_x, B_y \sim e^{i(kz - \omega t)} = e^{i[(k_{\text{real}} + i k_{\text{imag}})z - \omega t]} = e^{-k_{\text{imag}}z} e^{i(k_{\text{real}}z - \omega t)},$$

and the wave decays as it goes into the conductor. Thus, conductors such as metals are not transparent to light.