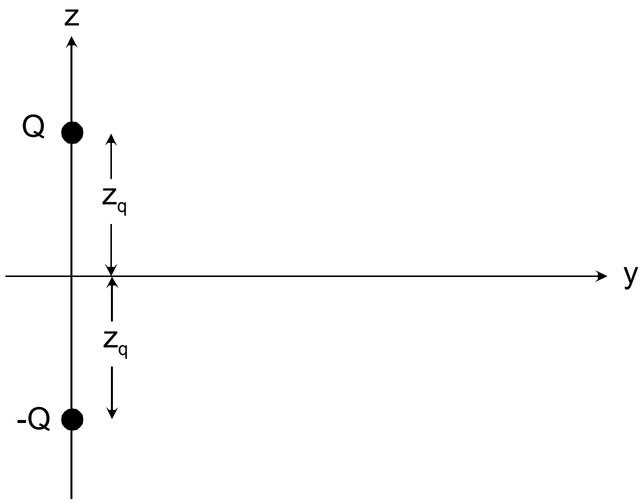
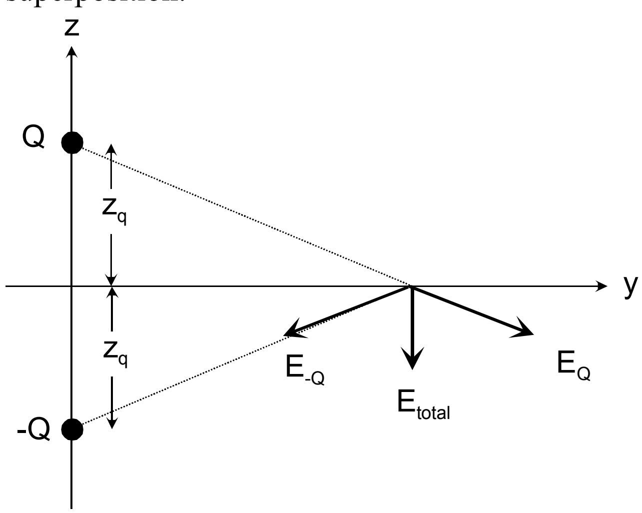
ELEG240- Spring, 2005
Homework 2
Due 3/2

1. Two point charges exist on the z-axis, a positive charge at $z=z_q$ and negative charge of equal magnitude at $z=-z_q$:



On the y-axis, what direction does the electric field point in? If the charges are oscillating, that is, $z_q = |z_q|\cos(\omega t)$, what direction does the magnetic field point in?
ANSWER:
To find the direction of the electric field, use Coulumb's Law and take the superposition:



Thus, the field points in the –z direction. If the charges are oscillating in time, the electric field is also oscillating in time. Thus, Maxwell's 3rd equation applies,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

One could just as easily use the 4^{th} equation to discern the direction of **B**. One needs to find the vector direction of ∇, and apply the cross product. Noting that there is symmetry around the z-axis, cylindrical coordinates are appropriate. There is variation in z, but as on the y-axis the field will be a maximum, the derivative with respect to z goes to zero on the y-axis. Thus, the vector direction of ∇ is in the -r direction, or on the y-axis in the -y direction. ∇ is in the –y direction since the fields decay as y increases. Thus, $\nabla \times \mathbf{E}$ and hence **B** point in the –x direction, or generally along the ϕ direction.

Note, this analysis implies that there is an electromagnetic wave flowing in the y direction. Thus, oscillating charges (as drawn) result in the production of an electromagnetic wave flowing away from them.

2. Show that the units of $\varepsilon \frac{\partial \mathbf{E}}{\partial t}$ are current/area.

ANSWER:
Applying Q=CV, and current=charge/time,

$$(\frac{\text{farads}}{\text{meter}})\text{x}(\frac{\text{volts/meter}}{\text{second}}) = \frac{\text{coulumbs}}{\text{meter}^2\text{second}} = \frac{\text{amps}}{\text{meter}^2}.$$

3. A conducting material, such as a metal, can be represented as having a conductivity σ and permittivity ε_0. Assuming that the solution for the electric field of a plane electromagnetic wave is still $Ae^{i(kz-\omega t)}$ (equation 3.15, forward going wave only), and the solution for the magnetic field of an electromagnetic wave is $\frac{k}{\omega}Ae^{i(kz-\omega t)}$ (equation 3.7, phasor notation), and letting *k* be complex, what is *k*? Note: you will need to redo equation 3.2 with a term including conductivity. Also, you may assume that $\sigma \gg \omega\varepsilon_0$. What does this imply about light propagation though a conductor?

ANSWER:
Leaving the current term in Maxwell's 4^{th} equation, and for the moment retaining the unit vectors in the $\nabla \times \mathbf{B}$ and $\frac{\partial \mathbf{E}}{\partial t}$ terms, equation 3.2 becomes

$$-\frac{\partial B_y}{\partial z}\mathbf{a_x} = \mu_0(\mathbf{j} + \varepsilon\frac{\partial E_x}{\partial t}\mathbf{a_x}).$$

At this point, one may infer that for the equation to hold, the current must be in the x direction, but also, one may deduce this by noting from Chapter 1 that

$$\mathbf{j} = \sigma\mathbf{E},$$

and hence for this problem $\mathbf{j} = \sigma E_x \mathbf{a_x}$. Plugging in, and dropping the unit vectors,

$$-\frac{\partial B_y}{\partial z} = \mu_0 (\sigma E_x + \varepsilon \frac{\partial E_x}{\partial t}).$$

Plugging in $E_x = Ae^{i(kz-\omega t)}$ and $B_y = \frac{k}{\omega} Ae^{i(kz-\omega t)}$, we obtain

$$-\frac{ik^2}{\omega} Ae^{i(kz-\omega t)} = \mu_0 (\sigma Ae^{i(kz-\omega t)} - i\omega\varepsilon Ae^{i(kz-\omega t)})\text{, or}$$

$$-\frac{ik^2}{\omega} = \mu_0 (\sigma - i\omega\varepsilon).$$

Using $\sigma \gg \omega\varepsilon_0$, this becomes

$$-\frac{ik^2}{\omega} \cong \mu_0 \sigma\text{, or}$$

$$k^2 \cong i\mu_0 \sigma\omega.$$

Noting that $\sqrt{i} = \frac{1+i}{\sqrt{2}}$ (square the right side to prove it to yourself),

$$k \cong (1+i)\sqrt{\frac{\mu_0 \sigma\omega}{2}}.$$

Since *k* has an imaginary component,

$$E_x, B_y \sim e^{i(kz-\omega t)} = e^{i[(k_{real}+ik_{imag})z-\omega t]} = e^{-k_{imag}z} e^{i[k_{real}z-\omega t]},$$

and the wave decays as it goes into the conductor. Thus, conductors such as metals are not transparent to light.