1. Actual solar cells contain parasitic circuit elements such as a shunt and series resistances as shown:

Assuming $I_0=10^{-14}$ amp., $I_p=0.0013$ amp., $R_S=200$ ohms, $R_{Sh}=10000$ ohms, and $R_L=500$ ohms, and at room temperature (300 K), calculate the power delivered to the load. Calculate it again for the ideal case of infinite shunt resistance and zero series resistance, and give the reduction in efficiency (the ratio of actual to ideal) due to the parasitic resistances.

**ANSWER:**

\[
-I_{\text{diodr}} = I_p - I_0(e^{V/k_BT} - 1) = \frac{V}{R_{Sh}} + \frac{V}{R_S + R_L} = \frac{V}{R_{\text{eff}}}.
\]

\[
\frac{1}{R_{\text{eff}}} = \frac{1}{R_{Sh}} + \frac{1}{R_S + R_L}, \quad R_{\text{eff}} = 654 \text{ ohms}.
\]

Solve for $V$, for example by solving for it on one side like shown in class:

\[
V = \frac{k_BT}{e} \ln\left[\frac{1}{I_0} \left(I_p - \frac{V}{R_{\text{eff}}} \right) + 1\right] = 0.026 \ln\left[\frac{1}{I_0} \left(I_p - \frac{V}{R_{\text{eff}}} \right) + 1\right] \text{ at } 300 \text{ K}.
\]
<table>
<thead>
<tr>
<th>$V_{\text{guess}}$</th>
<th>$0.026 \ln \left[ \frac{1}{I_0} \left( I_p - \frac{V}{R_{\text{eff}}} \right) + 1 \right]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.642</td>
</tr>
<tr>
<td>0.642</td>
<td>0.629</td>
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<tr>
<td>0.629</td>
<td>0.630</td>
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</table>

So the diode voltage is 0.630 volts. This means the load current is $0.630/(200+500)=0.0009$ amps, or the load power $(I^2R_L)$ is 0.000405 watts.

In the ideal case, $R_{\text{eff}}=500$ ohms, and solving $V=0.599$ volts, the load current is 0.0012 amps, and the load power is 0.000718 watts. Therefore the parasitic elements introduce a reduced efficiency of 0.56 or 56%.

2. An LED is a forward biased p-n junction that then emits light. If the junction is considered to be a point source of light inside the semiconductor, and the refractive index of the semiconductor is 3.5, calculate the fraction of light emitted that actually exits the device. Assume that the reflection coefficient of the surface is zero for rays less than the total internal reflection angle. Suggest a way to get more light out.

**ANSWER:**

For an index of 3.5, the total internal reflection angle is

$$\theta_c = \sin^{-1}(1/3.5) = 16.6^\circ.$$

Therefore, rays up to this angle will exit. Integrating,

$$\text{Fraction of emitted rays} = \frac{\int_0^{2\pi} \int_0^{\sin^{-1}(1/3.5)} \sin \theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^{\pi} \sin \theta \, d\theta \, d\phi} = \frac{2\pi(1-\cos \theta_c)}{4\pi} = 0.021.$$

One may increase the fraction of emitted rays by forming a hemispherical “bulb” on top of the diode.