ELEG240- Spring, 2005
Homework 1, solution

1. You are designing an integrated circuit and must have a 1000 ohm resistor defined on the chip. The resistor will have a length of 100 microns and a width of 5 microns. You have chromium available as a metal, whose conductivity is 7.7x10^6 1/(ohm-meter). What thickness must the resistor be?
ANSWER:
By equations 1.10 and 1.11 from the textbook,

$$R = \frac{\mathrm{L}}{\sigma \mathrm{S}} = \frac{\mathrm{L}}{\sigma \mathrm{Wh}},$$

where h is the thickness of the metal. Thus,

$$\mathrm{h} = \frac{\mathrm{L}}{\sigma \mathrm{W} R} = \frac{100x10^{-6}}{(7.7x10^{6})(5\mathrm{x}10^{-6})(1000)} = 2.6x10^{-9} \text{ meters} = 2.6 \text{ nanometers.}$$

2. From the periodic table of the elements and given that the density of chromium is 7.2 grams/cm^3, estimate the scattering time in chromium. Assume one free electron per atom.
ANSWER:
From the periodic table of the elements, chromium has an atomic mass of 52 atomic mass units, which are about the mass of a proton, which is 1.67x10^{-27} kg. Thus the mass of a chromium atom is 8.68x10^{-26} kg. Converting the density of chromium into MKS units,

$$7.2\frac{grams}{cm^3}x(\frac{1kg}{1000g})x(\frac{10^6 cm^3}{m^3}) = 7200\frac{kg}{m^3}.$$

Thus the density of free electrons is

$$N = 7200\frac{kg}{m^3}x\frac{1atom}{8.68x10^{-26}kg} = 8.3x10^{28}\frac{1}{m^3}.$$

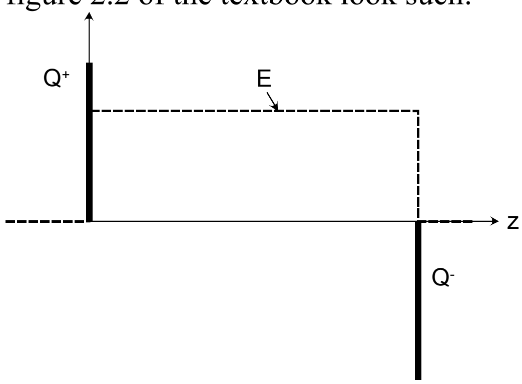
From equation 1.10 of the textbook,

$$\sigma = \frac{Ne^2\tau}{m},$$

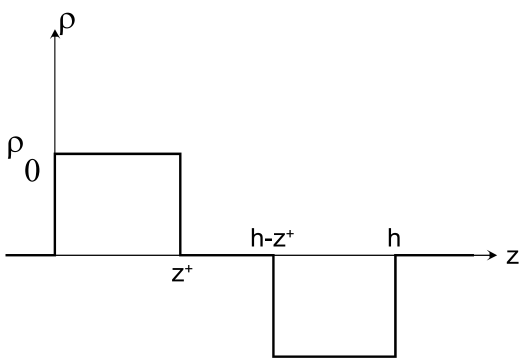
or

$$\tau = \frac{\sigma m}{Ne^2} = \frac{(7.7x10^6)(9.11x10^{-31})}{(8.3x10^{28})(1.6x10^{-19})^2} = 3.3x10^{-15} \text{ seconds.}$$

3. In lecture a capacitor is discussed, whose charge on its plates is taken to be infinitesimally thin. Even if the plate is not infinitesimally thin, the charge layer is, since charge cannot reside in a conducting medium, but will reside on the surface. Thus, the charge and electric field profile of the capacitor of figure 2.2 of the textbook look such:



Later in the course, we will show how sometimes an electronic element can have a distributed charge profile:



There is no variation in x or y. What is the electric field vs. z?

ANSWER:

The electric field is zero to the left of z=0 and to the right of z=h, since the total enclosed charge by such a volume is zero. Elsewhere, by the differential form of Gauss's Law, or Maxwell's first equation,

$$\nabla \bullet \mathbf{E} = \frac{\rho}{\varepsilon}.$$

Since there is no variation in x or y, this becomes for this problem

$$\frac{d\mathrm{E}_z}{dz} = \frac{\rho}{\varepsilon}.$$

Thus,

$$\frac{dE_z}{dz} = \frac{\rho_0}{\varepsilon} \qquad 0<z<z^+,$$

$$\frac{dE_z}{dz} = 0 \qquad z^+<z<h-z^+,$$

$$\frac{dE_z}{dz} = -\frac{\rho_0}{\varepsilon} \qquad h-z^+<z<h,$$

and

$$E_z = \frac{\rho_0}{\varepsilon} z \qquad 0<z<z^+,$$

$$E_z = \frac{\rho_0}{\varepsilon} z^+ \qquad z^+<z<h-z^+,$$

$$E_z = \frac{\rho_0}{\varepsilon} (h-z) \qquad h-z^+<z<h.$$