Abstract—Spatial reuse has been shown to be critical for efficient wireless networks. It is also well known that cooperative communication has the potential to significantly improve the network performance. How does cooperation affect the spatial reuse design in wireless networks? To answer this question, we investigate the impact of reuse on the throughput of multi-hop linear cooperative networks. The reuse that maximizes the network throughput is then derived. Simulation results are provided to illustrate the importance of choosing an appropriate reuse factor.

I. INTRODUCTION

In wireless multi-hop networks, spatial reuse is essential to balance the interference level and the channel utilization [1]. On the one hand, spatial reuse improves the channel utilization by enabling concurrent transmissions, which, in turn, can improve the network throughput. On the other hand, spatial reuse introduces co-channel interference that degrades the performance. Therefore, determining the optimal spatial reuse is critical for efficient multi-hop wireless transmission. There has been some research investigating optimal spatial reuse, particularly, in linear networks [2]-[4]. In addition, several practical schemes have been proposed to implement adaptive reuse by tuning the carrier-sensing thresholds [5]-[6].

Cooperative communication has been shown to be effective in combating fading on wireless channels, and thus has the potential to dramatically improve performance [7]-[9]. To exploit the potential benefits of cooperation in networks, several schemes have been proposed that make use of VMISO (Virtual Multiple-Input-Single-Output) [10] or VMIMO (Virtual Multiple-Input-Multiple-Output) [11]-[12] transmissions.

Although reuse in multi-hop linear networks is a well-studied topic, the impact of incorporating cooperation on spatial reuse scheduling is an open problem. In this paper, we investigate the impact of applying cooperative communication in multi-hop linear networks and derive the reuse scheduling that maximizes the network throughput. We show, through analysis and simulations, that cooperation with the appropriate reuse can dramatically improve the network throughput.

The rest of the paper is organized as follows: In Section II, a brief review of cooperative communication is provided. The analysis of spatial reuse in multi-hop linear cooperative networks is presented in Section III. In Section IV, we show the simulation results. Section V concludes the paper.

II. COOPERATIVE COMMUNICATION

Consider a system with one source (S) - destination (D) pair, as shown in Fig. 1. The traditional way to send information from S to D is by a direct link; if both terminals are equipped with a single antenna, this is a SISO (Single-Input-Single-Output) link. In contrast, S can first broadcast the message to its neighboring nodes, which are close enough and thus have good communication quality; then, together with the neighboring nodes, S can form a virtual multiple-antenna terminal to transmit the information to the destination. This cooperation among S and its neighboring nodes creates a VMISO channel and can provide a diversity gain.

Suppose there are L transmitters (including the source), and assume that Dis-STBC (Distributed Space-Time-Block-Coding) [13] is applied to coordinate the cooperation. Each node transmits a unique column of the L-column STBC matrix. Dis-STBC has been shown to provide full diversity in the number of cooperating nodes [13]. Denoting the channel coefficient from node i to D as $h_i$, the SNR of this link is

$$\gamma_i = \frac{P_i|h_i|^2}{P_N(d_i/d_0)\alpha}$$  \hspace{1cm} (1)

where $P_i$ is the transmit power at node i, $P_N$ is the noise power at the receiver, $d_i$ is the distance to the destination, $d_0$ is a reference distance (assumed here to be 1 meter), and $\alpha$ is the path-loss exponent. At the destination, using conventional coherent detection for STBC, the SNR is the sum of the SNRs of the individual links, that is, $\gamma = \sum_{i=1}^{L} \gamma_i$.

We assume identical transmit powers $P$ and transmission bit rates $r$ are used at all the nodes. We also assume the signal experiences i.i.d. Rayleigh fading; thus all the $|h_i|^2$ are exponentially distributed with mean 1. In general, the distances among neighboring nodes are much smaller than that between S and D. Hence, we assume that the distance from each node
in $S$’s neighborhood to $D$ is approximately the same (denote as $d$), which has been shown to be a reasonable assumption in practical wireless networks [10]. To demonstrate the benefits of cooperation, we compare the outage for traditional point-to-point communication (SISO) and cooperative communication (VMISO).

An outage occurs if the required transmission bit rate exceeds the channel capacity; hence, assuming additive white Gaussian noise, we have

$$p_{out} = Pr \left\{ B \log(1 + \gamma) < r \right\}$$

$$= Pr \left\{ B \log \left( 1 + \sum_{i=1}^{L} \frac{P|h_i|^2}{P_N d_i^\alpha} \right) < r \right\}$$

(2)

where $B$ is the available bandwidth, and $L = 1$ corresponds to the SISO link without cooperation. Since $|h_i|^2 \sim \text{Exp}(1)$, we have $\sum_{i=1}^{L} |h_i|^2 \sim \text{Erlang}(L, 1)$ [14]. Therefore

$$p_{out} = Pr \left\{ \sum_{i=1}^{L} |h_i|^2 < (\frac{2\pi}{P} - 1) P_N d_i^\alpha \right\}$$

$$= 1 - \sum_{j=0}^{L-1} \left( \frac{2\pi}{P} - 1 \right)^j P_N d_i^\alpha \left( \frac{2\pi}{P} - 1 \right)^j P_N d_i^\alpha$$

(3)

where $\eta = r/B$ is the normalized bit rate (spectral efficiency).

III. MULTI-HOP LINEAR COOPERATIVE NETWORKS

A. System Model

First, we consider one-dimensional wireless networks, also called linear or chain networks. Although linear networks are unrealistic, they are widely-used models to initiate the study of complicated network problems and have been shown to be capable of revealing insights that inspire practical solutions.

Multi-hop linear cooperative networks are constructed by applying cooperative routing algorithms that make use of VMISO (for example, see [10]). As shown in Fig. 3, we assume there are $N$ clusters and $N$ hops from $S$ to $D$. Each cluster is a group of neighboring nodes that cooperatively forwards the message to the next hop. Clearly, this cooperative network is based on a SISO route between $S$ and $D$, that is, $S \rightarrow 1 \rightarrow \cdots \rightarrow i \rightarrow \cdots \rightarrow D$. The intermediate nodes along the SISO route are the cluster-heads, which take charge of disseminating the information among the neighborhood.

![Multi-hop linear wireless network](image)

Fig. 3. A multi-hop linear wireless cooperative network

The data transmission from $S$ to $D$ works as follows: The source, $S$, broadcasts the message, which its neighboring nodes receive and decode. Since the distances among the neighboring nodes are usually quite small, we assume that, by applying error correction, the local information dissemination is perfect in the sense that all the nodes in the cluster will have a correct copy. The cluster, including $S$ itself and all its neighbors, then forwards the signal to the next cluster, specifically, node 1, in the Dis-STBC manner described in Section II. Upon receiving and decoding the message without outage, node 1 disseminates the message among its own neighborhood and, together with its neighboring nodes, forms a VMISO link to forward the message to the next cluster. This process continues until the message reaches the destination.

Clearly, every hop along this network is a VMISO channel, which reduces the outage significantly compared with a SISO channel. However, it also complicates the transmission by introducing an extra local message dissemination process, which doubles the per-hop transmission duration and thus halves the channel utilization (assuming the broadcast inside the cluster and the transmission between clusters use the same data rate). Therefore, a factor of $1/2$ is included in the following analysis to represent this efficiency loss.

B. Impact of Spatial Reuse on Outage

In the multi-hop linear cooperative network shown in Fig. 3, we denote the clusters as $C_0, C_1, \cdots, C_i, \cdots, C_{N-1}$ with cluster-heads, respectively, nodes $S$ (the source), $1, \cdots, i, \cdots, N-1$. Each cluster has $L$ members, which can be determined by the routing algorithm. We can then formulate the transmission between any two consecutive clusters as a VMISO channel with $L$ transmitters. As in Section II, assuming identical per-hop distance, $d$, and bit rate, $r$, with
i.i.d. Rayleigh fading among all the links, the transmission has the same outage $p_{out}$, as given in (3).

Given the per-hop data rate $r$, the per-hop throughput can be written as $\frac{1}{r}Pr(1-p_{out})$, which reflects the rate of successfully delivered bits. If no spatial reuse is applied, that is, at any time, there is only one active hop in the network, the throughput of the network is $1/N$ of the per-hop throughput. This is very inefficient, especially when $N$, the number of hops, is large.

To improve the throughput, a spatial reuse factor $K (2 \leq K \leq N)$ is introduced such that nodes $K$ hops away can transmit simultaneously. It is well understood that, in spite of the interference introduced by reuse, an appropriate reuse scheme could dramatically improve the network throughput. On the one hand, with a smaller $K$, more nodes can transmit concurrently, which increases the channel utilization, and, hence, has the potential to improve the throughput. On the other hand, a smaller $K$ suggests more “aggressive” reuse, which introduces more interference and has the potential to degrade the throughput.

In order to obtain a tractable, yet characteristic-conserving, model to investigate the impact of spatial reuse, we assume that (i) the interference coming from the adjacent two interferers is dominant (this is reasonable, especially when the path-loss exponent is large); and (ii) the network is so large that almost all nodes have more than one interferer, that is, the number of the cluster-heads that are close to the source or the destination and have only one interferer is much smaller than the number of all the intermediate cluster-heads. With these assumptions, only the interference coming from $K$ hops to the left of the transmitting cluster and $K$ hops to the right are considered. The per-hop outage is then

\[ p'_{out} = Pr \left\{ \log \left( 1 + \frac{Pr|g_1|^2 d^{-\alpha}}{Pr|g_2|^2 d^{-\alpha} + Pr|g_3|^2 d^{-\alpha} + Pr_N} \right) < \eta \right\} \approx Pr \left\{ |g_1|^2 < (2^{\eta - 1}) \left[ \frac{|g_2|^2}{(K-1)^\alpha} + \frac{|g_3|^2}{(K+1)^\alpha} \right] \right\} \]

(4)

where $|g_1|^2$ is the gain of the VMISO link under consideration, and $|g_2|^2$ and $|g_3|^2$ are the gains for the interfering links. The noise power is assumed negligible compared with the interference; the impact of $P_N$ is included in the simulations.

As discussed in Section II, for Dis-STBC, diversity gain is achieved and $|g_1|^2 \sim \text{Erlang}(L, 1)$; for the interfering links, no coordination is applied, so $|g_2|^2 \sim \text{Exp}(1/L)$ and $|g_3|^2 \sim \text{Exp}(1/L)$. Therefore,

\[ p'_{out} = \int_{x_1 < a_1 x_2 + a_2 x_3} x_1 L e^{-x_1} e^{-\frac{x_2}{L}} e^{-\frac{x_3}{L}} dx_1 dx_2 dx_3 \]

(5)

where $x_m = |g_m|^2, m = 1, 2, 3,$ and $a_1 = \frac{2^{\eta - 1}}{(K-1)^\alpha}, a_2 = \frac{2^{\eta - 1}}{(K+1)^\alpha}$. Note that $a_1 > a_2 > 0$, so,

\[
p'_{out} = Pr \{ x_1 < a_1 x_2 + a_2 x_3 \} \leq Pr \{ x_1 < a_1 (x_2 + x_3) \}
\]

\[
(i) \int_{x < a_1 y} x^{L-1} e^{-x} \frac{y e^{-\frac{y}{L}}}{(L-1)!} dx dy
\]

\[
(ii) \frac{1}{L^2 (L-1)!} \frac{a_1^L \Gamma(L+2)}{L(a_1 + \frac{1}{L})^{L+2}} \sum_{n=0}^{\infty} \frac{L + n}{L + 1} \left[ \frac{a_1}{a_1 + \frac{1}{L}} \right]^n
\]

\[
(iii) \frac{a_1^L (a_1 + 1 + \frac{1}{L})}{(a_1 + \frac{1}{L})^{L+1}} \triangleq p_{out}^\alpha
\]

(6)

where we substitute $x = x_1$ and $y = x_2 + x_3 \sim \text{Erlang}(2, \frac{1}{L})$ in step (i), $\Gamma(z)$ is the gamma function. Step (ii) comes from [16]. Note that $\frac{a_1}{a_1 + \frac{1}{L}} \in (0, 1)$, so, we can easily obtain the result in step (iii). Similarly,

\[
p'_{out} \geq Pr \{ x_1 < a_2 (x_2 + x_3) \} = \frac{a_2^L (a_2 + 1 + \frac{1}{L})}{(a_2 + \frac{1}{L})^{L+1}} \triangleq p_{out}^{lo}
\]

(7)

Recalling that

\[
a_2 = \frac{2^{\eta - 1}}{(K+1)^\alpha} \leq a = \frac{2^{\eta - 1}}{K^\alpha} \leq a_1 = \frac{2^{\eta - 1}}{(K-1)^\alpha}
\]

we apply the approximation

\[
\bar{p}_{out} = \frac{a^L (a + 1 + \frac{1}{L})}{(a + \frac{1}{L})^{L+1}}
\]

(8)

Clearly, $a = \frac{2^{\eta - 1}}{K^\alpha}$ increases when $\eta$ increases and/or $K$ decreases. From (8), given the fact that $L \geq 1$ and $a > 0$, $\bar{p}_{out}$ increases when $a$ increases. Therefore, with fixed $L$, $\bar{p}_{out}$ increases when $K$ decreases (more interference) and/or $\eta$ increases (higher channel quality requirement). In addition, with fixed $K$ and $\eta$, the outage decreases when $L$ increases, indicating the benefits of cooperation. These conclusions can be seen from the results in Fig. 4. We can also see that (8) is a good approximation, especially when $K$ is large.
C. Optimal Spatial Reuse Scheduling

In this section, we will determine the spatial reuse factor that maximizes the network throughput. As discussed before, in a network without reuse, the throughput is \( \frac{r(1 - p_{out})}{2N} \), which represents the number of bits successfully delivered from \( S \) to \( D \) per second. If reuse factor \( K \) is used, the throughput is
\[
\frac{N}{K} \frac{r(1 - p_{out})}{2N},
\]
where \( N \) is the average number of active hops and \( p_{out} \) is the outage including the interference. To find the optimal reuse factor, we formulate the following problem
\[
\max \frac{r(1 - p_{out})}{2K}, \quad \text{s.t.} \ K \in \mathbb{Z}, \ 2 \leq K < N \quad (9)
\]
Substituting (8) into (9), we have
\[
\max \frac{r}{2K} \left[ 1 - \frac{(\frac{2^{\alpha_L - 1}}{K + \beta})^L (\frac{2^{\alpha_L - 1}}{K + \beta})^{L+1}}{(\frac{2^{\alpha_L - 1}}{K + \beta})^{L+1}} \right] \quad (10)
\]
where \( \beta = \frac{2^{\alpha_L - 1}}{K+\beta} \). It is difficult to obtain a closed-form solution to (11), especially when \( L \) is large. Therefore, instead, we plot the numerical results to illustrate the properties of \( K^\dagger \).

Assuming \( \alpha = 4 \), the optimal reuse, computed via exhaustive search, and the approximation, \( K^\dagger \), are shown in Fig. 5. The approximation performs well, especially when the spectral efficiency is large. In addition, a larger \( K \) is required if the efficiency increases, which indicates a higher sensitivity to interference. Moreover, cooperation allows more efficient channel utilization, that is, the optimal reuse factor decreases when \( L \) increases. This means that the diversity advantage introduced by cooperation overcomes the disadvantage, i.e., more interference power due to more transmitting nodes.

IV. RESULTS AND DISCUSSIONS

Considering a multi-hop linear network with a large enough number of hops (that is, the reuse factor is much smaller than the number of hops), we assume the transmit power at each node is 20 dBm, the noise power spectral density is \( -174 \) dBm/Hz, and \( B = 10 \) MHz. We study the impact of cooperation and reuse on the network throughput.

A. Impact of Cooperation

We have shown that cooperation can effectively reduce the outage. However, incorporating cooperation also incurs inefficiency, specifically, the factor of 1/2. A natural question arises: Is it always beneficial to apply cooperative communication? Intuitively, when the SISO link is good enough, cooperation will degrade the throughput because the inefficiency will be greater than the benefits; when the SISO link is poor, the improvement provided by cooperation has the potential to overcome the inefficiency and thus increase the throughput. There has been a rich body of literature investigating when to apply cooperation (e.g., see [17]). To illustrate, we plot, in Fig. 6, the normalized throughput when the optimal reuse factor is used.

As shown in Fig. 6, when the channel suffers large attenuation resulting from the large partition \( (d \geq 300 \) m), cooperation can provide a significant increase in the network throughput, and the improvement increases as \( d \) increases. On the other hand, when the channel has acceptable performance \( (d \leq 280 \) m), the inefficiency from the extra local information dissemination surpasses the outage reduction provided by cooperation. The best achievable throughput with cooperation is actually smaller than that without cooperation. Although the crosspoint \( (d \approx 280 \) m in Fig. 6) might change if different system parameters (transmit power, required spectral efficiency, and so on) are used, there is always an operating point beyond which cooperation will provide significant benefit.

B. Impact of Spatial Reuse

Assuming the per-hop distance \( d = 400 \) m and the per-hop efficiency is 4 bits/s/Hz, the impact of reuse on the network throughput with different degrees of cooperation is presented in Fig. 7. Even with the efficiency loss from the local message dissemination, cooperation significantly improves the throughput. The optimal throughput with \( L = 8 \) is more than 10 times that without cooperation \( (L = 1) \). In addition, we observe that, given \( L \), there is a reuse factor that maximizes the throughput. For example, if \( L = 8 \), the optimal throughput is achieved when \( K = 5 \). The optimal reuse factor balances the channel utilization and the interference level.

Comparing the results in Figs. 5 and 7, the values of the optimal reuse factors are slightly larger than what we derived.
that (i) appropriate reuse is essential for efficient networks, and (ii) incorporating cooperation is effective in improving the network performance when the SISO links are suffering high outages. In addition, the reuse factor that maximizes the network throughput was derived, the performance of which was illustrated in the simulation results.

Linear networks are good models to begin with, but more practical network models will be considered in the following research, for example, grid networks. Furthermore, the practical implementation of cooperation and spatial reuse in wireless networks is also an interesting and important topic, in particular, the trade-off between the overhead incurred in the practical implementations and the performance improvement.

REFERENCES


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Fig. 6. Normalized network throughput as a function of per-hop distance in Section III-C. For example, when $\eta = 4$ bits/s/Hz and $L = 8$, from Fig. 5, $K^\dagger = 4$; while, from Fig. 7, the optimum is 5. The difference comes from the assumption that the noise power is negligible compared with the interference. In realistic scenarios, of course, the optimal reuse factor might be greater than $K^\dagger$ because of the increased power of the interference and noise; however, the difference is usually small, and thus, $K^\dagger$ still provides an effective guideline for choosing the reuse factor.

V. CONCLUSIONS

We investigated the spatial reuse scheduling in multi-hop linear cooperative networks. The impact of enabling spatial reuse and incorporating cooperation on the network throughput was addressed through analysis and simulations. We showed