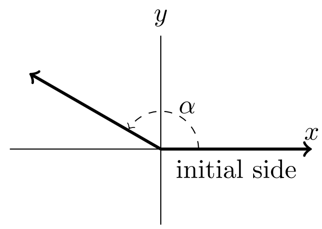
Mr. ROARabaugh's Predraculus Class

Halloween 2013

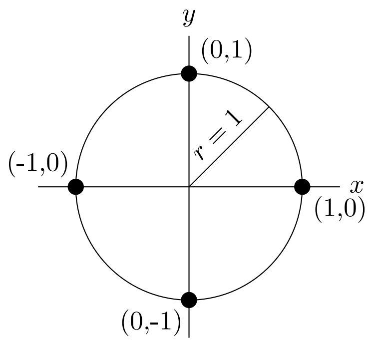
Disection 5.2 - The Sine and Cosine Monsters

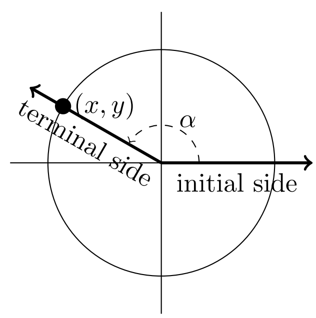
In yesterday's creepy adventure (Disection 5.1) we learned about angles, which can be measured in *degrees* (page 406) or in *radians* (page 410). In particular, we talked about our angles being in *standard position*, with the vertex on the origin, the initial side on the positive x-axis, and the rotation counterclockwise.



#1. The angle in the image above has a measure of 150°. What is its measure in radians?

Now we are going to construct a rickety ferris wheel with a circle of radius 1, centered at the origin. Half of our ferris wheel is in the dark where it goes underground - that is, has negative y-values. When we put an angle in standard position, let us look at the point where the *terminal side* intersects the unit circle; we can pretend that point is a frightened person riding the ferris wheel.





Out jump two monsters, named **Sine** and **Cosine**! These functions eat angles - and their associated ferris wheel riders - and spit out information about the victims:

<u>Definition</u> (page 421): If α is an angle in standard position and (x, y) is the point of intersetion of the terminal side and the unit circle, then

$$\sin(\alpha) = y \quad \text{and} \quad \cos(\alpha) = x.$$

#2. The terminal side of the angle with measure 90° (or $\pi/2$ in radians), intersects the unit circle at $(0, 1)$. From the definition, $\sin(90°) = 1$ and $\cos(\pi/2) = 0$. Find the function output for each of the following:
(a) $\sin(0)$ (b) $\cos(180°)$ (c) $\sin(-\pi/2)$ (d) $\cos(720°)$

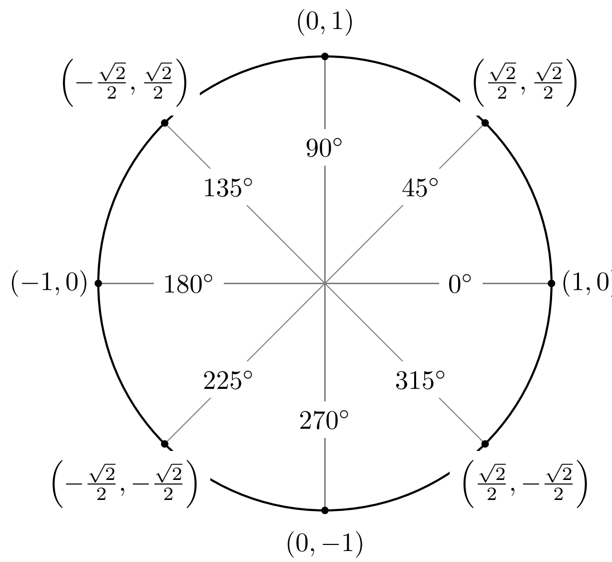
#3. Suppose angle α has measure 45°, then the intersection point (x, y) is such that $x = y$.
(a) In the margin below, draw a unit circle centered at the origin with this angle α. Then using the terminal side of the angle as a hypotenuse, draw two legs of equal length, forming a right triangle.
(b) The horizontal leg has length x and the vertical leg has length y. Use $x = y$ and the Pythagorean Theorem to find (x, y).

Using what we learned from #3, we can figure out what our monsters expectorate after masticating other riders on the unit ferris wheel:

$$\sin(135°) = \frac{\sqrt{2}}{2} \text{ and } \cos(135°) = \frac{-\sqrt{2}}{2}$$

$$\sin(225°) = \frac{-\sqrt{2}}{2} \text{ and } \cos(225°) = \frac{-\sqrt{2}}{2}$$

$$\sin(315°) = \frac{-\sqrt{2}}{2} \text{ and } \cos(315°) = \frac{\sqrt{2}}{2}$$



#4. Use the farris wheel (unit circle) above to find the function output for each of the following:
(a) $\cos(5\pi/4)$ (b) $\sin(-135°)$ (c) $\cos(405°)$ (d) $\sin(-\pi/4)$

#5. Now suppose α has measure 60°.
(a) As done in #3, draw a unit circle centered at the origin with this angle α. Then using the terminal side of the angle as a hypotenuse, draw two legs of equal length, forming a right triangle.
(b) What is the sum of the measures of the three angles of a triangle?
(c) In the right triangle from (a), what is the measures of each of the three angles?
(d) What are the angles in an equilateral triangle?
(e) By now, you might notice that your triangle is half of an equilateral triangle. Thus the horizontal leg has half the length of the hypotenuse. Calculate the length of the vertical leg. See page 423 in the textbook if you get stuck.
(f) Use (e) to find $\sin(60°)$ and $\cos(60°)$.

Using what we learned from #5, we can figure out what our monsters expectorate after masticating other riders on the unit ferris wheel:

$$\sin(120°) = \frac{\sqrt{3}}{2} \text{ and } \cos(120°) = \frac{-1}{2}$$

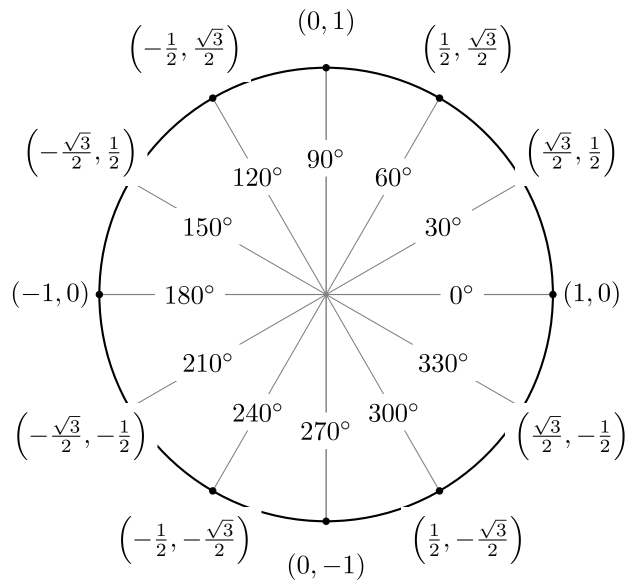
$$\sin(150°) = \frac{1}{2} \text{ and } \cos(150°) = \frac{-\sqrt{3}}{2}$$

$$\sin(210°) = \frac{-1}{2} \text{ and } \cos(210°) = \frac{-\sqrt{3}}{2}$$

$$\sin(240°) = \frac{-\sqrt{3}}{2} \text{ and } \cos(240°) = \frac{-1}{2}$$

$$\sin(300°) = \frac{-\sqrt{3}}{2} \text{ and } \cos(300°) = \frac{1}{2}$$

$$\sin(330°) = \frac{-1}{2} \text{ and } \cos(330°) = \frac{\sqrt{3}}{2}$$



#6. Use the unit circle (ferris wheel) above to find the function output for each of the following:
(a) $\sin(-\pi/6)$ (b) $\cos(-120°)$ (c) $\sin(420°)$ (d) $\cos(4\pi/6)$

We can now say what the sine and cosine funtions do to quite a few angles in standard position. The next page shows a summary of our current knowledge. On your quiz on Monday, one of the questions will be $\sin(\alpha)$ and $\cos(\alpha)$ for one of the angles shown, with α in degrees or radians.

We know that when α is in regular position, its terminal side intersects the unit circle at $(x, y) = (\cos(\alpha), \sin(\alpha))$. To help you remember the x- and y-values on the next page, you might want to use this list of numbers:

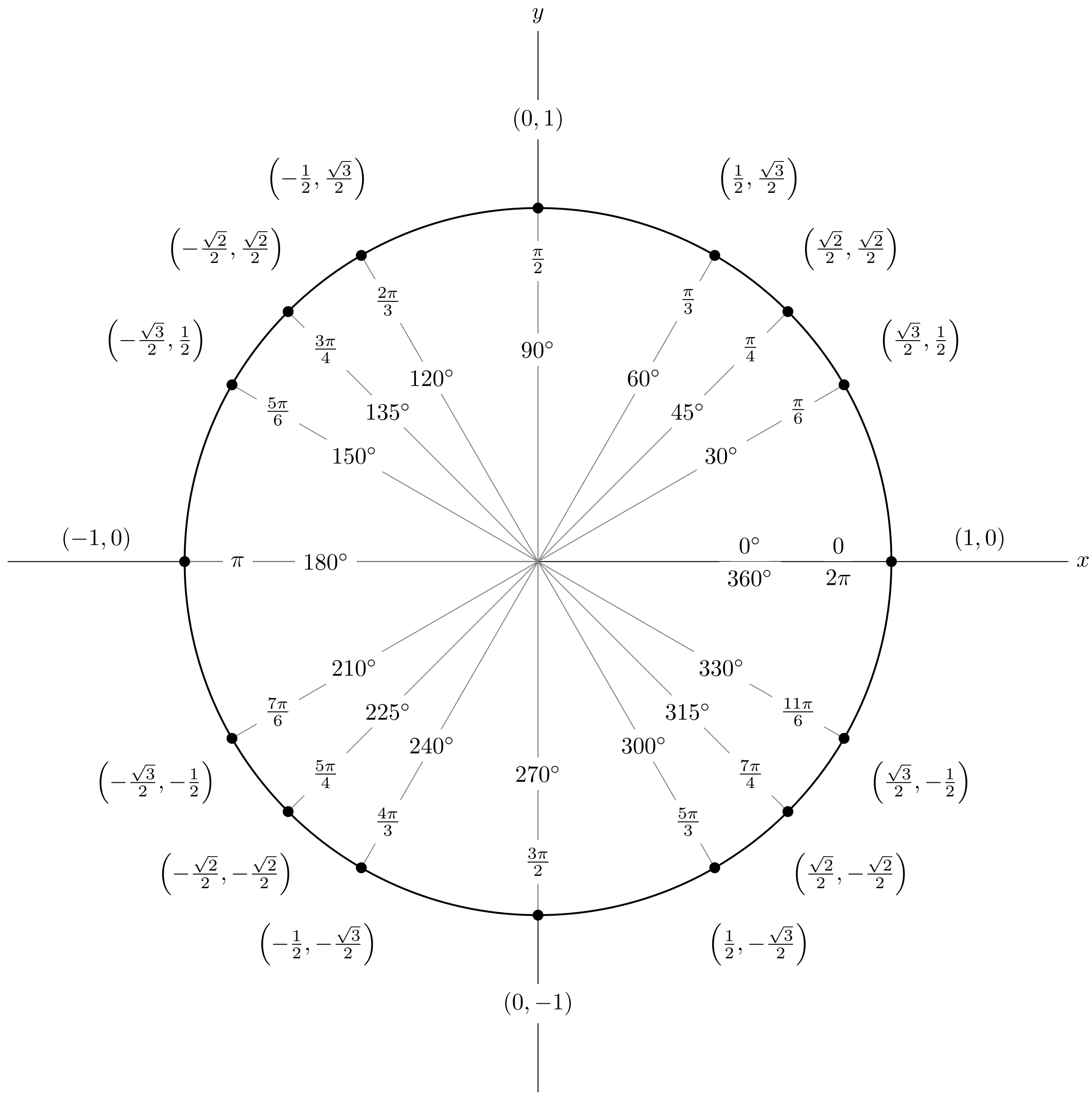
$$\frac{\sqrt{0}}{2} = 0 \quad , \quad \frac{\sqrt{1}}{2} = \frac{1}{2} \quad , \quad \frac{\sqrt{2}}{2} \quad , \quad \frac{\sqrt{3}}{2} \quad , \quad \frac{\sqrt{4}}{2} = 1 \quad .$$

By thinking about where your angle is in the unit circle, you can figure out which of the five numbers to use, and whether it should be positive or negative.

Now let us arm ourselves with one more definition to help fight against trignometric monsters and weeping angles:

Definition (page 425): If angle θ is in standard position and its measure is not a multiple of $\pi/2$, then its reference angle θ' is the positive *acute* angle formed by the terminal side of θ and either the positive or negative x-axis, whichever is closer.

#7. For each of the angles below, find the reference angle (in the same units). For examples, see page 425.
(a) $\pi/4$ (b) $100°$ (c) $10\pi/3$ (d) $-281°$



To understand this picture:

- First, go around the circle just looking at the angle measures; make sure you understand the values given in degrees and in radians.
- Second, go around the circle just looking at y values, watching how they increase and decrease as the height of your point changes; this is what your sine function looks at.
- Third, go around the circle just looking at the x values; this is what the cosine function looks at.

To help you prepare for Disection 5.3 on Monday:

- On a graphing calculator, find out how to change it between the degree setting and the radian setting.
- With your calculator set to radians, graph $y = \sin(x)$ and draw a copy of the graph below. From the unit circle, you can see that the range of your graph will be $[-1, 1]$.
- Look at the Wikipedia page for *sine* and find the animated gif that shows the relationship between the unit circle (above) and the graph of $y = \sin(x)$ (below).