A Simple Distributed Autonomous Power Control Algorithm and its Convergence

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***Abstract*— For wireless cellular communication systems, one seeks a simple effective means of power control of signals associated with randomly dispersed users that are reusing a single channel in different cells. By effecting the lowest interference environment, in meeting a required minimum signal-to-interference ratio of ρ per user, channel reuse is maximized. Distributed procedures for doing this are of special interest, since the centrally administered alternative requires added infrastructure, latency, and network vulnerability. Successful distributed powering entails guiding the evolution of the transmitted power level of each of the signals, using only *local* measurements, so that eventually all users meet the ρ requirement. The local per channel power measurements include that of the intended signal as well as the undesired interference from other users (plus receiver noise). For a certain simple distributed type of algorithm, whenever power settings exist for which all users meet the ρ requirement, we demonstrate exponentially fast convergence to these settings.**

I. Introduction

For wireless cellular communication systems, one seeks a simple effective means of power control of the base-to-mobile signals, for a channel that is being reused by J randomly dispersed customers in different cells. (For example, the channel can be a TDMA or FDMA channel.) For expositional simplicity, we will concentrate here on the down-link power control problem; however, it will be made clear that what we have to say generalizes easily to the combined up-link–down-link problem.

Using no more power than is required in meeting a minimum signal-to-interference ratio constraint of ρ per user is critical for efficient channel use, since by effecting the lowest interference environment, channel reuse is maximized. Our consideration here of power control for a static propagation model is a step toward eventually understanding algorithms for users moving about in a time-varying propagation environment.

Distributed power control is of special interest, since the alternative of centrally orchestrated control involves added infrastructure, latency, and network vulnerability. Successful distributed power control entails guiding the evolution of the power level of each of the J base-launched signals, using only *local* measurements, so that eventually all users meet the ρ requirement. The local per channel power measurements, made at each user's site and relayed to the corresponding base, include that of the intended signal as well as the combined interference from other users (plus receiver noise). Interesting questions include: Given J deployed users and bases, is it possible to meet the ρ constraint even with a central controller of power levels? When it is possible, can these same power levels be achieved with a distributed algorithm? If so, how fast can such a distributed power control respond? We aim to help develop answers to these questions here.

Specifically, we show that for a certain simple type of distributed algorithm that we will specify, whenever there exist power settings that meet the ρ constraint for all users, exponentially fast convergence to these desired settings occurs. Perhaps the very simplest instance of the algorithm type, and an associated convergence result, goes as follows: Each user proceeds to iteratively reset its power level to what it needs to be to have acceptable performance as if the other users were not going to change their power level. Yet the other users are following the same algorithm and, therefore, are changing their power levels. No matter; for each user, exponentially fast convergence to the desired performance occurs with this local distributed algorithm any time a central controller can achieve the desired performance.

For analytical work on power control in wireless systems see [1]–[9].[1] J. Zander, in [3], (which builds on the basic reference [2]) as well as Zander's companion paper, [4], report interesting initial work on distributed power control.[2] Namely, an iterative scheme is presented that operates under the assumption that the transmitter power is sufficiently high in order to allow receiver noise to be neglected. The power of the signals for the J users is evolved to achieve the greatest signal-to-interference ratio that they are jointly capable of achieving (the same ratio for all J users). The algorithm is provably optimum in terms of maximizing the minimum signal to interference ratio. The *absolute* power setting turns out to be problematic. Zander raises the issue of how it should be set as an open topic that needs to be addressed to establish whether the scheme can be completed in a fully distributed way. As will be apparent to those readers familiar with the iterative scheme in [3] and [4], while there are strong similarities, key features differ from the algorithm that will occupy us here. Particularly, our inclusion of receiver noise in the definition of interference avoids the difficulty with absolute power settings. Here we will stress defining an algorithm that is totally local in that only

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[1] References [3], [7], and [8] provide an up-to-date listing of references, including several of special interest for the area of spread spectrum communications. The papers [3]–[9] include numerical studies for various cellular propagation models. Papers [6]–[9] also include a preliminary look at some dynamic channel allocation considerations.

[2] Reference [5] reports an improved variant of Zander's scheme that has superior convergence properties.

the power and interference measurements of each base–mobile link is used to evolve power levels on that link. Also, as we have already mentioned, we will work with a prefixed target ρ that our algorithm will succeed or fail in attaining for all users, rather than the interesting, but different, floating "best-we-can-do-under-the-circumstances" type of objective considered in [3] and [4].

II. Notation

In preparation for introducing the algorithm that is the topic of the present paper and establishing its convergence, we introduce the following notation:

- *Base-to-user propagation matrix:* A. In this report, A is a random constant $J \times J$ matrix having only nonnegative entries that are strictly positive along the diagonal. We use $a_{[ij]}$ for the ijth entry of A so that $A = [a_{ij}]$. Let σ_{ij} be one when $i = j$ and $-\rho$ otherwise. The matrix A and the diagonally normalized variant that we will employ, $B = [\sigma_{ij} a_{ij}/a_{ii}]$, are each assumed to be sufficiently random to have J distinct eigenvalues with probability one. For example, users could be randomly placed uniformly, over space of such a geographical scope as to include a large number of cells, and signal loss could follow a decay law complicated by lognormally distributed fading. (In more refined studies, A can be taken to be a matrix random process incorporating the element of Raleigh fading.)
- *J-dimensional power vector:* $p(t)$. The jth component, $p_j(t)$, denotes the power of the signal launched at the jth base intended for the jth user.
- *Power of the signal at the* ith *user site from the* jth *base:* $a_{ij}p_j(t)$.
- *Power of the additive receiver noise at the user sites:* ν.
- *Interference power at the* ith *user:* $\iota_i(t) = \Sigma_{j \neq i} a_{ij} p_j(t) + \nu$. As already mentioned, we are taking the liberty of using the term "interference power" to include receiver noise as well as intercell interference.
- *Signal-to-interference ratio at the* ith *user:* $\rho_i(t) = a_{ii}p_i(t)/\iota_i(t)$.

III. Distributed Algorithms: Definitions and Convergence Theory

Our focus here is on defining the dynamics of a simple class of power control algorithms and analyzing convergence properties. These algorithms seek to effect, the quicker the better, the determination of whether or not J users can achieve a signal-to-interference ratio of ρ.

In Section III.A., primarily for the purpose of motivating a type of power control algorithm, we start with a simple, idealized, continuous time view of power level evolution. Aside from its motivational feature, the continuous time view also serves to offer a simple model of power control for future, high-level, wireless system studies. Then, in Section III.B., we shift to a discrete time version of this type of algorithm. The more realistic discrete time model exhibits a stability issue that is glossed over when the continuous time model is used.

A. Differential Equation Form of Algorithm

A differential dynamic that might seem desirable (if it was attainable) is to have the ith user evolve his signal-to-interference ratio, $\rho_i(t)$, to drive it towards the desired amount ρ by an amount proportional to the offset from ρ. Expressing this dynamic using β to denote the (necessarily positive) proportionality constant, we have

$$\dot{\rho}_i(t) = -\beta[\rho_i(t) - \rho]. \tag{1}$$

This dynamic cannot be implemented in a distributed manner: While the ith user is assumed to have direct control over his own power, $p_i(t)$, he is assumed to have no *direct* control over the power level for the other users.

In place of (1), we need a dynamic that is easily synthesized using only local measurements. To get such a substitute for (1), we take a cue from the circumstance that the ith user does not have direct knowledge or control of how $\iota_i(t)$ will change. So we simply have the ith user strive to evolve ρ_i as if ι_i were not going to change. (Despite this method of devising a local autonomous dynamic, we stress that interference levels can indeed change, and those changes will be fully accounted for in our algorithm analysis.) In other words, we compose a dynamic by replacing the derivative of ρ_i in equation (1) by a "surrogate derivative of ρ_i," computed as if the ι_i denominator held constant. The equation for this new dynamic is, therefore,

$$a_{ii}\dot{p}_i(t)/\iota_i(t) = -\beta[\rho_i(t) - \rho]. \tag{2}$$

We stress that we are aiming for a distributed control law so that only ρ_i and p_i (and ρ) appear on the right-hand side of the equation. Multiplying through by ι_i/a_{ii} enables us to achieve this. Specifically, we get the key differential equation for guiding the evolution of $p_i(t)$:

$$\dot{p}_i(t) = -\beta\left[p_i(t) - (\rho/a_{ii})\left[\nu + \sum_{j \neq i}(a_{ij})p_j(t)\right]\right], \quad (i = 1, 2, \cdots, J). \tag{3}$$

As with (1), this dynamic cannot stop unless $\rho_i(t) = \rho$ for each i.

As background for investigating convergence of $p(t)$, we need the following result paraphrased from reference [11]:

Lemma: If a $J \times J$ matrix has nonnegative entries off the diagonal, and if it maps a J-vector, all of whose entires are positive, into a J-vector, all of whose entires are negative, then each of the J eigenvalues of the matrix has negative real parts.

With this Lemma we can now prove:

Proposition: If there is a power vector p^*, for which the desired ρ_i values are attained, then no matter what the initial $p_i(0)$, each of the $p_i(t)$ evolving according to (3) will converge to p^*.

Proof: To explore the convergence properties of (3) it is expendient to convert to vector form. For that, the following definition is useful:

$$\eta = \rho[(\nu/a_{11}), (\nu/a_{22}), \cdots (\nu/a_{JJ})]'. \tag{4}$$

We can now rewrite (3) as a vector differential equation

$$\dot{p}(t) = -\beta B p(t) + \beta\eta. \tag{5}$$

At the desired p^*, we have

$$Bp^* = \eta. \tag{6}$$

The solution to such a linear, constant coefficient, differential equation with initial setting of $p(0)$ is, [10],

$$p(t) = e^{-\beta Bt} \int_0^t e^{\beta B\tau}\, d\tau \beta\eta + e^{-\beta Bt} p(0). \tag{7}$$

(Recall that the exponential function of a matrix is defined by the standard power series for the exponential function.) The lemma, coupled with (6), enables us to conclude that all the eigenvalues of $-B$ have negative real parts. Since B is diagonalizable, we can factor B as

$$B = Q\Lambda Q^{-1} \tag{8}$$

were Λ is a diagonal matrix, all of whose entries have positve real parts. By analyticity of the matrix exponential function

$$e^{-\beta Bt} = \sum_0^\infty (-\beta t Q\Lambda Q^{-1})^n / n! = Qe^{-\beta\Lambda t}Q^{-1}. \tag{9}$$

We use (9) to substitute into (7), and then take a limit to get

$$\lim_{t\to\infty} p(t) = \lim_{t\to\infty} Qe^{-\beta\Lambda t} \int_0^t e^{\beta\Lambda\tau}\, d\tau Q^{-1}\beta\eta. \tag{10}$$

The $p(0)$ term is absent since $\lim_{t\to\infty} e^{-\beta Bt} p^{(0)}$ is the all zero vector by virtue of the positivity of the real parts of the eigenvalues of B. Equation (10), after integrating, becomes

$$\lim_{t\to\infty} p(t) = \lim_{t\to\infty} Qe^{-\beta\Lambda t}(e^{\beta\Lambda t} - I)\beta^{-1}\Lambda^{-1}Q^{-1}\beta\eta \tag{11}$$

where I denotes the identity matrix. It is easy to see that the evaluated limit is

$$= (Q\Lambda^{-1}Q^{-1})\eta = B^{-1}\eta = p^* \tag{12}$$

demonstrating the required convergence.

From (11), it is evident that the "time constant" of the $p(t)$ evolution is $(\beta \times \alpha_B)^{-1}$, where α_B is the real part of the eigenvalue of the B matrix that possesses the smallest real part. So the marginality of the convergence situation is measured by real part of $(\beta \times \alpha_\beta)^{-1}$: as $\beta \times \alpha_\beta$ tends to zero, convergence is slower and slower.

The convergence result generalizes easily: in (3) the thermal noise level, ν, the desired signal-to-interference ratio, ρ, and the proportionality factor, β, can instead be functions of the user index, i; so we have ν_i, ρ_i and β_i in (3) in place of ν, ρ and β. We are still assured of convergence. This is demonstrated in essentially the same way as when these vectors were required to have equal entries. The generality has various applications. For example, allowance for different ρ_k is useful, since different ρ_k can be associated with different services or grades of service. Also, different β_k can be used for discriminatory endowing of user responsiveness to changing conditions.

B. More Precise Difference Equation Form of Algorithm

In the discrete time set-up that we investigate next, we conveniently define the time coordinate so that unity is the time between consecutive power vector iterations. A substantial multiple of the transmitted data symbol period could be used for this unit time. We employ a captial letter for the iterated power vector so that $P(k)$ is the power vector at the time $k = 0, 1, 2 \cdots$.

In correspondence with (5), we write the difference equation

$$P(k+1) - P(k) = -\beta BP(k) + \beta\eta. \tag{13}$$

As with (3), the distributed control law must have it that $P(k+1)$ is determined in terms of $P(k)$ and ρ_i (and ρ). We stress that this is the case by rewriting (13) in the form

$$P_i(k+1) = (1-\beta)P_i(k)[1 + (\beta/(1-\beta))(\rho/\rho_i)] \tag{13a}$$

which also serves to reveal best the differences with references [4] and [5]. See, for example, the component form of the iteration appearing in the first sentence of the proof in Section III of [4]. See also Equation (6) of [5].

Proposition: Whenever a centralized "genie" can find a power vector p^* meeting the desired criterion, then so long as β is appropriately chosen, the solution to (13a) starting from any initial vector $P(0)$ converges to p^*.

Proof: A substitution into (13a) verifies that the solution of this equation starting from the initial vector $P(0)$, is

$$P(k) = (I + C + C^2 + \cdots C^k)\beta\eta + C^k P(0),$$
$$(C = I - \beta B). \tag{14}$$

From (8) we can write $C^k = Q(I - \beta\Lambda)^k Q^{-1}$, where the diagonal entries of $I - \beta\Lambda$ are the eigenvalues of C. The Q matrix diagonalizes all powers of C. Therefore, as long as the moduli of each of the eigenvalues of C are strictly less than one, the geometric series formula allows us to conclude

$$\lim_{k\to\infty} P(k) = (I - C)^{-1}\beta\eta = B^{-1}\eta = p^*. \tag{15}$$

This is the limit desired, but just when are the moduli of the eigenvalues of C all strictly less than one? This question, which brings us to the subject of algorithm stability, is shown to have a very satisfactory answer. The answer, so long as β is set correctly, is: when the centralized "genie" can find a power vector p^* meeting the desired criterion.

When the "genie" can find such a power vector p^*, it is easy to show that, so long as we choose β appropriately, the moduli of the eigenvalues of C are all less than one. As mentioned in Section III.A., for a "genie" to be successful, the eigenvalues of B must have positive real parts. From this positivity, it follows directly that for β, a sufficiently small positive number, all the moduli of the eigenvalues of C are less than one. Indeed, as β approaches zero, the moduli of the eigenvalues of $C = I - \beta B$ approach one as approximately linear functions of β with negative slopes.

Next we take up the issue of the *universality.* Is there a *universal* setting of β, such that, whenever the "genie" has a solution, by using (13a), $P(k)$ will converge to p^* no matter what A is? This universality question is important because the users do not know the A matrix. They cannot even be expected to know the number of interfering users J. We show that $\beta = 1$ is the largest possible universal constant, from which it follows immediately that any value of β on the half-open interval (0, 1] is universal.

To demonstrate the universality of $\beta = 1$, we need some additional notation. Define R_0 by reversing the sign of the corresponding entry of B and taking the diagonal terms of R_0 to all be zero: $R_0 = I - B$. When, for a vector x, we write $x < 0\,(x \leq 0)$ we mean that the strict (nonstrict) inequality constraint is imposed on each component of x. The meaning of a vector being strictly greater than (and greater than) zero is defined in the corresponding way. We also need the notation, X_+, for the set of all J vectors, x, other than the zero vector, that satisfy $x \geq 0$.

Furthermore, we need the following result from [11], [12] that holds for any square matrix with nonnegative entries.

Lemma: R_0 possesses a nonnegative eigenvalue μ that is larger in modulus than the modulus of any of its other eigenvalues. This eigenvalue is given by

$$\mu = \min_{x \in X_+} \min\,[\text{real}\,\gamma\,\text{such that}\,R_0 x - \gamma x \leq 0]. \quad (16)$$

With the lemma we can now prove the following.

Proposition: With the choice $\beta = 1$, whenever the "genie" has a solution, by using (13a), $P(k)$ will converge to p^* no matter what A is. Also $\beta = 1$ is the best (largest) universal constant possible.

Proof: If in (15) we replace X_+ by the smaller set containing only p^*, we can write

$$\mu \leq \min\,[\text{real}\,\gamma\,\text{such that}\,R_0 p^* - \gamma p^* \leq 0]. \quad (17)$$

From (6), $R_0 p^* - p^* < 0$, so in inequality (17) the minimizing γ must be strictly less than one. We can, therefore, conclude that $\mu < 1$. With $\beta = 1, C = R_0$. Because of the strictness of this bound on μ, if we take $\beta = 1$, (13a) exhibits universal convergence to p^* whenever p^* exists.

That $\beta = 1$ is the best (largest) universal constant possible can be seen by considering the case $J = 2$. Suppose A is such that two users are essentially isolated from the others. To study the interaction of these two users, we employ the notation $\alpha = +(a_{12}a_{21}/a_{11}a_{22})^{1/2}$. The eigenvalues of C are $\lambda_+ = (1-\beta) + \rho\alpha\beta$ and $\lambda_- = (1-\beta) - \rho\alpha\beta$. The constraints that the absolute value of the eigenvalues be strictly less than one lead directly to the requirement that one strictly exceeds $\rho\alpha$, and when it does, $2/(1 + \rho\alpha)$ strictly exceeds β. It is evident from these two inequalities that, as we wanted to show, a β exceeding one is not universal.

(Since this two-user case is so easy to analyze explicitly, it is worthwhile doing so even though the remainder of this paragraph is subsumed by the preceding much more abstract general development in this subsection.) When these two conditions hold, the algorithm converges as desired to the "genie's" solution. As can be shown directly from (6), the first of these two conditions, $1 > \rho\alpha$, is essential for the "genie" to have a solution in the first place. This condition can be interpreted as requiring that the geometric mean of the two "raw signal interference ratios" a_{11}/a_{12} and a_{22}/a_{21} exceed the requirement ρ. The selection of β giving the fastest convergence, that is, the choice that gives the minimum maximum modulus, is easily seen to be $\beta = 1$. This (universal) choice of β gives $|\lambda_+| = |\lambda_-| = \alpha\rho < 1$.

With the largest *universal* value of β determined to be $\beta = 1$, (13) becomes

$$P(k+1) = R_0 P(k) + \eta. \quad (18)$$

Recall that R_0 has $\rho a_{ij}/a_{ii}$ in each nondiagonal ijth position and zero along the diagonal, while the generic (ith) component of v is $\rho\nu/a_{ii}$. We see then, that according to (18), each user simply proceeds to reset its power level to what it needs to be to have acceptable performance when the users are not changing their power level. Yet the other users *are* changing their power levels. No matter, for each user, convergence to the desired performance occurs with this local distributed algorithm any time a central controller can achieve the desired performance.

While we see from our theoretical study so far that there is a strong preference for $\beta = 1$, follow-up work may reveal that there are practical tracking considerations which could make it advisable to back off to a somewhat lower β value. In Section IV we will get an indication of the effect of varying β.

C. Generalizing to Inlcude Up-Link Power Control

In the development so far we have dealt exclusively with the down-link dynamic. If a "genie" can find both up-link and down-link power settings, p^* and q^* respectively, to achieve the signal-to-interference requirements, then the solution to (5) (or (13) or (18)) together with its up-link counterpart will converge exponentially fast to the (p^*, q^*) pair, no matter how the power settings are initiated. The explanation of this convergence is essentially the same as for the down-link only case that we have treated. In the simplest case, the up-link and down-link channels are assumed to be identical from a propagation standpoint—the user-to-base channel is the same as the base-to-user channel; it is just used in the reverse direction at a different time. In this case, the up-link power control dynamical equation differs from that for the down-link dynamic only in that B is replaced by its transpose. This is assumed for the way that the up-link power control is treated in the numerical examples that follow next.

IV. Numerical Example of Improved Responsiveness as β Increases

In this section, we report a power control simulation result for an idealized, regular, two-dimensional, hexagonal cellular array. Single channel reuse was simulated to take an initial look at the effect of increasing β.

Channel request arrivals are taken to be uniformly distributed over 64 contiguous cells arranged in a locally planar fashion, but as a square (8×8) toroid to encourage interference. In each hexagonal cell, a base is situated at the center. The propagation environment model includes a distance from transmitter decay law with exponent $\gamma = 4$. The model also includes a shadow fading component which is assumed to be normally distributed with a mean of 0 dB and a standard deviation of 8 dB. The transmitter power was normalized to unity at (1/100)th the distance from the base (located at cell center) to a cell corner. The receiver noise level of -100 dB is expressed relative to this normalization. The up-link and down-link propagation matrices are assumed to be transposes of each other. In the example, the required ρ is 12 dB. A user gets assigned to the base with the strongest signal-to-interference ratio at time of arrival, which is not necessarily the base for the hexagon that the user is situated in.

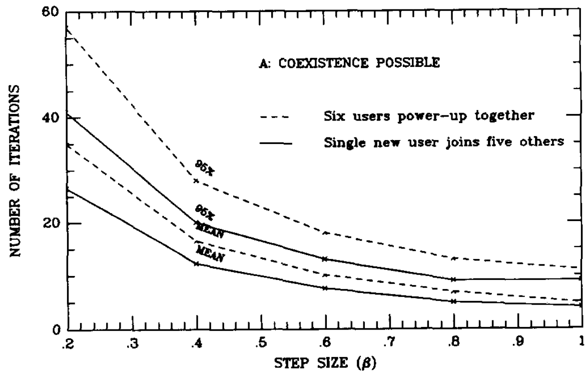
Moreover, the simulations also incorporate the practical constraint that the received power cannot exhibit gain. Some 10000 independent experiments involving six randomly placed users were conducted. About 48% of the experiments permitted coexistence with 12 dB performance for all users. Only these cases where convergence is possible are represented in Fig. 1(a), which, therefore, represents compatible situations where the users have a significant tendency to interfere with each other. Note that, as predicted in Section III.B., as β increases, the mean time to convergence is reduced.

Fig. 1(a) includes two cases. In one case, all six users power-up together, while, in the second case, a sixth user jockeys in, powering up after the other five users have converged to an equilibrium (which is upset by the sixth user). In the simulation, "convergence" was considered to have occured when all users attained performance within 1% of the 12 dB objective; in other words a signal-to-interference ratio of $15.85 \pm .16$. Intuitively, there is typically more to be done in the first case where all six users power-up. Therefore, it is not surprising the graphs show that the case where the five ongoing users are perturbed by a sixth takes fewer iterations to converge.

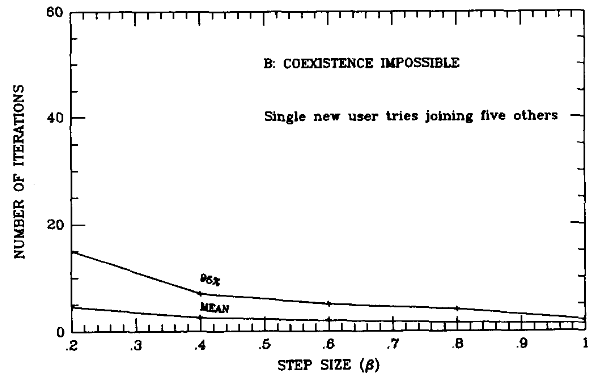
Secondly, we looked at those cases where a sixth user cannot join five users that coexist with $\rho = 12$ dB and allow all six to achieve this ρ value. The number of iterations until at least one user learns that coexistence is impossible is shown. In these examples, the need to exceed the maximum allowable transmitter power provides the knowledge that coexistence is impossible. With $\beta = 1$, this is seen to have a mean of 1.3 iterations and a 95% point of two iterations. In actual situations, a user recognizing the impossibility of coexistence might use this information to trigger leaving the channel for another, or, as a last resort, dropping the call in progress.

In the above examples, the departure of users is not modeled. However, imagining an application with a level of user interaction as in the example, it is interesting to consider whether the intursion to accommodate a new user is an acceptable disturbance of a call in progress. If a call lasts an average of 100 s, a mean of five steps (and a 95 percentile of 10 steps) at $\beta = 1$ might be interpreted as quite reasonable if, say, the time per iteration was about .01 s. If, on the other hand, the time per iteration were, say 0.1 s, it is not clear if that would be acceptable; further study would be required.

While the figures all correspond to a ρ of 12 dB some spot examples for 9, 15 and 18 dB were also studied. The results are not very sensitive to changes in ρ. For example from figure B at $\beta = 1$ we see that an average of about 1.3 iterations are needed until some user learns coexistence is impossible. As ρ ranges from 9 dB to 18 dB this mean changes from 1.5 to 1.2.



(a)



(b)

Fig. 1. Six tightly coupled users controlling their power: 10 000 independent computer experiments were performed in which six users were randomly (uniformly) situated in 64 contiguous hexagonal cells on a two-dimensional surface. The users share a single reused channel on each of the up and down links. They interfere with each other according to a standard propagation model, incorporating an attenuation with distance effect as well as shadow fading. A signal-to-interference requirement of 12 dB was required on both the up-link and down-link for each of the six users in order for them to coexist. The "interference" included -100 dB of receiver noise relative to the maximum allowable transmitter power. Graphs of number of iterations versus β are shown. (a) The approximately 48% of cases where the six users can coexist is represented. Two situations are depicted, one where all six users power-up together from zero power, and the other where five of the users are coexisting at the required 12 dB performance level and are then disturbed by a sixth user that can be accommodated. (b) Cases where it is impossible for a sixth user to join five users that are meeting the 12 dB requirement are represented. The number of iterations until at least one of the users learns of the impossibility of the situation is shown.

We emphasize that the above examples employ a very preliminary model. In more detailed quantitative studies[3] the effects of Rayleigh fading would also be included. Mobility is another complicating feature to be considered for future investigations [13].

V. Conclusion

Equation (3) (or (5), or better (13a)) expresses a simple distributed autonomous means of power control, in that only local power and interference measurements are used to effect the control dynamic. In Section III, we proved convergence

[3] References [3] and [4] take a preliminary look at the effect of fast fading on the power control scheme reported there, and suggest that it does not undermine control accuracy very much.

of the algorithm: When an all-knowing "genie" can set each of the transmitted signals at power levels needed to achieve a required signal-to-interference ratio of ρ dB for each user, these settings can be made to arise exponentially fast from scratch, by evolution in accordance with (3) (or (5) or (13a)). These equations deal with only the down-link, but, as indicated in Section III.C., the algorithm and convergence result extend in a straightforward manner to include controlling both the up-link and down-link power together.

For the more practical discrete form of the algorithm given by (13a), the question arises as to just how fast the algorithm can be made to evolve. The issue of pacing the power control evolution is a concern, since instability is a risk if the pace is too great. The fastest version of the algorithm for which convergence is assured was found in Section III.B. The effect of improving the convergence speed was illustrated in the simulation example of Section IV.

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Gerald J. Foschini for biography and photograph, see p. 424 of this issue.

Zoran Miljanic for biography and photograph, see p. 424 of this issue.