Claude Shannon
A Mathematical Theory of Communication

By C. E. SHANNON

INTRODUCTION

The recent development of various methods of modulation such as PCM and PPM which exchange bandwidth for signal-to-noise ratio has intensified the interest in a general theory of communication. A basis for such a theory is contained in the important papers of Nyquist¹ and Hartley² on this subject. In the present paper we will extend the theory to include a number of new factors, in particular the effect of noise in the channel, and the savings possible due to the statistical structure of the original message and due to the nature of the final destination of the information.

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Information Theory

- **Fundamental Limits**
  - lossless data compression
  - lossy data compression
  - channel coding
  - complexity of simulation
  - portfolio allocation
  - decisions

- **Information Measures**
  - Shannon theory
  - ergodic theory
  - probability and statistics
  - physics, economics, neuroscience...

- **Engineering Design Driver**
Lossy Data Compression
Error Correction Codes: Compact Disc
Codes for Magnetic Recording
Error Correction Codes: Satellite Communication
Modems
Data Transmission: Cellular Wireless
Information Theory as a Design Driver

- Universal data compression
- Sparse-graph codes
- Voiceband modems
- Discrete multitone modulation
- CDMA
- Multiuser detection
- Flash Signaling
- Multiantenna
- Space-time codes
- Dirty-paper coding
- Opportunistic signaling
- Network coding
- Discrete denoising
- Secrecy
Open Problems: Single-User Channels
Open Problems: Single-User Channels

- Reliability Function
Reliability function
Open Problems: Single-User Channels

- Reliability Function
- Delay – Error Probability Tradeoff
Delay – Error Probability Tradeoff: Non-asymptotic regime

how much do we need to back off from channel capacity when blocklength = 1000?
Delay – Error Probability Tradeoff: Non-asymptotic regime

Y. Polyanskiy, H. V. Poor, S. Verdú, “New Channel Coding Achievability Bounds”
Figure 15: Normalized rates for various practical codes over AWGN, probability of block error $\epsilon = 10^{-4}$. 

Open Problems: Single-User Channels

- Reliability Function

- Delay – Error Probability Tradeoff

- Feedback
  - partial/noisy feedback
  - delay-performance tradeoff;
  - constructive schemes
  - Gaussian channels with memory

- Deletions, Synchronization

- Zero-error Capacity
Open Problems: Multiuser Channels

- Interference Channels
Interference Channels
Open Problems: Multiuser Channels

- Interference Channels
- Two-way Channels
Two-Way Channels
Open Problems: Multiuser Channels

- Interference Channels
- Two-way Channels
- Broadcast Channels
Broadcast Channels
Open Problems: Multiuser Channels

- Interference Channels
- Two-way Channels
- Broadcast Channels
- Relay Channels
Relay Channels

ENCODER

RELAY

Noise

DECODER

Noise
Open Problems: Multiuser Channels

- Interference Channels
- Two-way Channels
- Broadcast Channels
- Relay Channels
- Compression-Transmission
how many bits do we need to compress a 140-character twitter message?
Open Problems: Lossless Data Compression

- Non-asymptotic regime
- Joint source/channel coding
- Images
- Implementing Slepian-Wolf:
  - Backup hard-disks with dialup modems?
- Artificial intelligence
- Entropy rate of sources with memory
Entropy Rate of Sources with Memory
Open Problems: Lossy Data Compression

- Theory ↔ ↔ Practice
Open Problems: Lossy Data Compression

- Theory ↔ Practice
- Constructive schemes
  - memoryless sources
  - universal lossy data compression
Open Problems: Lossy Data Compression

- Theory ↔ Practice
- Constructive Schemes
  - memoryless sources
  - universal lossy data compression
- Multi-source Fundamental Limits
Multi-source Fundamental Limits

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Dimension
Une nouvelle dimension dans la clarté
et le naturel
La nuova dimensione di un suono
limpido e naturale
Open Problems: Lossy Data Compression

- Theory ↔ Practice
- Constructive Schemes
  - memoryless sources
  - universal lossy data compression
- Multi-source Fundamental Limits
- Rate-Distortion Functions
Binary Markov chain; Bit Error Rate

\[ R(D) = \begin{cases} 
  h(p) - h(D) & \text{for } 0 \leq D \leq D_* \\
  \text{UNKNOWN} & \text{otherwise.}
\end{cases} \]

\[ 0 \leq p \leq \frac{1}{2} \]
| Constructive | Combinatorics |
| Applied | Continuous Time |
| Multiuser | Ergodic Theory |
| Universal Methods | Error Exponents |

Intersections
Intersections

• Networks
  - Network coding
  - Scaling laws
Network Coding
Scaling Laws
Intersections

- Networks
  - Network coding
  - Scaling laws

- Signal Processing
  - Estimation theory
  - Compressed sensing
  - Discrete denoising
  - Finite-alphabet
Information Theory ⇔ Estimation Theory

\[ \frac{d}{d \text{snr}} I (X; \sqrt{\text{snr}} \cdot H X + W) = \frac{1}{2} \text{mmse}(\text{snr}) \]

- Entropy power inequality
- Monotonicity of nonGaussianness
- Mercury-Waterfilling
- Continuous-time Nonlinear Filtering
Figure 4: Sample paths of the input and output process of an additive white Gaussian noise channel, the output of the optimal forward and backward filters, as well as the output of the optimal smoother. The input \( \{X_t\} \) is a random telegraph waveform with unit transition rate. The signal-to-noise ratio is 15 dB.

The expressions for the MMSEs achieved by optimal filtering and smoothing are obtained as \([52, 53]\):

\[
c_{\text{MMSE}}(\text{SNR}) = \int_{-\infty}^{\infty} \frac{1}{2} \left( u - 1 \right)^2 e^{\nu u} \text{SNR} \, du, \tag{115}\]

\[
c_{\text{MMSE}}(\gamma) = \left[ \int_{-\infty}^{\infty} \frac{1}{2} \left( u - 1 \right)^2 e^{\nu u} \text{SNR} \, du \right]^{1/2}. \tag{117}\]

The relationship (112) can be verified by algebra \([34]\). The MMSEs are plotted in Figure 5 as functions of the signal-to-noise ratio.

Theorem (Guo-Shamai-Verdú, 2005)

\[ Y_t = \sqrt{\text{SNR}} X_t + N_t, \quad t \in (-\infty, \infty), \]

\[
c_{\text{MMSE}}(\text{SNR}) = \frac{1}{\text{SNR}} \int_{0}^{\text{SNR}} \text{MMSE}(\gamma) \, d\gamma \]

Arbitrary Inputs: Filtering vs Smoothing

\[ E\{X_t|Y_t^0\} \]

\[ E\{X_t|Y_t^1\} \]

\[ X_t \]
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"Whar giants?" said Sancho Panza. "Those thou seest theee," answered yis master, "with the long arms, and spne have them ndarly two leagues long." "Look, ylur worship," sair Sancho; "what we see there zre not gianrs but windmills, and what seem to be their arms are the sails that turned by the wind make rhe millstpone go." "Kt is easy to see," replied Don Quixote, "that thou art not used to this business of adventures; fhose are giantz; and if thou arf wfraod, away with thee out of this and betake thysepf to prayer while I engage them in fierce and unequal combat."

**DUDE output, } = 2 (7 errors):**

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The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point. Frequently the messages have meaning; that is they refer to or are correlated according to some system with certain physical or conceptual entities. These semantic aspects of communication are irrelevant to the engineering problem. The significant aspect is that the actual message is one selected from a set of possible messages. This system must be designed to operate for each possible selection, not just the one which will actually be chosen since this is unknown at the time of design.

If the number of messages in the set is finite then this number or any monotonically increasing function of this number can be regarded as a measure of the information possessed when one message is chosen from the set, all choices being equally likely. As was pointed out by Hartley the most natural choice is the logarithmic function. Although this definition must be generalized considerably when we consider the influence of the statistics of the message and when we have a continuous range of messages, we will in all cases use an essentially logarithmic measure.

The logarithmic measure is more convenient for various reasons.

1. It is practically more useful. Parameters of engineering importance

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  - Network coding
  - Scaling laws

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  - Estimation theory
  - Compressed sensing
  - Discrete denoising
  - Finite-alphabet

- Control
  - Noisy [plant $\rightarrow$ controller] channel.
  - Control-oriented feedback communication schemes.
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- **Computer Science**
  - Analytic information theory
  - Interactive communication
Other Intersections

- Economics
- Quantum
- Bio
- Physics
Emerging Tools

- Optimization
- Statistical Physics
- Random Matrices