Games, Uncertainty. (9 points) The minimax algorithm is used to find the best move in a game tree. Suppose however that the game involves an element of chance, in which a player's possible moves depend on the throw of a dice, the spin of a wheel, or some other phenomenon whose result is uncertain.



Consider the above game tree, in which Player-1 (who wants to maximize the value of the resulting state of the game) can choose to move to A or to B. At this point, Player-2's possible moves are uncertain and depend on a chance event, as indicated by the labelled chance nodes (shown as circles in the above game tree)—for example, from A, the probability is .3 that Player-2 can choose to move to C or to D, and the probability is .7 that Player-2 can choose to move to E or to F. Similarly, if Player-2 moves to C, the probability is .4 that Player-1 can choose to move to G or H, and the probability is .6 that Player-1 can choose to move to I or J.

Given this game tree, determine what the backed-up value of node B must be if the best decision for Player-1 is to move to A. You must explain your answer in detail.

Search. (16 points) In recreational mathematics, a **magic square** of order *n* is an arrangement of n^2 numbers, usually distinct integers, in a square, such that the *n* numbers in all rows, all columns, and both diagonals sum to the same constant. A **normal** magic square contains the integers from 1 to n^2 .

Normal magic squares exist for all orders $n \ge 1$ except n = 2, although the case n = 1 is trivial, consisting of a single cell containing the number 1. The smallest nontrivial case, shown below, is of order 3.



The constant sum in every row, column and diagonal is called the <u>magic con-</u> <u>stant</u> or magic sum, M. The magic constant of a normal magic square depends only on *n* and has the value

$$M = \frac{n(n^2 + 1)}{2}$$

- (a) [6 pts] Formulate the construction of a normal magic square of size n = 3 as a constraint satisfaction problem. Specify what would be taken as variables, and (informally) specify the constraints on those variables.
- (b) [10 pts] Set up the problem and show the first steps of processing using backtracking with forward checking. Show the assignments and variable domains through either 4 assignments or until the algorithm needs to backtrack – whichever comes first.