Individual Assignment 2

Each problem is worth 10 points.

1. We implemented disjoint sets in such a way that the time cost of an m-element sequence of operators Make-Set, Find-Set and Union is $O(m*\log^*(n))$ where n is the number of Make-Set operators. Prove that if all the Unions precede all of the Find-Sets, the time cost is only $O(m)$.

2. Prove that given a connected undirected graph, an edge (u,v) cannot belong to any minimum spanning tree for the graph if and only if there is a path from u to v such that every edge in the path has less weight than the weight of edge (u,v). Be sure to prove both directions of this biconditional.

3. Do problem 21-3 (page 584), parts a, c, and d.

4. Suppose that the weight function w assigns values to the edges in an undirected graph $G= (V,E)$ that are integers in the range 0, 1, ..., $|V|$. How can we find a minimum spanning tree in $O(|E|*\log^*(|V|))$ time? Prove your answer.

5. Suppose that the weights of the edges in a directed graph $G=(V,E)$ are integers in the range 0, 1, ..., W for some small integer W. How can Dijkstra's algorithm be improved to find the shortest paths faster than $O(|E| + |V|*\lg(|V|))$? What is the time complexity of your algorithm? Prove it.

6. Suppose that the weights of the edges in a directed graph $G=(V,E)$ are all positive numbers. Write an algorithm that will find the length of the shortest cycle in the graph if there is one and returns 0 if there is no cycle. Your algorithm should be at most $O(|V|^3)$. Determine the complexity of your algorithm and prove it.