CISC 404/604 Homework assignment 8.

Each problem is worth 10 points.

1. Do exercise 81 (page 89) in the Schöning book.

2. Do exercise 85 (page 96) in the Schöning book. Assume that everything in the universe of discourse is a dragon. That means that you do not need a D(x) predicate that asserts that x is a dragon.

3. Compute a most general unifier for the following pairs of clauses, if possible. Show your work. The symbol a is a constant.

i) $P(a,x,f(g(y)))$, $P(y,f(z),f(z))$
ii) $P(x,g(f(a)),f(x))$, $P(f(a),y,y)$
iii) $P(x,g(f(a)),f(x))$, $P(f(y),z,y)$
iv) $P(a,x,f(g(y)))$, $P(z,h(z,u),f(u))$

4. Show that the following clauses are unsatisfiable by writing a refutation (derivation of the empty clause) in the style used in class. I have listed the clauses as the first four lines of the refutation you are to write:

1. $\{P(x), Q(f(x))\}$
2. $\{P(x), \neg Q(f(x))\}$
3. $\{\neg P(a), R(x)\}$
4. $\{\neg P(x), \neg R(b)\}$

5. Write a refutation in the style used in class starting with the following clauses:

1. $\{P(x,f(x),a)\}$
2. $\{\neg Q(x), \neg Q(y), \neg P(x,f(y),z), Q(z)\}$
3. $\{Q(b)\}$
4. $\{\neg Q(a)\}$

6. Let's solve a simple puzzle using resolution. A farmer, goat and cabbage are on the left side of a river. Using a boat, the farmer can move the goat or the cabbage to the other side of the river, but no more than one of them at a time. The goal is to get the farmer, goat and cabbage to the right side of the river. To solve this problem, we will use one predicate, S, where S(x,y,z,w) means that at time w, the goat is on side x, the cabbage is on side y and the farmer is on side z of the river. Here are the conditions of the problem, stated as predicate logic formulas and what they mean. The symbols l, r and s are constants.

1. $S(l,l,l,s)$ - At the start time s, the goat, cabbage and farmer are all on the left side of the river.
2. $(\forall y)(\exists z)(\forall x)(S(l,x,l,y) \Rightarrow S(r,x,r,z))$ - for any time y, there is a time z such that if both the goat and the farmer are on the left side of the river at time y, the farmer can move the goat to the right side of the river by time z no matter where the cabbage is.
3. $(\forall y)(\exists z)(\forall x)(S(x,l,l,y) \Rightarrow S(x,r,r,z))$ - for any time y, there is a time z such that if both the

cabbage and the farmer are on the left side of the river, the farmer can move the cabbage to the right side of the river by time z, no matter where the goat is.

4. $(\forall z)(\exists w)(\forall x)(\forall y)(S(x,y,r,z) \Rightarrow S(x,y,l,w))$ - for any time z, if the farmer is on the right side of the river, the farmer can boat across the river to the other side alone, no matter where the goat and cabbage are.

The objective is to get the farmer, goat and cabbage all on the right side of the river at the same time. In other words, we want to prove goal $(\exists x)S(r,r,r,x)$, which means that there is a time when the goat, cabbage and farmer can all be on the right side of the river.

Write a refutation that shows that the goal formula is logically implied by the four axioms just listed. To do this, transform the four axioms into clauses written as disjunctions of literals. Axiom 1 is already a clause. Axioms 2, 3 and 4 will introduce three different Skolem functions. To prove the goal formula, first negate it and then transform it into a clause. You will now have five clauses that are unsatisfiable. The refutation you write should begin with these five clauses and end with the empty clause.
Note: If you look at the substitution to the variable in the clause that was generated from the goal formula and combine it with the other substitutions you made, you will find a representation of how the farmer can get the goat and cabbage to the other side of the river.