CISC 404/604 homework 4.

Prove the following theorems using the Natural Deduction Rules as specified in lecture slides ln6.

1. (10 pts) $(\forall x)(B(x) \Rightarrow C(x)) \Rightarrow ((\forall x)B(x) \Rightarrow (\forall x)C(x))$
2. (10 pts) $(\forall x)(B(x) \Rightarrow C(x)) \Rightarrow ((\exists x)B(x) \Rightarrow (\exists x)C(x))$
3. (10 pts) $(\forall x)(B(x) \wedge C(x)) \Rightarrow (\forall x)B(x) \wedge (\forall x)C(x)$
4. (10 pts) $(\exists x)\neg B(x) \Leftrightarrow \neg(\forall x)B(x)$
5. (10 pts) $((\forall x)C(x) \Rightarrow D) \Leftrightarrow (\exists y)(C(y) \Rightarrow D)$

Caution: the following deduction is not valid with our rules:

1. * $((\forall x)C(x) \Rightarrow D)$	Pr
2. ** $C(x)$	Pr
3. ** $(\forall x)C(x)$	UG 2
4. ** D	TF 1,3
5. * $C(x) \Rightarrow D$	Cd

6. (10 pts) $((\exists x)C(x) \Rightarrow D) \Leftrightarrow (\forall y)(C(y) \Rightarrow D)$
7. (10 pts) $(\forall x)\neg B(x) \Leftrightarrow \neg(\exists x)B(x)$
8. (10 pts) $(D \Rightarrow (\forall x)C(x)) \Leftrightarrow (\forall y)(D \Rightarrow C(y))$
9. (10 pts) Describe an interpretation in which $(D \Rightarrow (\forall x)C(x)) \Leftrightarrow (\exists y)(D \Rightarrow C(y))$ is false.
10. (10 pts) Describe an interpretation in which $(\forall x)(B(x) \vee C(x))$ is true, but $(\forall x)B(x) \vee (\forall x)C(x)$ is false.