Vision Review: Classification

Course web page:

www.cis.udel.edu/~cer/arv

Announcements

- Homework 2 due next Tuesday
- Project proposal due next Thursday, Oct. 10. Please make an appointment to discuss before then

Computer Vision Review Outline

- Image formation
- Image processing
- Motion & Estimation
- **Classification**

Outline

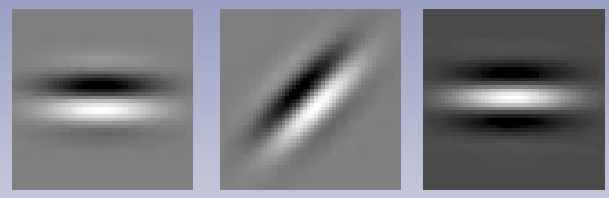
- Classification terminology
- Unsupervised learning (clustering)
- Supervised learning
  - *k*-Nearest neighbors
  - Linear discriminants
    - Perceptron, Relaxation, modern variants
  - Nonlinear discriminants
    - Neural networks, etc.
- Applications to computer vision
- Miscellaneous techniques

Classification Terms

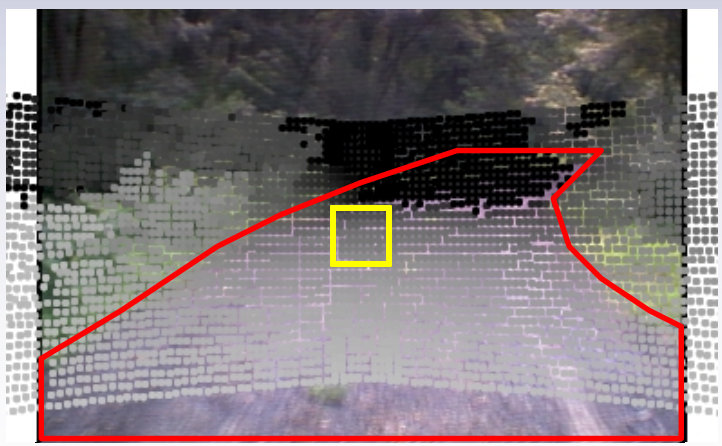
- **Data**: A set of *N* vectors **x**
  - Features are parameters of **x**; **x** lives in *feature space*
  - May be whole, raw images; parts of images; filtered images; statistics of images; or something else entirely
- **Labels**: *C* categories; each **x** belongs to some c_i
- **Classifier**: Create formula(s) or rule(s) that will assign unlabeled data to correct category
  - Equivalent definition is to parametrize a *decision surface* in feature space separating category members

Features and Labels for Road Classification

- Feature vectors: 410 features/point over 32 x 20 grid
  - **Color histogram** [Swain & Ballard, 1991] (24 features)
    - 8 bins per RGB channel over surrounding 31 x 31 camera subimage
  - **Gabor wavelets** [Lee, 1996] (384)
    - Characterize *texture* with 8-bin histogram of filter responses for 2 phases, 3 scales, 8 angles over 15 x 15 camera subimage
  - **Ground height, smoothness** (2)
    - Mean, variance of laser height values projecting to 31 x 31 camera subimage
- Labels derived from inside/ outside relationship of feature point to road-delimiting polygon



0° even 45° even 0° odd



from Rasmussen, 2001

Key Classification Problems

- What features to use? How do we extract them from the image?
- Do we even have labels (i.e., examples from each category)?
- What do we know about the structure of the categories in feature space?

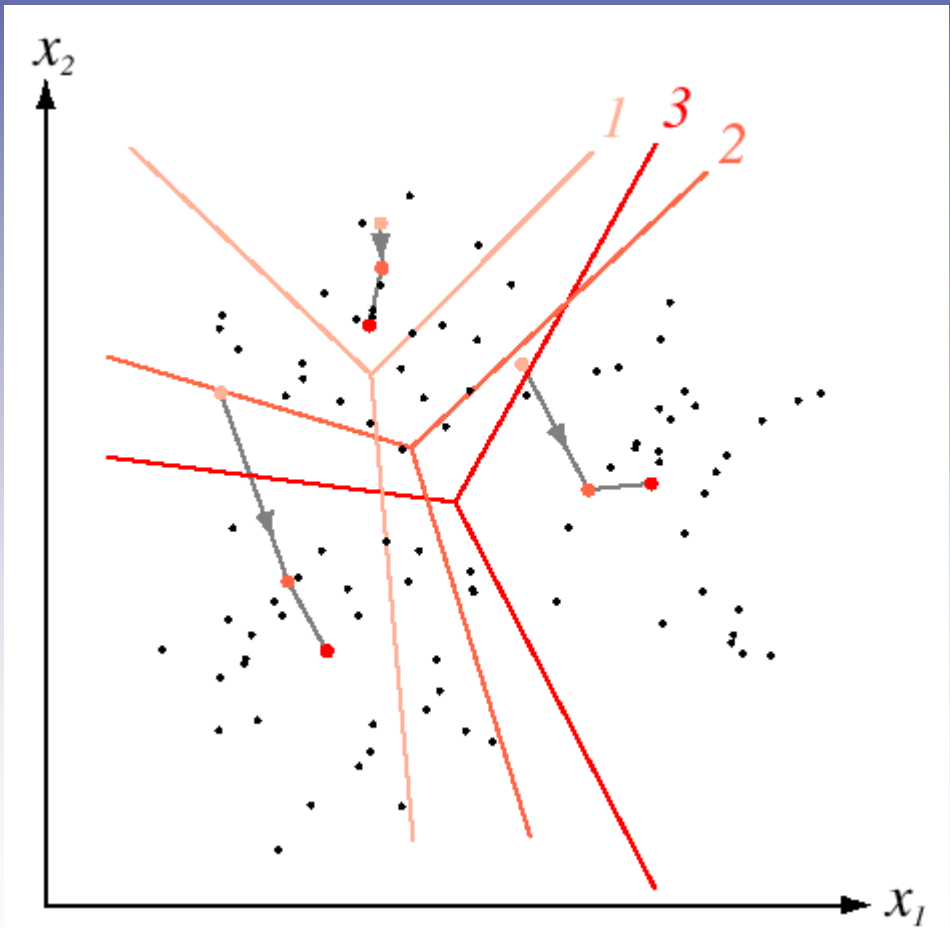
Unsupervised Learning

- May know number of categories *C*, but not labels
- If we don't know *C*, how to estimate?
  - Occam's razor (formalized as Minimum Description Length, or MDL, principle): Favor simpler classifiers over more complex ones
  - Akaike Information Criterion (AIC)
- Clustering methods
  - k-means
  - Hierarchical
  - Etc.

k-means Clustering

- Initialization: Given *k* categories, *N* points. Pick *k* points randomly; these are initial means $\mu_1, \ldots, \mu_k$
- (1) Classify *N* points according to nearest μ_i
- (2) Recompute mean μ_i of each cluster from member points
- (3) If any means have changed, goto (1)

Example: 3-means Clustering



from
Duda et al.

Convergence in 3 steps

Supervised Learning: Assessing Classifier Performance

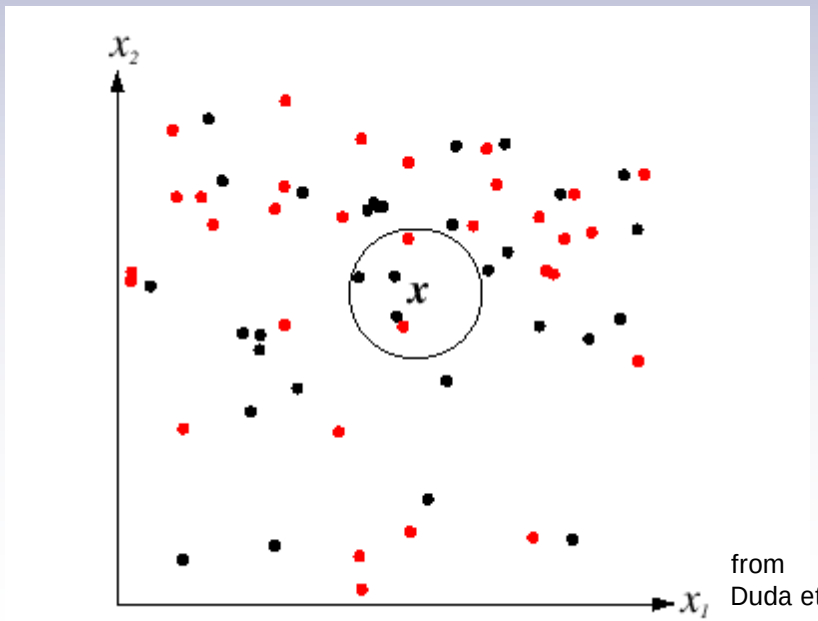
- Bias: Accuracy or quality of classification
- Variance: Precision or specificity—how stable is decision boundary for different data sets?
  - Related to generality of classification result
    $\rightarrow$ Overfitting to data at hand will often result in a very different boundary for new data

Supervised Learning: Procedures

- Validation: Split data into training and test set
  - Training set: Labeled data points used to guide parametrization of classifier
    - % misclassified guides learning
  - Test set: Labeled data points left out of training procedure
    - % misclassified taken to be overall classifier error
- *m*-fold Cross-validation
  - Randomly split data into *m* equal-sized subsets
  - Train *m* times on *m* - 1 subsets, test on left-out subset
  - Error is mean test error over left-out subsets
- Jackknife: Cross-validation with 1 data point left out
  - Very accurate; variance allows confidence measuring

k-Nearest Neighbor Classification

- For a new point, grow sphere in feature space until *k* labeled points are enclosed
- Labels of points in sphere vote to classify
- Low bias, high variance: No structure assumed

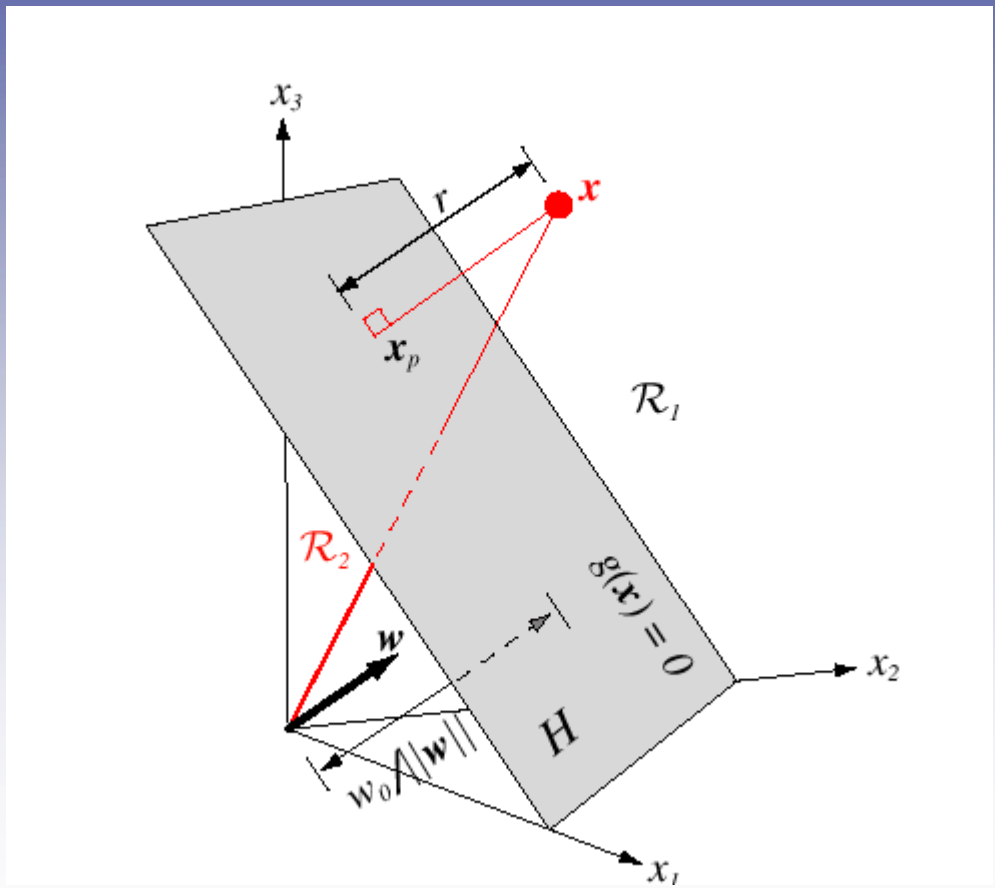


from Duda et al.

Linear Discriminants

- Basic: $g(\mathbf{x}) = \mathbf{w}^T \mathbf{x} + w_0$
  - **w** is weight vector, **x** is data, w_0 is bias or threshold weight
  - Number of categories
    - **Two**: Decide c_1 if $g(\mathbf{x}) < 0$, c_2 if $g(\mathbf{x}) > 0$. $g(\mathbf{x}) = 0$ is decision surface—a hyperplane when $g(\mathbf{x})$ linear
    - **Multiple**: Define C functions $g_i(\mathbf{x}) = \mathbf{w}^T \mathbf{x}_i + w_{i0}$. Decide c_i if $g_i(\mathbf{x}) > g_j(\mathbf{x})$ for all $j \neq i$
- Generalized: $g(\mathbf{x}) = \mathbf{a}^T \mathbf{y}$
  - Augmented form: $\mathbf{y} = (1, \mathbf{x}^T)^T$, $\mathbf{a} = (w_0, \mathbf{w}^T)^T$
  - Functions $\mathbf{y}_i = y_i(\mathbf{x})$ can be nonlinear—e.g., $\mathbf{y} = (1, x, x^2)^T$

Separating Hyperplane in Feature Space



from Duda et al.

Computing Linear Discriminants

- Linear separability: Some **a** exists that classifies all samples correctly
- Normalization: If $\mathbf{y}_i$ is classified correctly when $\mathbf{a}^T \mathbf{y}_i < 0$ and its label is c_1, simpler to replace all c_1-labeled samples with their negation
  - This leads to looking for an **a** such that $\mathbf{a}^T \mathbf{y}_i > 0$ for all of the data
- Define a criterion function J(**a**) that is minimized if **a** is a solution. Then gradient descent on J (for example) leads to a discriminant

Criterion Functions

- Idea: J = Number of misclassified data points. But only piecewise continuous $\rightarrow$ Not good for gradient descent
- Approaches
  - **Perceptron**: $J_p(\mathbf{a}) = \sum_{y \in Y} (-\mathbf{a}^T \mathbf{y})$, where $Y(\mathbf{a})$ is the set of samples misclassified by **a**
    - Proportional to sum of distances between misclassified samples and decision surface
  - **Relaxation**: $J_r(\mathbf{a}) = \frac{1}{2} \sum_{y \in Y} (\mathbf{a}^T \mathbf{y} - b)^2 / ||y||^2$, where $Y(\mathbf{a})$ is now set of samples such that $\mathbf{a}^T \mathbf{y} \leq b$
    - Continuous gradient; J not so flat near solution boundary
    - Normalize by sample length to equalize influences

Non-Separable Data: Error Minimization

- Perceptron, Relaxation assume separability—won't stop otherwise
  - Only focus on erroneous classifications
- Idea: Minimize error over all data
- Try to solve linear equations rather than linear inequalities: $\mathbf{a}^T \mathbf{y} = \mathbf{b} \rightarrow$ Minimize $\sum_i (\mathbf{a}^T \mathbf{y}_i - b_i)^2$
- Solve batch with pseudoinverse or iteratively with Widrow-Hoff/LMS gradient descent
- Ho-Kashyap procedure picks **a** and **b** together

Other Linear Discriminants

- Winnow: Improved version of Perceptron
  - Error decreases monotonically
  - Faster convergence
- Appropriate choice of **b** leads to Fisher's Linear Discriminant (used in "Vision-based Perception for an Autonomous Harvester," by Ollis & Stentz)
- Support Vector Machines (SVM)
  - Map input nonlinearly to higher-dimensional space (where in general there is a separating hyperplane)
  - Find separating hyperplane that maximizes distance to nearest data point

Neural Networks

- Many problems require a nonlinear decision surface
- Idea: Learn linear discriminant and nonlinear mapping functions $y_i(\mathbf{x})$ simultaneously
- Feedforward neural networks are multi-layer Perceptrons
  - Inputs to each unit are summed, bias added, put through nonlinear *transfer* function
- Training: *Backpropagation*, a generalization of LMS rule

Neural Network Structure

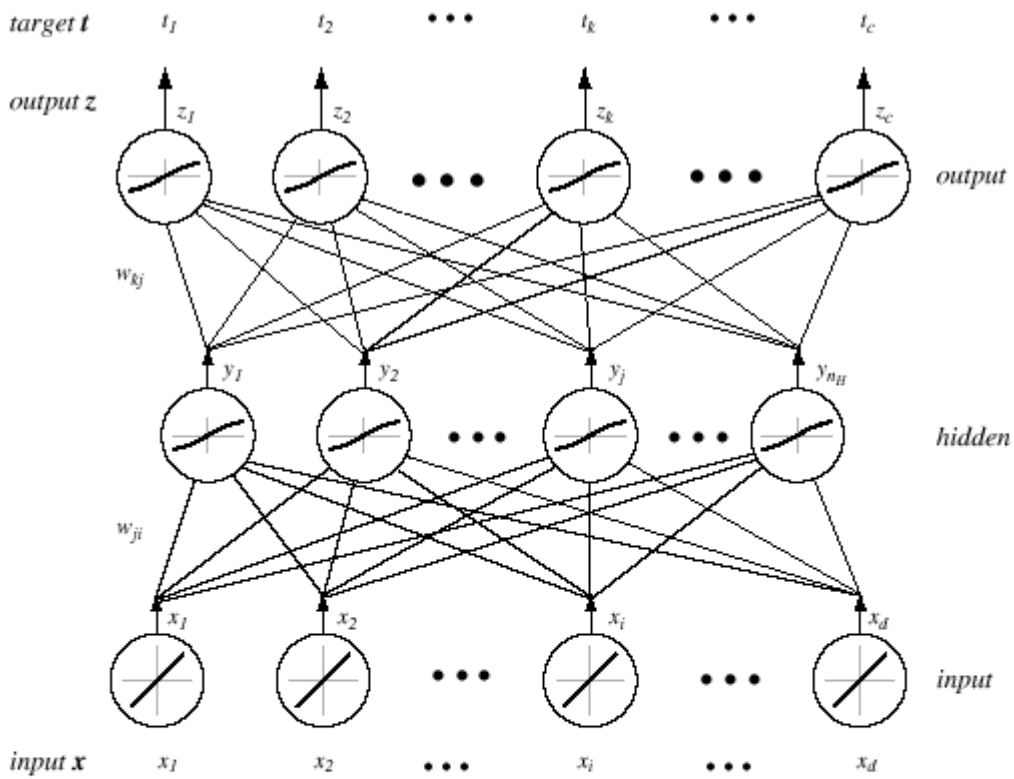


FIGURE 6.4. A d-n_H-c fully connected three-layer network and the notation we shall use. During feedforward operation, a d-dimensional input pattern $\mathbf{x}$ is presented to the input layer; each input unit then emits its corresponding component x_i. Each of the n_H hidden units computes its net activation, net_j, as the inner product of the input layer signals with weights w_{ji} at the hidden unit. The hidden unit emits $y_j = f(net_j)$, where $f(\cdot)$ is the nonlinear activation function, shown here as a sigmoid. Each of the c output units functions in the same manner as the hidden units do, computing net_k as the inner product of the hidden unit signals and weights at the output unit. The final signals emitted by the network, $z_k = f(net_k)$, are used as discriminant functions for classification. During network training, these output signals are compared with a teaching or target vector $\mathbf{t}$, and any difference is used in training the weights throughout the network. From: Richard O. Duda, Peter E. Hart, and David G. Stork, *Pattern Classification*. Copyright © 2001

Neural Networks in Matlab

```
net = newff(minmax(D), [h o], {'tansig', 'tansig'}, 'traincgf');
net = train(net, D, L);
test_out = sim(net, testD);
```

where:

D is training data feature vectors (row vector)

L is labels for training data

testD is testing data feature vectors

h is number of hidden units

o is number of outputs

Dimensionality Reduction

- Functions $\mathbf{y}_i = y_i(\mathbf{x})$ can reduce dimensionality of feature space $\rightarrow$ More efficient classification
- If chosen intelligently, we won't lose much information and classification is easier
- Common methods
  - Principal components analysis (PCA): Maximize total "scatter" of data

$$S_T = \sum_{k=1}^{N} (x_k - \mu)(x_k - \mu)^T$$

  - Fisher's Linear Discriminant (FLD): Maximize ratio of between-class scatter to within-class scatter

Principal Component Analysis

- Orthogonalize feature vectors so that they are uncorrelated
- Inverse of this transformation takes zero mean, unit variance Gaussian to one describing covariance of data points
- Distance in transformed space is *Mahalanobis distance*
- By dropping eigenvectors of covariance matrix with low eigenvalues, we are essentially throwing away least important dimensions

PCA

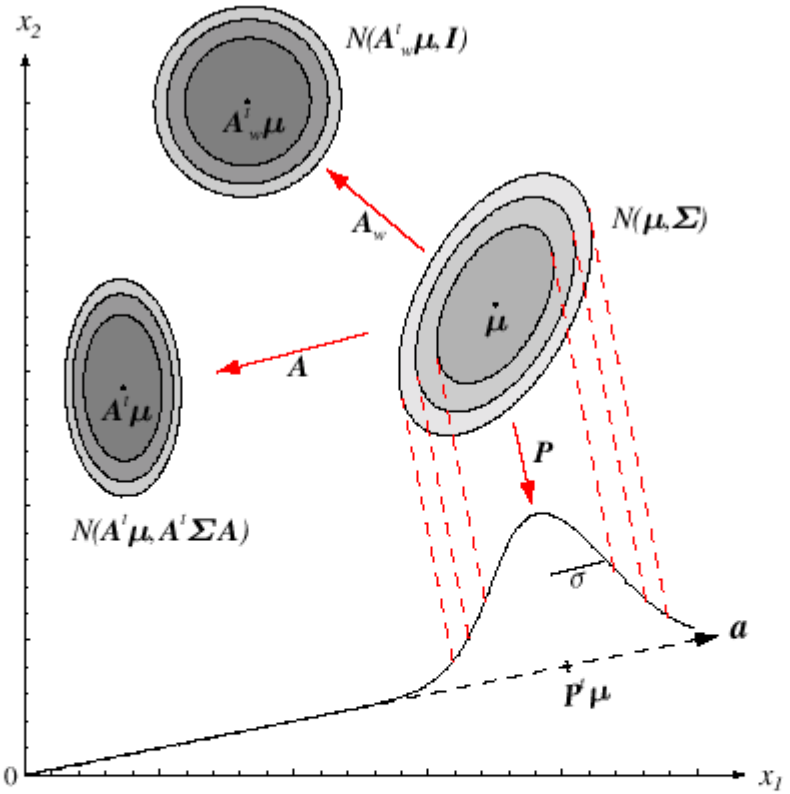
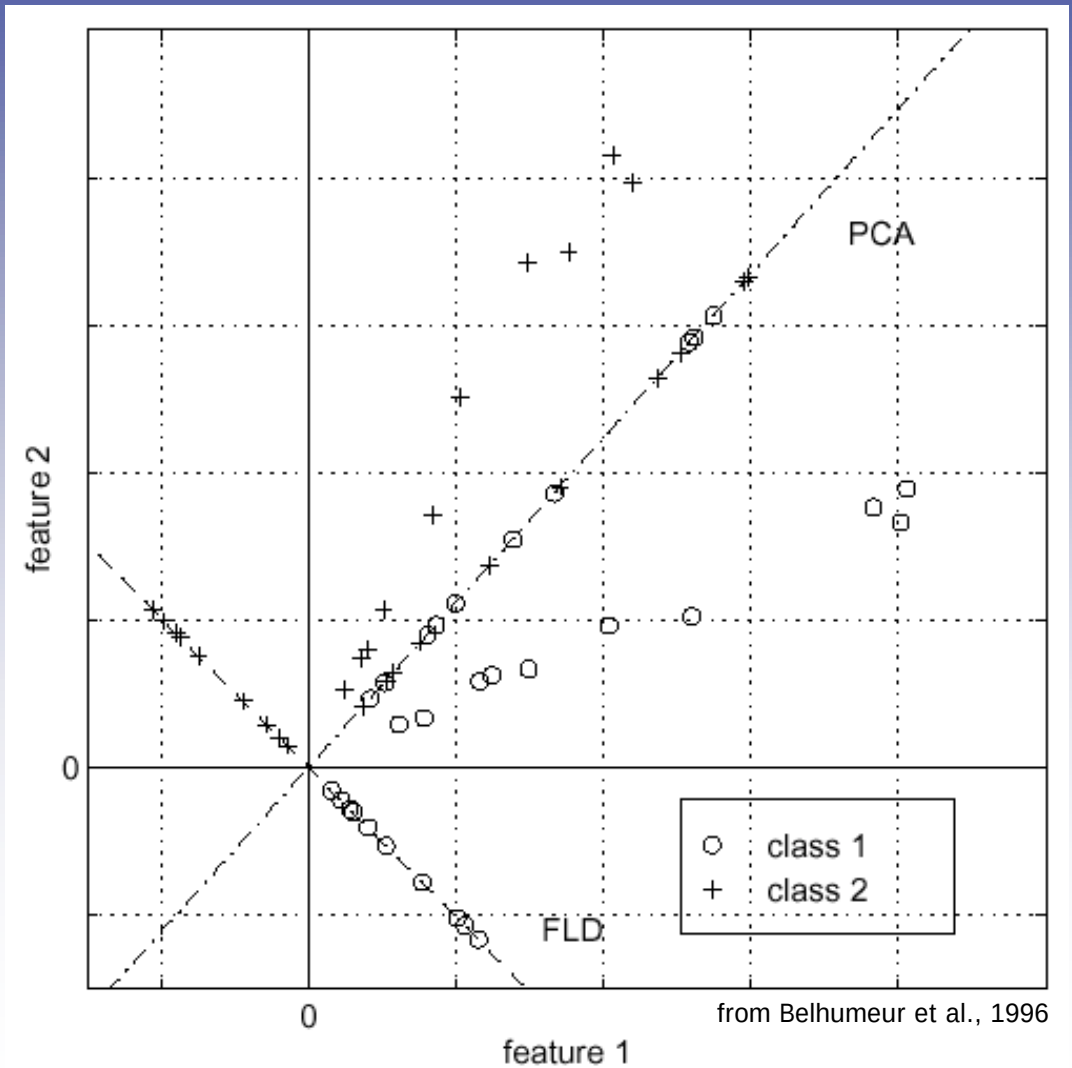


FIGURE 2.8. The action of a linear transformation on the feature space will convert an arbitrary normal distribution into another normal distribution. One transformation, **A**, takes the source distribution into distribution $N(\mathbf{A}^t\boldsymbol{\mu}, \mathbf{A}^t\boldsymbol{\Sigma}\mathbf{A})$. Another linear transformation—a projection **P** onto a line defined by vector **a**—leads to $N(\mu, \sigma^2)$ measured along that line. While the transforms yield distributions in a different space, we show them superimposed on the original x_1x_2-space. A whitening transform, $\mathbf{A}_w$, leads to a circularly symmetric Gaussian, here shown displaced. From: Richard O. Duda, Peter E. Hart, and David G. Stork, *Pattern Classification*. Copyright © 2001 by John Wiley & Sons, Inc.

Dimensionality Reduction: PCA vs. FLD



from Belhumeur et al., 1996

Face Recognition (Belhumeur et al., 1996)

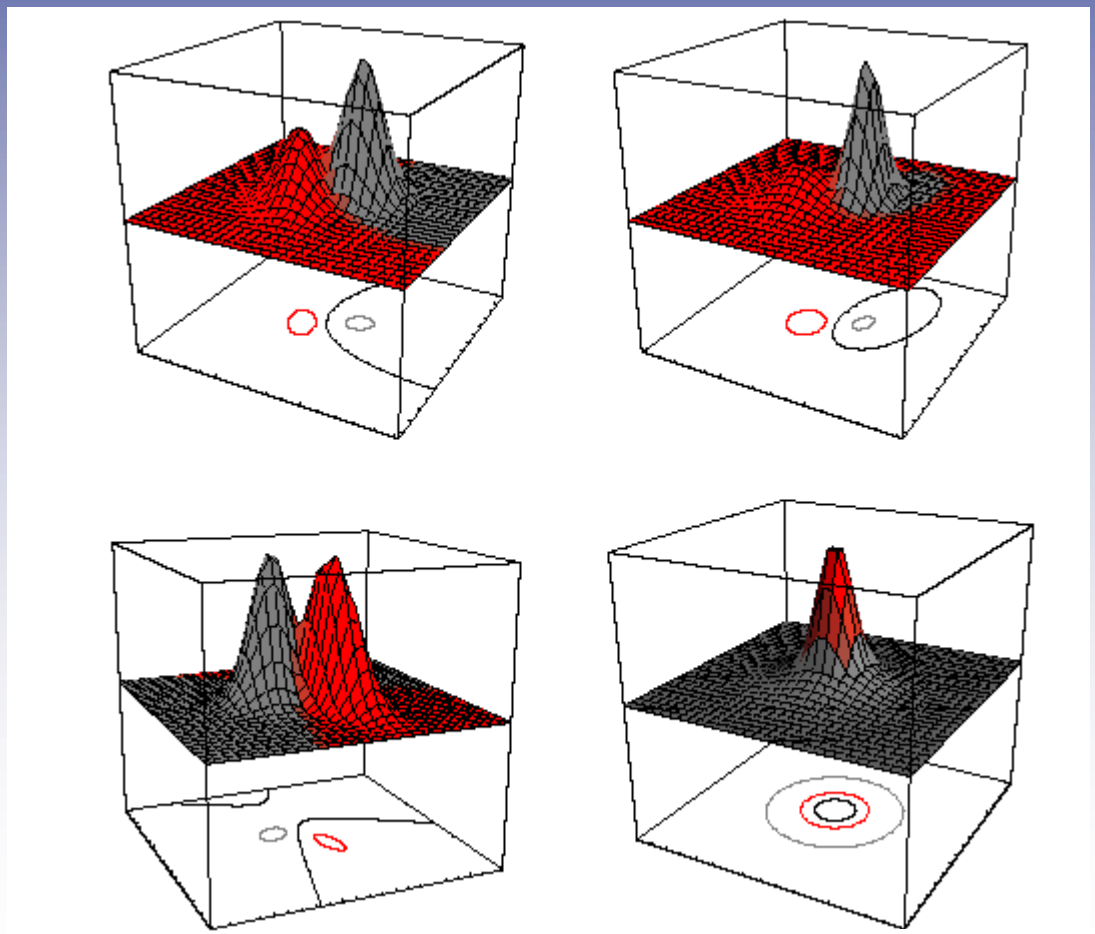
- Given cropped images {**I**} of faces with different lighting, expressions

- Nearest neighbor approach equivalent to correlation (**I**'s normalized to 0 mean, variance 1)
  - Lots of computation, storage
- PCA projection ("Eigenfaces")
  - Better, but sensitive to variation in lighting conditions
- FLD projection ("Fisherfaces")
  - Best (for this problem)

Bayesian Decision Theory: Classification with Known Parametric Forms

- Sometimes we know (or assume) that the data in each category is drawn from a distribution of a certain form—e.g., a Gaussian
- Then classification can be framed as simply a nearest-neighbor calculation, but with a different distance metric to each category—i.e., the Mahalanobis distance for Gaussians

Decision Surfaces for Various 2-Gaussian Situations



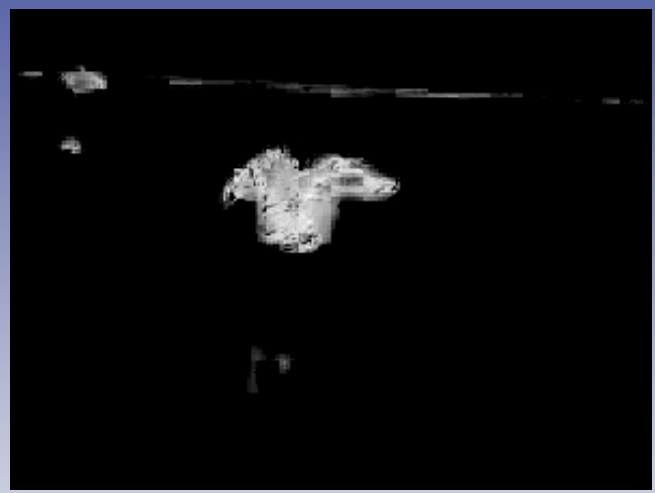
from Duda *et al.*

Example: Color-based Image Comparison

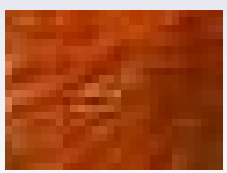
- Per image: e.g., histograms from Image Processing lecture
- Per pixel: Sample homogeneously-colored regions...
  - Parametric: Fit model(s) to pixels, threshold on distance (e.g., Mahalanobis)
  - Non-parametric: Normalize accumulated array, threshold on likelihood

Color Similarity: RGB Mahalanobis Distance

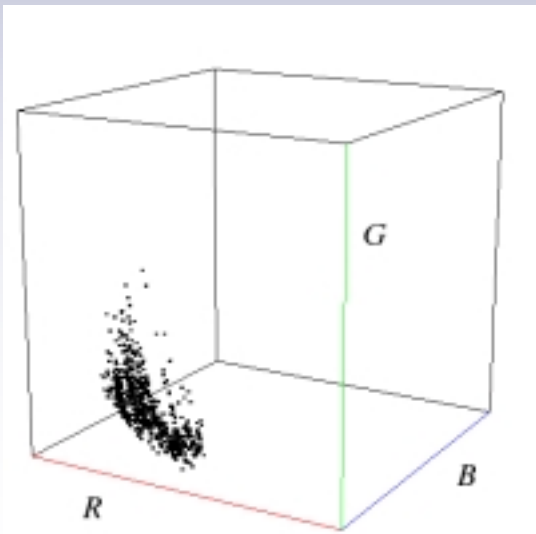
$\mathbf{I}$

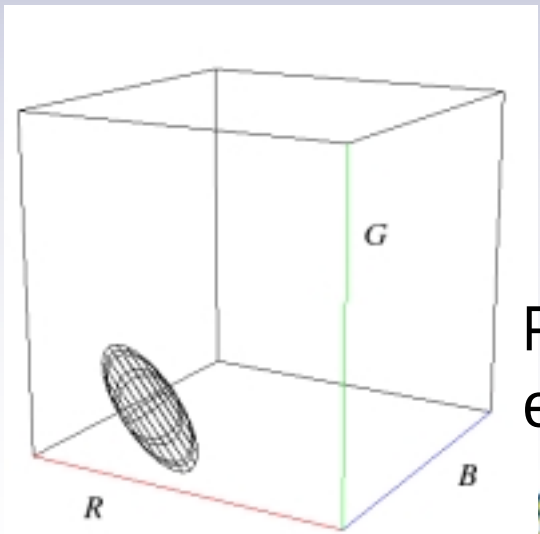


$\gamma(\mathbf{I})$



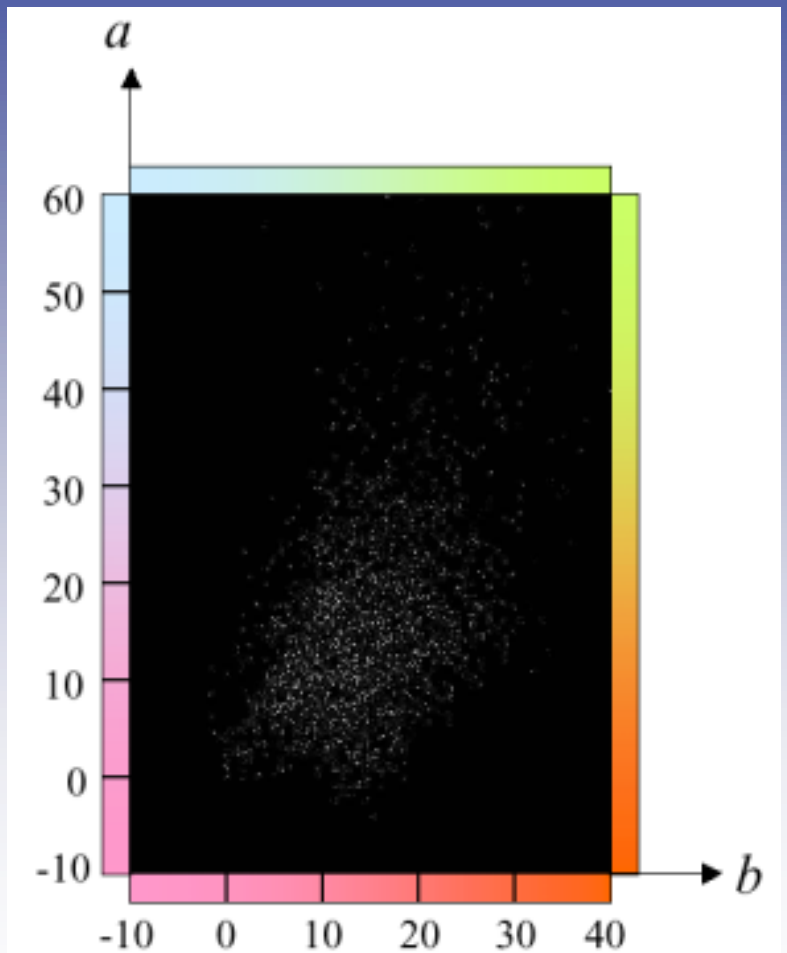
Sample



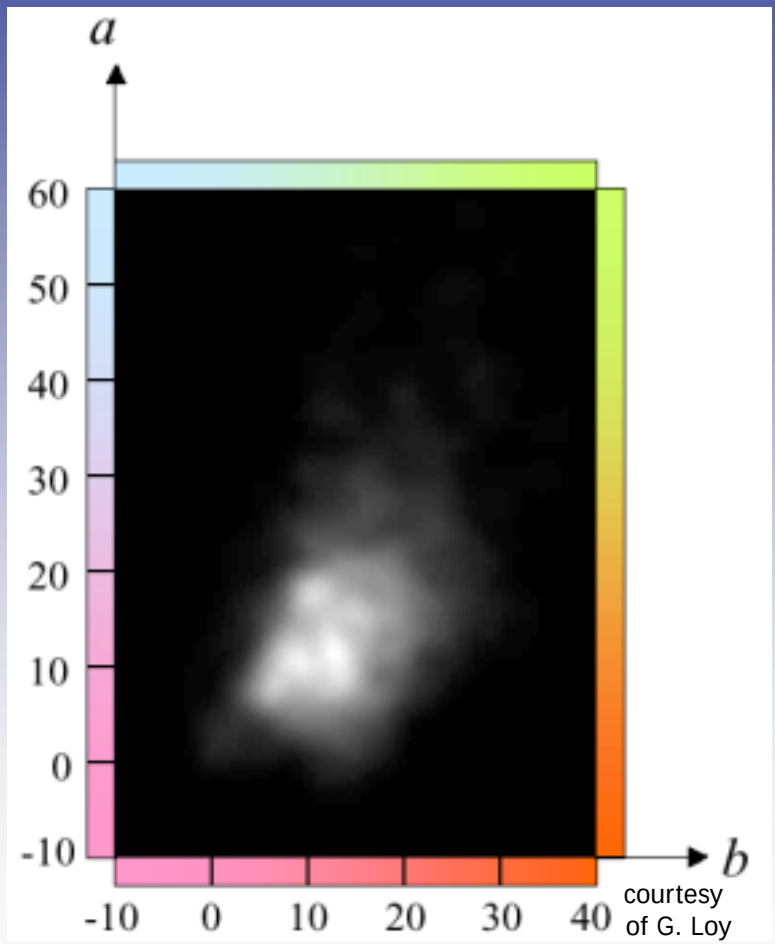


PCA-fitted ellipsoid

Non-parametric Color Models



Skin chrominance points



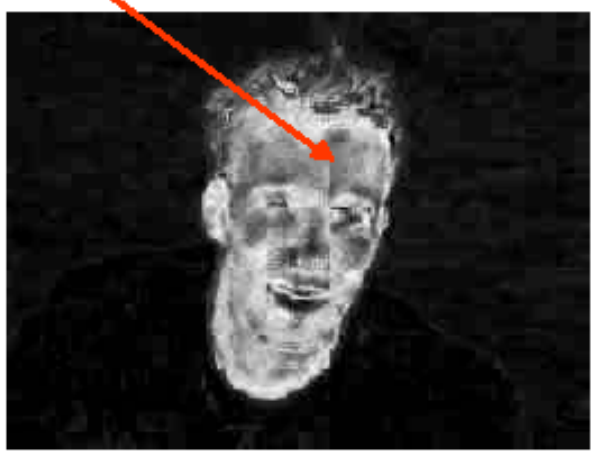
Smoothed, [0,1]-normalized

Non-parametric Skin Classification

For every pixel $\mathbf{p}_i$ in $\mathbf{I}_{test}$

- Determine the chrominance values (a_i, b_i) of $\mathbf{I}_{test}(\mathbf{p}_i)$
- Lookup the skin likelihood for (a_i, b_i) using the skin chrominance model.
- Assign this likelihood to $\mathbf{I}_{skin}(\mathbf{p}_i)$

$\mathbf{I}_{test}$



$\mathbf{I}_{skin}$

courtesy of G. Loy

Other Methods

- Boosting
  - AdaBoost (Freund & Schapire, 1997)
    - "Weak learners": Classifiers that do better than chance
    - Train *m* weak learners on successive versions of data set with misclassifications from last stage emphasized
    - Combined classifier takes weighted average of *m* "votes"
- Stochastic search (think particle filtering)
  - Simulated annealing
  - Genetic algorithms