Lexical Analysis:
DFA Minimization & Wrap Up

Automating Scanner Construction

PREVIOUSLY

RE→NFA *(Thompson's construction)*

- Build an NFA for each term
- Combine them with ε-moves

NFA →DFA *(subset construction)*

- Build the simulation

TODAY

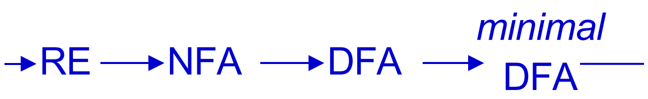
DFA →Minimal DFA

- Hopcroft's algorithm

DFA →RE *(not really part of scanner construction)*

- All pairs, all paths problem
- Union together paths from s_0 to a final state

The Cycle of Constructions



DFA Minimization

The Big Picture

- Discover sets of equivalent states
- Represent each such set with just one state

DFA Minimization

The Big Picture

- Discover sets of equivalent states
- Represent each such set with just one state

Two states are equivalent if and only if:

- The set of paths leading to them are equivalent
- $\forall\ \alpha \in \Sigma$, transitions on α lead to equivalent states (DFA)
- α-transitions to distinct sets $\Rightarrow$ states must be in distinct sets

DFA Minimization

The Big Picture

- Discover sets of equivalent states
- Represent each such set with just one state

Two states are equivalent if and only if:

- The set of paths leading to them are equivalent
- $\forall\ \alpha \in \Sigma$, transitions on α lead to equivalent states (DFA)
- α-transitions to distinct sets $\Rightarrow$ states must be in distinct sets

A partition P of S

- Each $s \in S$ is in exactly one set $p_i \in P$
- The algorithm iteratively partitions the DFA's states

DFA Minimization

Details of the algorithm

- Group states into maximal size sets, *optimistically*
- Iteratively subdivide those sets, as needed
- States that remain grouped together are equivalent

Initial partition, P_0, has two sets: $\{D_F\}$ & $\{D-D_F\}$

$(DFA = (Q,\Sigma,\delta,q_0,F))$

Splitting a set ("partitioning a set by <u>a</u>")

- Assume q_i, & $q_j \in s$, and $\delta(q_i,\underline{a}) = q_x$, & $\delta(q_j,\underline{a}) = q_y$
- If q_x & q_y are not in the same set, then s must be split
  - → q_i has transition on a, q_j does not ⇒ <u>a</u> splits s
- One state in the final DFA cannot have two transitions on <u>a</u>

DFA Minimization

The algorithm

$P \leftarrow \{ D_F, \{D-D_F\}\}$

while (P is still changing)

$T \leftarrow \varnothing$

for each set $p \in P$

$T \leftarrow T \cup Split(p)$

$P \leftarrow T$

Split(S)

for each $\alpha \in \Sigma$

if α *splits S into* s_1 *and* s_2

then return $\{ s_1, s_2\}$

return S

Why does this work?

- Partition $P \in 2^D$
- Starts with 2 subsets of D $\{D_F\}$ and $\{D-D_F\}$
- *While* loop takes $P_i \rightarrow P_{i+1}$ by splitting 1 or more sets
- P_{i+1} is at least one step closer to the partition with $|D|$ sets
- Maximum of $|D|$ splits

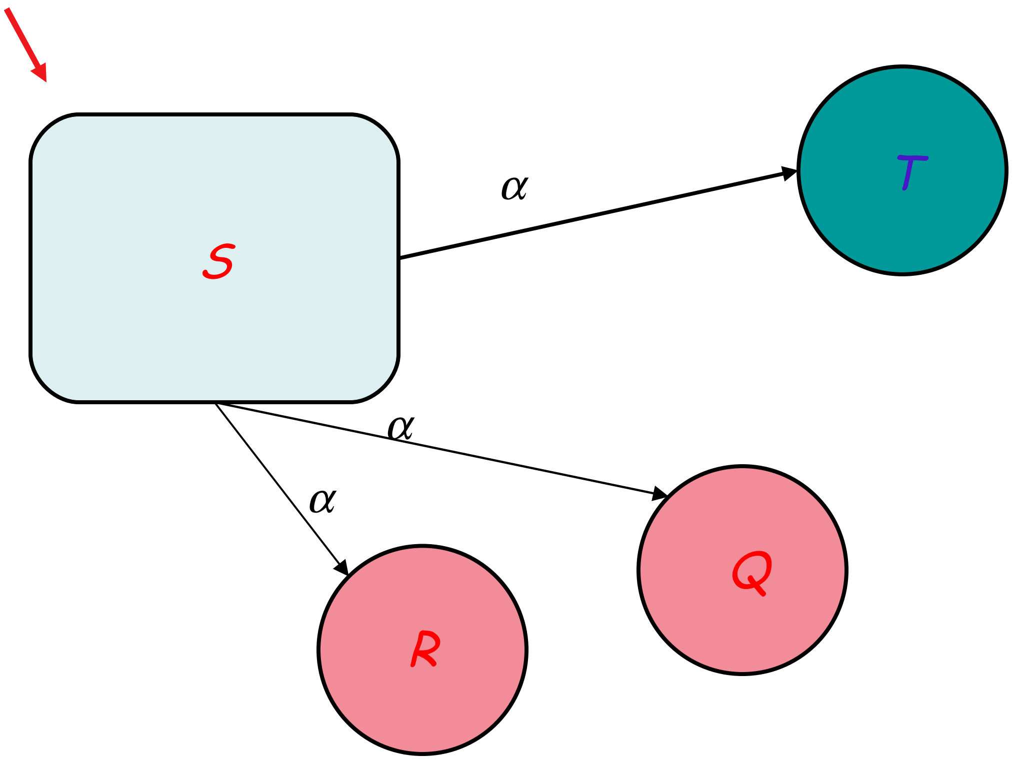
Note that

- Partitions are <u>never</u> combined
- Initial partition ensures that final states are intact

This is a fixed-point algorithm!

Key Idea: Splitting S around α

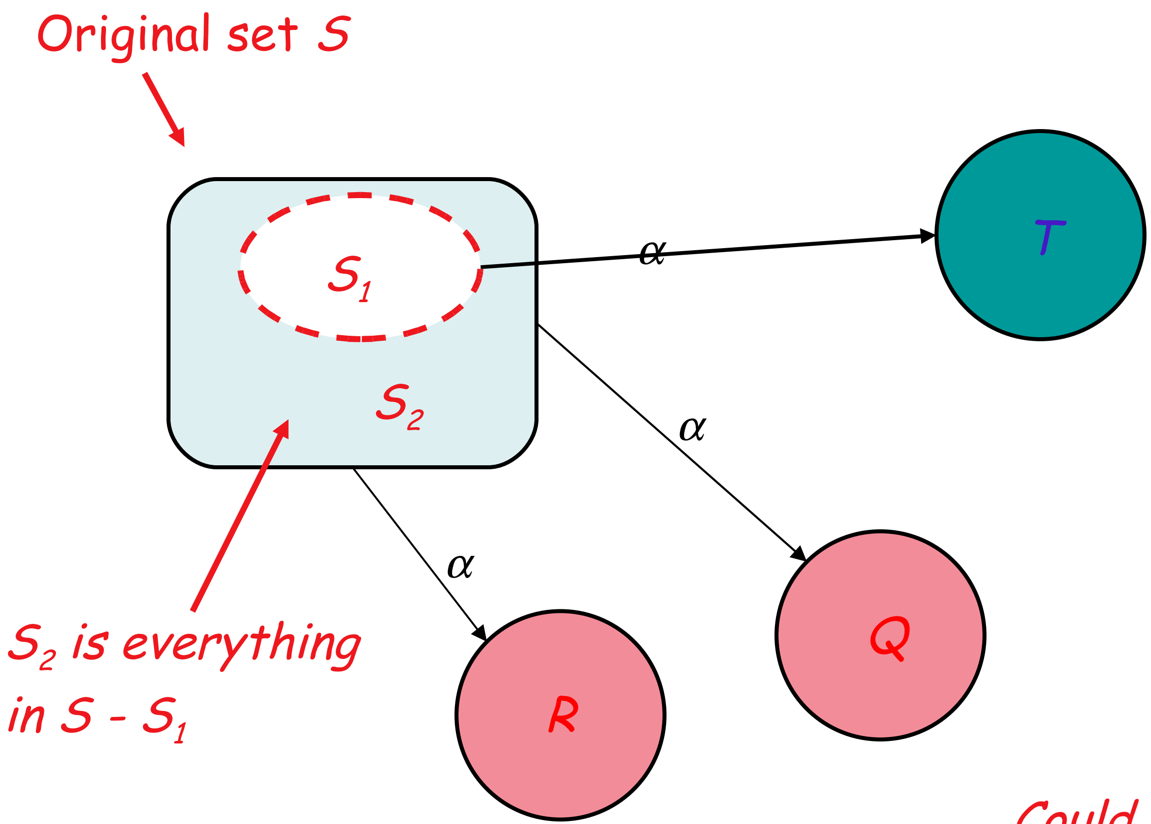
Original set S



S has transitions on α to R, Q, & T

The algorithm partitions S around α

Key Idea: Splitting S around α

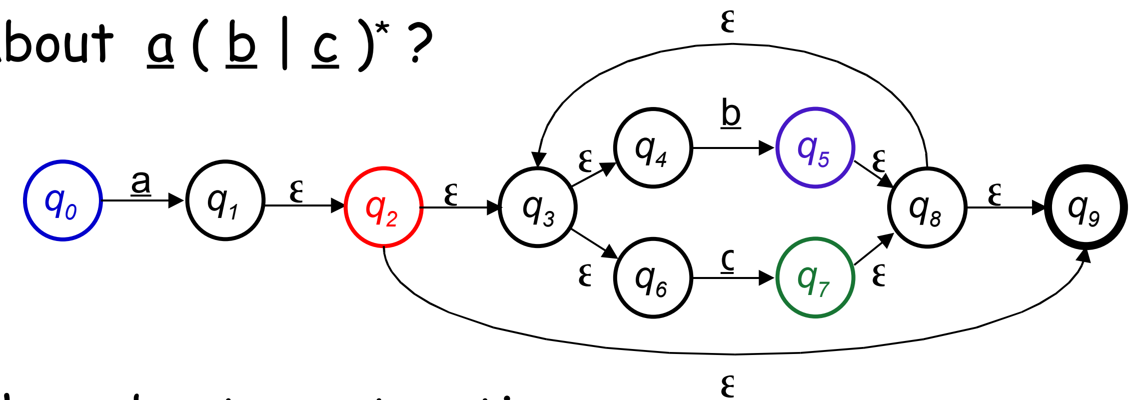


Could we split S_2 further?

Yes, but it does not help asymptotically

DFA Minimization

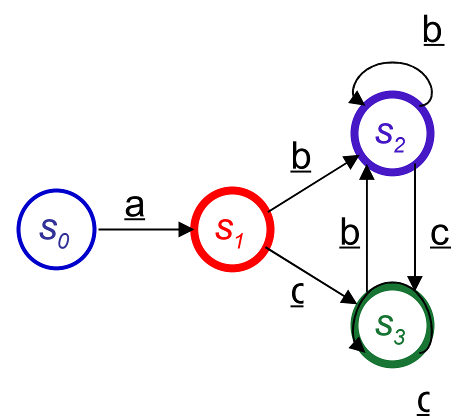
What about $\underline{a}\,(\,\underline{b}\mid\underline{c}\,)^*$?



First, the subset construction:

		ε-closure(move(s,*))		
	NFA states	a	b	c
s_0	q_0	$q_1, q_2, q_3, q_4, q_6, q_9$	none	none
s_1	$q_1, q_2, q_3, q_4, q_6, q_9$	none	$q_5, q_8, q_9, q_3, q_4, q_6$	$q_7, q_8, q_9, q_3, q_4, q_6$
s_2	$q_5, q_8, q_9, q_3, q_4, q_6$	none	s_2	s_3
s_3	$q_7, q_8, q_9, q_3, q_4, q_6$	none	s_2	s_3

Final states

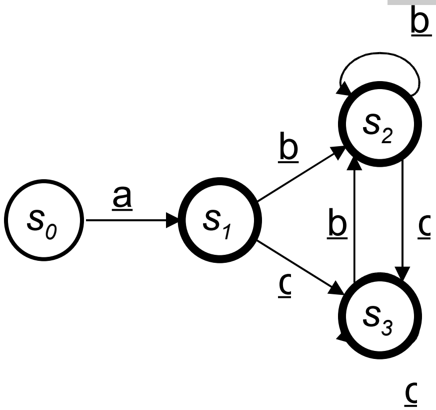


DFA Minimization

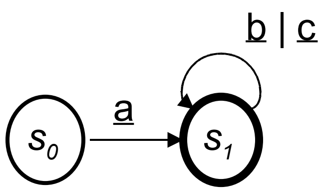
Then, apply the minimization algorithm

		Split on		
	Current Partition	a	b	c
P_0	$\{s_1, s_2, s_3\}$ $\{s_0\}$	*none*	*none*	*none*

final states



To produce the minimal DFA



In lecture 4, we observed that a human would design a simpler automaton than Thompson's construction & the subset construction did.

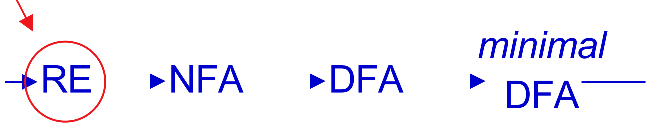
Minimizing that DFA produces the one that a human would design!

Abbreviated Register Specification

Start with a regular expression

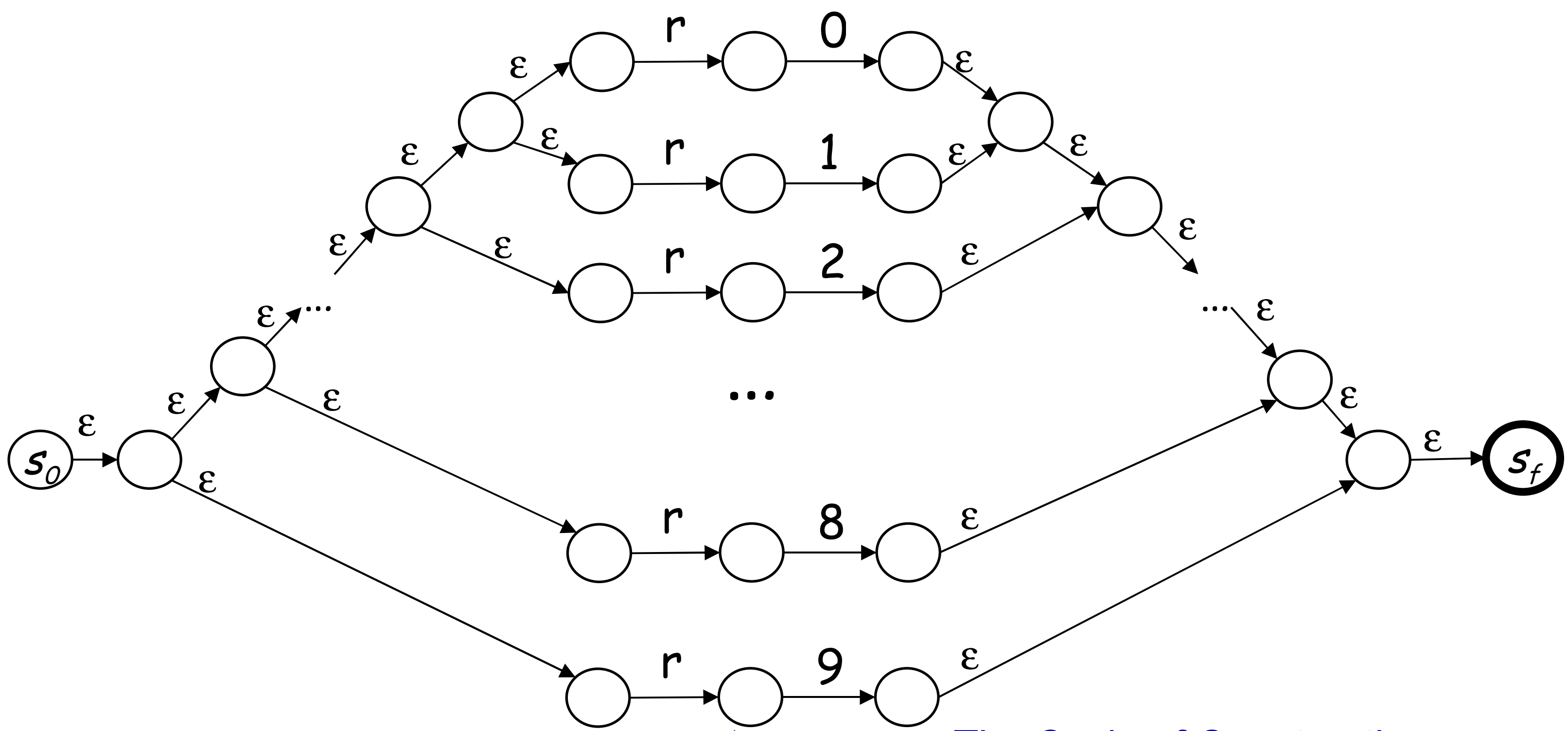
r0 | r1 | r2 | r3 | r4 | r5 | r6 | r7 | r8 | r9

The Cycle of Constructions



Abbreviated Register Specification

Thompson's construction produces



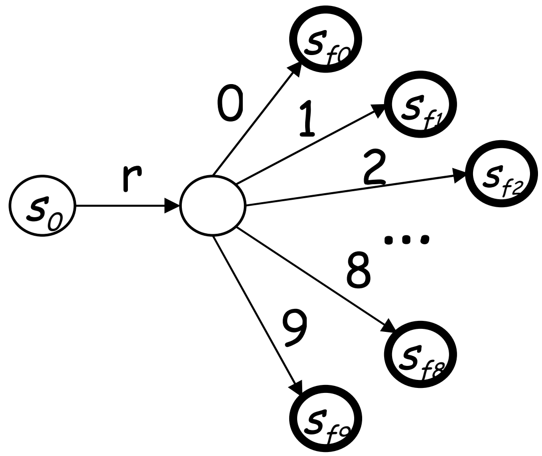
To make it fit, we've eliminated the ε-transition between "r" and "0…9".

The Cycle of Constructions

→ RE → NFA → DFA → *minimal* DFA

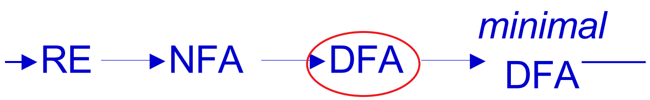
Abbreviated Register Specification

The subset construction builds



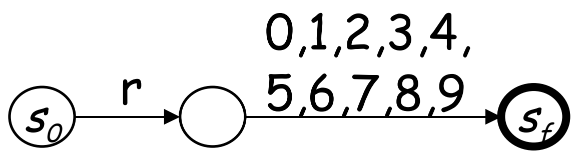
This is a DFA, but it has a lot of states …

The Cycle of Constructions



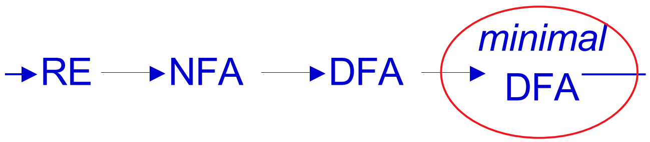
Abbreviated Register Specification

The DFA minimization algorithm builds



This looks like what a skilled compiler writer would do!

The Cycle of Constructions



Limits of Regular Languages

Advantages of Regular Expressions

- Simple & powerful notation for specifying patterns
- Automatic construction of fast recognizers
- Many kinds of syntax can be specified with REs

Example — an expression grammar

Term → [a-zA-Z] ([a-zA-z] | [0-9])*

Op → + | - | * | /

Expr → (*Term Op*)* *Term*

Of course, this would generate a DFA …

If REs are so useful …

Why not use them for everything?

Limits of Regular Languages

Not all languages are regular

RL's $\subset$ CFL's $\subset$ CSL's

You cannot construct DFA's to recognize these languages

- $L = \{ p^k q^k \}$ *(parenthesis languages)*
- $L = \{ wcw^r \mid w \in \Sigma^* \}$

Neither of these is a regular language *(nor an RE)*

But, this is a little subtle. You can construct DFA's for

- Strings with alternating 0's and 1's

  $(\varepsilon \mid 1) (01)^* (\varepsilon \mid 0)$
- Strings with an even number of 0's and 1's

RE's can count bounded sets and bounded differences

What can be so hard?

Poor language design can complicate scanning

- Reserved words are important

  if then then then = else; else else = then (PL/I)

- Insignificant blanks (Fortran & Algol68)

  do 10 i = 1,25

  do 10 i = 1.25

- String constants with special characters (C, C++, Java, …)

  newline, tab, quote, comment delimiters, …

- Finite closures (Fortran 66 & Basic)
  - → Limited identifier length
  - → Adds states to count length

Building Faster Scanners from the DFA

Table-driven recognizers waste effort

- Read (& classify) the next character
- Find the next state
- Assign to the state variable
- Trip through case logic in *action()*
- Branch back to the top

We can do better

- Encode state & actions in the code
- Do transition tests locally
- Generate ugly, spaghetti-like code
- Takes (many) fewer operations per input character

```
char ← next character;
state ← s0 ;
call action(state,char);
while (char ≠ eof)
   state ← δ(state,char);
   call action(state,char);
   char ← next character;

if T(state) = final then
   report acceptance;
else
    report failure;
```

Building Faster Scanners from the DFA

A direct-coded recognizer for r Digit Digit*

```
      goto s0;
   s0: word ← Ø;
      char ← next character;
      if (char = 'r')
         then goto s1;
         else goto se;
   s1: word ← word + char;
      char ← next character;
      if ('0' ≤ char ≤ '9')
         then goto s2;
         else goto se;
   s2: word ← word + char;
      char ← next character;
      if ('0' ≤ char ≤ '9')
         then goto s2;
         else if (char = eof)
            then report success;
            else goto se;
   se: print error message;
      return failure;
```

- Many fewer operations per character
- Almost no memory operations
- Even faster with careful use of fall-through cases

Building Faster Scanners

Hashing keywords versus encoding them directly

- Some *(well-known)* compilers recognize keywords as identifiers and check them in a hash table
- Encoding keywords in the DFA is a better idea
  - → O(1) cost per transition
  - → Avoids hash lookup on each identifier

It is hard to beat a well-implemented DFA scanner

Building Scanners

The point

- All this technology lets us automate scanner construction
- Implementer writes down the regular expressions
- Scanner generator builds NFA, DFA, minimal DFA, and then writes out the (table-driven or direct-coded) code
- This reliably produces fast, robust scanners

For most modern language features, this works

- You should think twice before introducing a feature that defeats a DFA-based scanner
- The ones we've seen (e.g., insignificant blanks, non-reserved keywords) have not proven particularly useful or long lasting