



Parsing VI

The Last Parsing Lecture

Quiz : The Left Recursive SheepNoise Grammar



1. Goal $\rightarrow$ SheepNoise
2. SheepNoise $\rightarrow$ SheepNoise baa
3. | baa



Example From SheepNoise

Initial step builds the item $[Goal \rightarrow \cdot SheepNoise, EOF]$
and takes its *closure*()

Closure($[Goal \rightarrow \cdot SheepNoise, EOF]$)

<i>Item</i>	<i>From</i>
$[Goal \rightarrow \cdot SheepNoise, \underline{EOF}]$	Original item
$[SheepNoise \rightarrow \cdot SheepNoise \underline{baa}, \underline{EOF}]$	1, $\delta \underline{a}$ is <u>EOF</u>
$[SheepNoise \rightarrow \cdot \underline{baa}, \underline{EOF}]$	1, $\delta \underline{a}$ is <u>EOF</u>
$[SheepNoise \rightarrow \cdot SheepNoise \underline{baa}, \underline{baa}]$	2, $\delta \underline{a}$ is <u>baa</u> <u>EOF</u>
$[SheepNoise \rightarrow \cdot \underline{baa}, \underline{baa}]$	2, $\delta \underline{a}$ is <u>baa</u> <u>EOF</u>

So, S_0 is

{ $[Goal \rightarrow \cdot SheepNoise, \underline{EOF}]$, $[SheepNoise \rightarrow \cdot SheepNoise \underline{baa}, \underline{EOF}]$,
 $[SheepNoise \rightarrow \cdot \underline{baa}, \underline{EOF}]$, $[SheepNoise \rightarrow \cdot SheepNoise \underline{baa}, \underline{baa}]$,
 $[SheepNoise \rightarrow \cdot \underline{baa}, \underline{baa}]$ }



Example from SheepNoise

S_0 is { [*Goal* → • *SheepNoise*, EOF], [*SheepNoise* → • *SheepNoise* baa, EOF],
[*SheepNoise* → • baa, EOF], [*SheepNoise* → • *SheepNoise* baa, baa],
[*SheepNoise* → • baa, baa] }

Goto(S_0 , baa)

- Loop produces

<i>Item</i>	<i>From</i>
[<i>SheepNoise</i> → <u>baa</u> • , <u>EOF</u>]	Item 3 in s_0
[<i>SheepNoise</i> → <u>baa</u> • , <u>baa</u>]	Item 5 in s_0

- Closure adds nothing since • is at end of *rhs* in each item

In the construction, this produces s_2

{ [*SheepNoise* → baa• , {EOF, baa}] }

New, but *obvious*, notation
for two distinct items
[*SheepNoise* → baa• , EOF] &
[*SheepNoise* → baa• , baa]



Example from SheepNoise

Starts with S_0

$S_0 : \{ [Goal \rightarrow \cdot SheepNoise, \underline{EOF}], [SheepNoise \rightarrow \cdot SheepNoise \underline{baa}, \underline{EOF}],$
 $[SheepNoise \rightarrow \cdot \underline{baa}, \underline{EOF}], [SheepNoise \rightarrow \cdot SheepNoise \underline{baa}, \underline{baa}],$
 $[SheepNoise \rightarrow \cdot \underline{baa}, \underline{baa}] \}$



Example from SheepNoise

Starts with S_0

$S_0 : \{ [Goal \rightarrow \cdot SheepNoise, \underline{EOF}], [SheepNoise \rightarrow \cdot SheepNoise \underline{baa}, \underline{EOF}], [SheepNoise \rightarrow \cdot \underline{baa}, \underline{EOF}], [SheepNoise \rightarrow \cdot SheepNoise \underline{baa}, \underline{baa}], [SheepNoise \rightarrow \cdot \underline{baa}, \underline{baa}] \}$

Iteration 1 computes

$S_1 = Goto(S_0, SheepNoise) =$

$\{ [Goal \rightarrow SheepNoise \cdot, \underline{EOF}], [SheepNoise \rightarrow SheepNoise \cdot \underline{baa}, \underline{EOF}], [SheepNoise \rightarrow SheepNoise \cdot \underline{baa}, \underline{baa}] \}$

$S_2 = Goto(S_0, \underline{baa}) = \{ [SheepNoise \rightarrow \underline{baa} \cdot, \underline{EOF}], [SheepNoise \rightarrow \underline{baa} \cdot, \underline{baa}] \}$



Example from SheepNoise

Starts with S_0

$S_0 : \{ [Goal \rightarrow \cdot SheepNoise, \underline{EOF}], [SheepNoise \rightarrow \cdot SheepNoise \underline{baa}, \underline{EOF}],$
 $[SheepNoise \rightarrow \cdot \underline{baa}, \underline{EOF}], [SheepNoise \rightarrow \cdot SheepNoise \underline{baa}, \underline{baa}],$
 $[SheepNoise \rightarrow \cdot \underline{baa}, \underline{baa}] \}$

Iteration 1 computes

$S_1 = Goto(S_0, SheepNoise) =$

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 $[SheepNoise \rightarrow SheepNoise \cdot \underline{baa}, \underline{baa}] \}$

$S_2 = Goto(S_0, \underline{baa}) = \{ [SheepNoise \rightarrow \underline{baa} \cdot, \underline{EOF}],$
 $[SheepNoise \rightarrow \underline{baa} \cdot, \underline{baa}] \}$

Iteration 2 computes

$S_3 = Goto(S_1, \underline{baa}) = \{ [SheepNoise \rightarrow SheepNoise \underline{baa} \cdot, \underline{EOF}],$
 $[SheepNoise \rightarrow SheepNoise \underline{baa} \cdot, \underline{baa}] \}$



Example from SheepNoise

Starts with S_0

$S_0 : \{ [Goal \rightarrow \cdot SheepNoise, \underline{EOF}], [SheepNoise \rightarrow \cdot SheepNoise \underline{baa}, \underline{EOF}], [SheepNoise \rightarrow \cdot \underline{baa}, \underline{EOF}], [SheepNoise \rightarrow \cdot SheepNoise \underline{baa}, \underline{baa}], [SheepNoise \rightarrow \cdot \underline{baa}, \underline{baa}] \}$

Iteration 1 computes

$S_1 = Goto(S_0, SheepNoise) =$

$\{ [Goal \rightarrow SheepNoise \cdot, \underline{EOF}], [SheepNoise \rightarrow SheepNoise \cdot \underline{baa}, \underline{EOF}], [SheepNoise \rightarrow SheepNoise \cdot \underline{baa}, \underline{baa}] \}$

$S_2 = Goto(S_0, \underline{baa}) = \{ [SheepNoise \rightarrow \underline{baa} \cdot, \underline{EOF}], [SheepNoise \rightarrow \underline{baa} \cdot, \underline{baa}] \}$

Iteration 2 computes

$S_3 = Goto(S_1, \underline{baa}) = \{ [SheepNoise \rightarrow SheepNoise \underline{baa} \cdot, \underline{EOF}], [SheepNoise \rightarrow SheepNoise \underline{baa} \cdot, \underline{baa}] \}$

Nothing more to compute, since $\cdot$ is at the end of every item in S_3 .



Example from SheepNoise

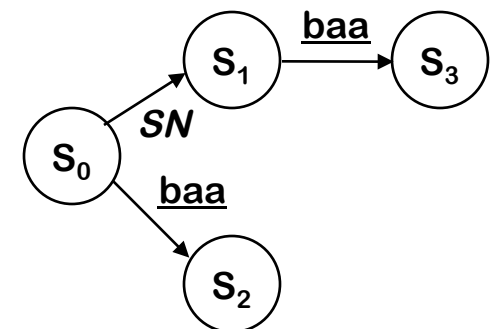
$S_0 : \{ [Goal \rightarrow \cdot SheepNoise, \underline{EOF}], [SheepNoise \rightarrow \cdot SheepNoise \underline{baa}, \underline{EOF}],$
 $[SheepNoise \rightarrow \cdot \underline{baa}, \underline{EOF}], [SheepNoise \rightarrow \cdot SheepNoise \underline{baa}, \underline{baa}],$
 $[SheepNoise \rightarrow \cdot \underline{baa}, \underline{baa}] \}$

$S_1 = Goto(S_0, SheepNoise) =$
 $\{ [Goal \rightarrow SheepNoise \cdot, \underline{EOF}], [SheepNoise \rightarrow SheepNoise \cdot \underline{baa}, \underline{EOF}],$
 $[SheepNoise \rightarrow SheepNoise \cdot \underline{baa}, \underline{baa}] \}$

$S_2 = Goto(S_0, \underline{baa}) = \{ [SheepNoise \rightarrow \underline{baa} \cdot, \underline{EOF}],$
 $[SheepNoise \rightarrow \underline{baa} \cdot, \underline{baa}] \}$

$S_3 = Goto(S_1, \underline{baa}) = \{ [SheepNoise \rightarrow SheepNoise \underline{baa} \cdot, \underline{EOF}],$
 $[SheepNoise \rightarrow SheepNoise \underline{baa} \cdot, \underline{baa}] \}$

Control DFA for SN

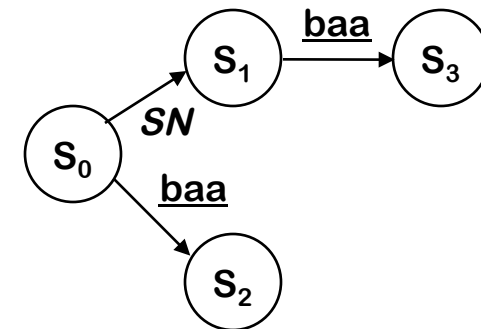




Filling in the ACTION and GOTO Tables

$\forall$ set $s_x \in S$
 $\forall$ item $i \in s_x$
 if i is $[A \rightarrow \beta \cdot \underline{a}d, \underline{b}]$ and $\text{goto}(s_x, \underline{a}) = s_k, \underline{a} \in T$
 then $\text{ACTION}[x, \underline{a}] \leftarrow \text{"shift k"}$
 else if i is $[S' \rightarrow S \cdot, \text{EOF}]$
 then $\text{ACTION}[x, \underline{a}] \leftarrow \text{"accept"}$
 else if i is $[A \rightarrow \beta \cdot, \underline{a}]$
 then $\text{ACTION}[x, \underline{a}] \leftarrow \text{"reduce } A \rightarrow \beta \text{"}$
 $\forall n \in NT$
 if $\text{goto}(s_x, n) = s_k$
 then $\text{GOTO}[x, n] \leftarrow k$

Control DFA for SN



$S_0 : \{ [Goal \rightarrow \cdot \text{SheepNoise}, \underline{EOF}], [SheepNoise \rightarrow \cdot \text{SheepNoise } \underline{baa}, \underline{EOF}],$
 $[SheepNoise \rightarrow \cdot \underline{baa}, \underline{EOF}], [SheepNoise \rightarrow \cdot \text{SheepNoise } \underline{baa}, \underline{baa}],$
 $[SheepNoise \rightarrow \cdot \underline{baa}, \underline{baa}] \}$

$S_1 = \text{Goto}(S_0, \text{SheepNoise}) =$
 $\{ [Goal \rightarrow \text{SheepNoise } \cdot, \underline{EOF}], [SheepNoise \rightarrow \text{SheepNoise } \cdot \underline{baa}, \underline{EOF}],$
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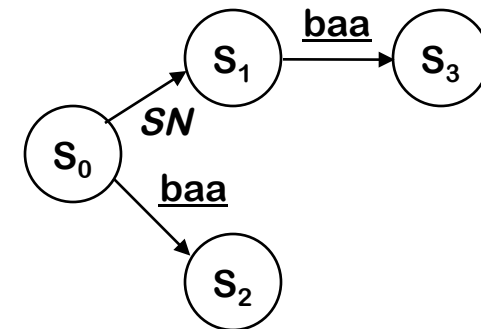
$S_2 = \text{Goto}(S_0, \underline{baa}) = \{ [SheepNoise \rightarrow \underline{baa} \cdot, \underline{EOF}], [SheepNoise \rightarrow \underline{baa} \cdot, \underline{baa}] \}$



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 $\forall$ item $i \in s_x$
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 then $\text{ACTION}[x, \underline{a}] \leftarrow \text{"shift } k\text{"}$
 else if i is $[S' \rightarrow S \cdot, \text{EOF}]$
 then $\text{ACTION}[x, \underline{a}] \leftarrow \text{"accept"}$
 else if i is $[A \rightarrow \beta \cdot, \underline{a}]$
 then $\text{ACTION}[x, \underline{a}] \leftarrow \text{"reduce } A \rightarrow \beta\text{"}$
 $\forall n \in NT$
 if $\text{goto}(s_x, n) = s_k$
 then $\text{GOTO}[x, n] \leftarrow k$

Control DFA for SN



$S_1 = \text{Goto}(S_0, \text{SheepNoise}) =$
 $\{ [\text{Goal} \rightarrow \text{SheepNoise} \cdot, \underline{\text{EOF}}], [\text{SheepNoise} \rightarrow \text{SheepNoise} \cdot \underline{\text{baa}}, \underline{\text{EOF}}],$
 $[\text{SheepNoise} \rightarrow \text{SheepNoise} \cdot \underline{\text{baa}}, \underline{\text{baa}}] \}$

$S_2 = \text{Goto}(S_0, \underline{\text{baa}}) = \{ [\text{SheepNoise} \rightarrow \underline{\text{baa}} \cdot, \underline{\text{EOF}}],$
 $[\text{SheepNoise} \rightarrow \underline{\text{baa}} \cdot, \underline{\text{baa}}] \}$

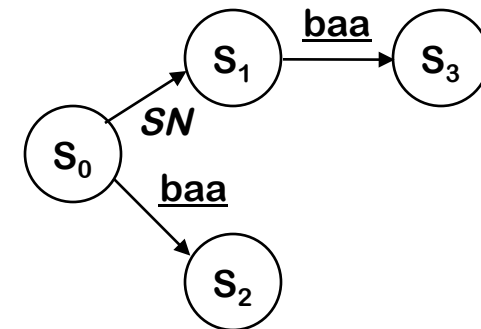
$S_3 = \text{Goto}(S_1, \underline{\text{baa}}) = \{ [\text{SheepNoise} \rightarrow \text{SheepNoise} \underline{\text{baa}} \cdot, \underline{\text{EOF}}],$
 $[\text{SheepNoise} \rightarrow \text{SheepNoise} \underline{\text{baa}} \cdot, \underline{\text{baa}}] \}$



Filling in the ACTION and GOTO Tables

$\forall$ set $s_x \in S$
 $\forall$ item $i \in s_x$
 if i is $[A \rightarrow \beta \cdot \underline{a}, \underline{b}]$ and $\text{goto}(s_x, \underline{a}) = s_k, \underline{a} \in T$
 then $\text{ACTION}[x, \underline{a}] \leftarrow \text{"shift } k\text{"}$
 else if i is $[S' \rightarrow S \cdot, \text{EOF}]$
 then $\text{ACTION}[x, \underline{a}] \leftarrow \text{"accept"}$
 else if i is $[A \rightarrow \beta \cdot, \underline{a}]$
 then $\text{ACTION}[x, \underline{a}] \leftarrow \text{"reduce } A \rightarrow \beta\text{"}$
 $\forall n \in NT$
 if $\text{goto}(s_x, n) = s_k$
 then $\text{GOTO}[x, n] \leftarrow k$

Control DFA for SN



ACTION		
State	EOF	<u>baa</u>
0	—	shift 2
1	accept	shift 3
2	reduce 3	reduce 3
3	reduce 2	reduce 2

GOTO	
State	<i>SheepNoise</i>
0	1
1	-
2	-
3	-



Shrinking the Tables

Three options:

- Combine terminals such as number & identifier, + & -, * & /
 - Directly removes a column, may remove a row
 - For expression grammar, 198 (vs. 384) table entries
- Combine rows or columns *(table compression)*
 - Implement identical rows once & remap states
 - Requires extra indirection on each lookup
 - Use separate mapping for ACTION & for GOTO
- Use another construction algorithm
 - Both LALR(1) and SLR(1) produce smaller tables
 - Implementations are readily available



What can go wrong with LR table construction?

What if set s contains $[A \rightarrow \beta \cdot \underline{a} \gamma, \underline{b}]$ and $[B \rightarrow \beta \cdot, \underline{a}]$?

- First item generates "shift", second generates "reduce"
- Both define $\text{ACTION}[s, \underline{a}]$ — cannot do both actions
- This is a fundamental ambiguity, called a *shift/reduce error*
- Modify the grammar to eliminate it (if-then-else)
- Shifting will often resolve it correctly

EaC includes a
work example

What if set s contains $[A \rightarrow \gamma \cdot, \underline{a}]$ and $[B \rightarrow \gamma \cdot, \underline{a}]$?

- Each generates "reduce", but with a different production
- Both define $\text{ACTION}[s, \underline{a}]$ — cannot do both reductions
- This fundamental ambiguity is called a *reduce/reduce error*
- Modify the grammar to eliminate it

In either case, the grammar is not LR(1)



Summary of top-down and LR(1) parsing

	<i>Advantages</i>	<i>Disadvantages</i>
Top-down recursive descent	Fast Good locality Simplicity Good error detection	Hand-coded High maintenance Right associativity
LR(1)	Fast (Direct encoding) Deterministic langs. Automatable Left associativity	Large working sets Poor error messages Large table sizes



CYK Parser

- Simple context-free-language parser
 - running time is $O(n^3)$, space is $O(n^2)$
- Shunned for many years

“Even tabular methods [CYK, Earley] should be avoided if the language at hand has a grammar for which more efficient algorithms [LL, LALR] are available.” The Theory of Parsing, Aho, Ullman, 1972
- But in practice, running time is more like $O(n^{\approx 1.2})$
 - Plus computers are now 1,000,000-times faster than in 1972

Source: Ras Bodik, Slides: Browsing Web 3.0 on 3.0 Watts

CYK Parser (Sequential Version)

b	a	a	b	a

$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow AB \mid a$

CYK Parser (Sequential Version)

b	a	a	b	a
{B}	{A,C}	{A,C}	{B}	{A,C}

$$\begin{aligned} S &\rightarrow AB \mid BC \\ A &\rightarrow BA \mid a \\ B &\rightarrow CC \mid b \\ C &\rightarrow AB \mid a \end{aligned}$$

CYK Parser (Sequential Version)

b	a	a	b	a
{B}	{A,C}	{A,C}	{B}	{A,C}
BA,BC				
{S,A}				

$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow AB \mid a$

CYK Parser (Sequential Version)

b	a	a	b	a
{B}	{A,C}	{A,C}	{B}	{A,C}
BA,BC	AA, AG GA,CC			
{S,A}	{B}			

$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow AB \mid a$

CYK Parser (Sequential Version)

b	a	a	b	a
{B}	{A,C}	{A,C}	{B}	{A,C}
BA,BC	AA,AG GA,CC	AB,CB		
{S,A}	{B}	{S,C}		

$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow AB \mid a$

CYK Parser (Sequential Version)

b	a	a	b	a
{B}	{A,C}	{A,C}	{B}	{A,C}
BA,BC	AA, AC GA,CC	AB, CB	BA,BC	
{S,A}	{B}	{S,C}	{S,A}	

$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow AB \mid a$

CYK Parser (Sequential Version)

b	a	a	b	a
{B}	{A,C}	{A,C}	{B}	{A,C}
BA,BC	AA, AC CA,CC	AB,CB	BA,BC	
{S,A}	{B}	{S,C}	{S,A}	
BB				
{}				

$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow AB \mid a$

CYK Parser (Sequential Version)

b	a	a	b	a
{B}	{A,C}	{A,C}	{B}	{A,C}
BA,BC	AA,AG GA,CC	AB,CB	BA,BC	
{S,A}	{B}	{S,C}	{S,A}	
BA,BC SA,SC AA,AG {}				

$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow AB \mid a$

CYK Parser (Sequential Version)

b	a	a	b	a
{B}	{A,C}	{A,C}	{B}	{A,C}
BA,BC	AA,AG GA,CC	AB,CB	BA,BC	
{S,A}	{B}	{S,C}	{S,A}	
BB SA,SC AA,AG	AS,AG CS,CC,			
{}	{B}			

$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow AB \mid a$

CYK Parser (Sequential Version)

b	a	a	b	a
{B}	{A,C}	{A,C}	{B}	{A,C}
BA,BC	AA, AG CA,CC	AB,CB	BA,BC	
{S,A}	{B}	{S,C}	{S,A}	
BB SA,SC AA,AG	AS,AG CS,CC BB			
{}	{B}			

$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow AB \mid a$

CYK Parser (Sequential Version)

b	a	a	b	a
{B}	{A,C}	{A,C}	{B}	{A,C}
BA,BC	AA,AG GA,CC	AB,GB	BA,BC	
{S,A}	{B}	{S,C}	{S,A}	
BB SA,SC AA,AG	AS,AG CS,CC, BB	AS,AA CS,CA		
{}	{B}	{}		

$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow AB \mid a$

CYK Parser (Sequential Version)

b	a	a	b	a
{B}	{A,C}	{A,C}	{B}	{A,C}
BA,BC	AA,AG GA,CC	AB,GB	BA,BC	
{S,A}	{B}	{S,C}	{S,A}	
BB SA,SG AA,AG	AS,AG CS,CC, BB	AS,AA CS,GA SA,SG CA,CC		
{}	{B}	{B}		

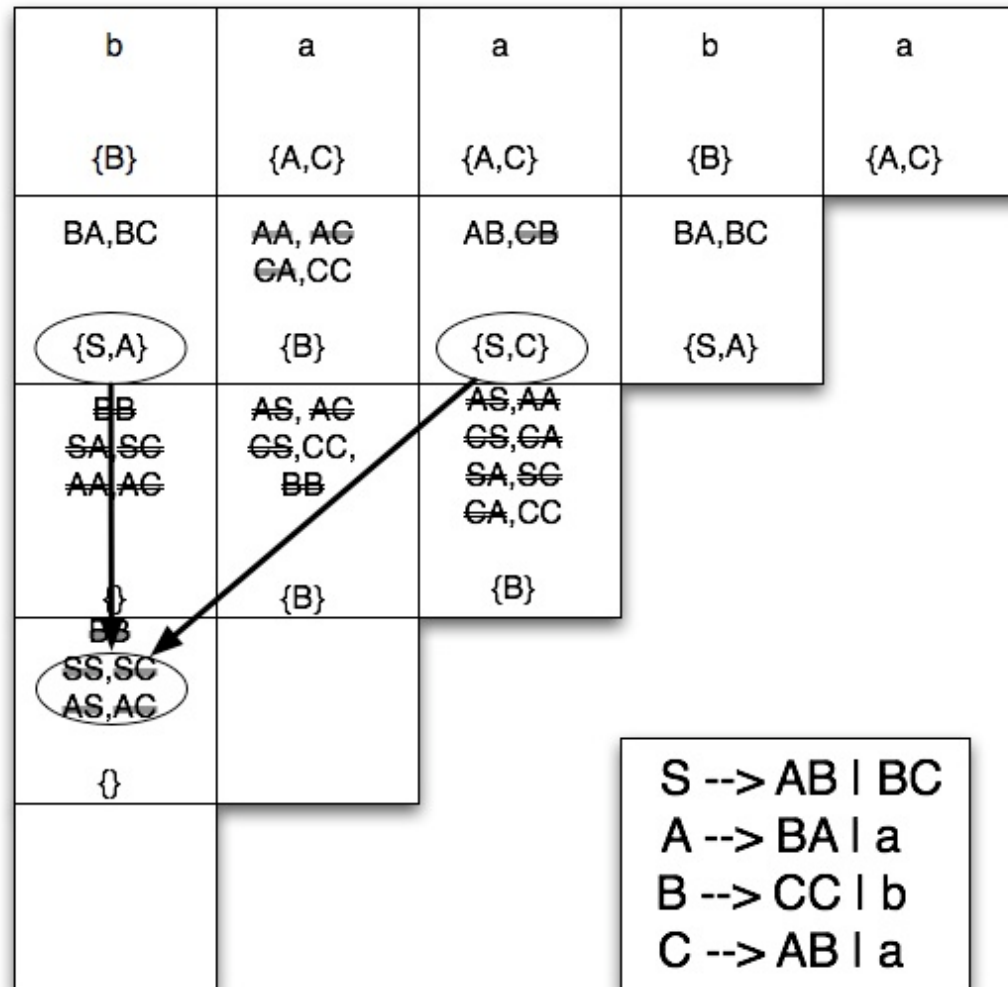
$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow AB \mid a$

CYK Parser (Sequential Version)

b	a	a	b	a
{B}	{A,C}	{A,C}	{B}	{A,C}
BA, BC {S,A}	AA, AG CA, CC {B}	AB, CB {S,C}	BA, BC {S,A}	
EB SA, SC AA, AG	AS, AG CS, CC, BB	AS, AA CS, CA SA, SC CA, CC {B}		
BB	{B}			
{}				

$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow AB \mid a$

CYK Parser (Sequential Version)



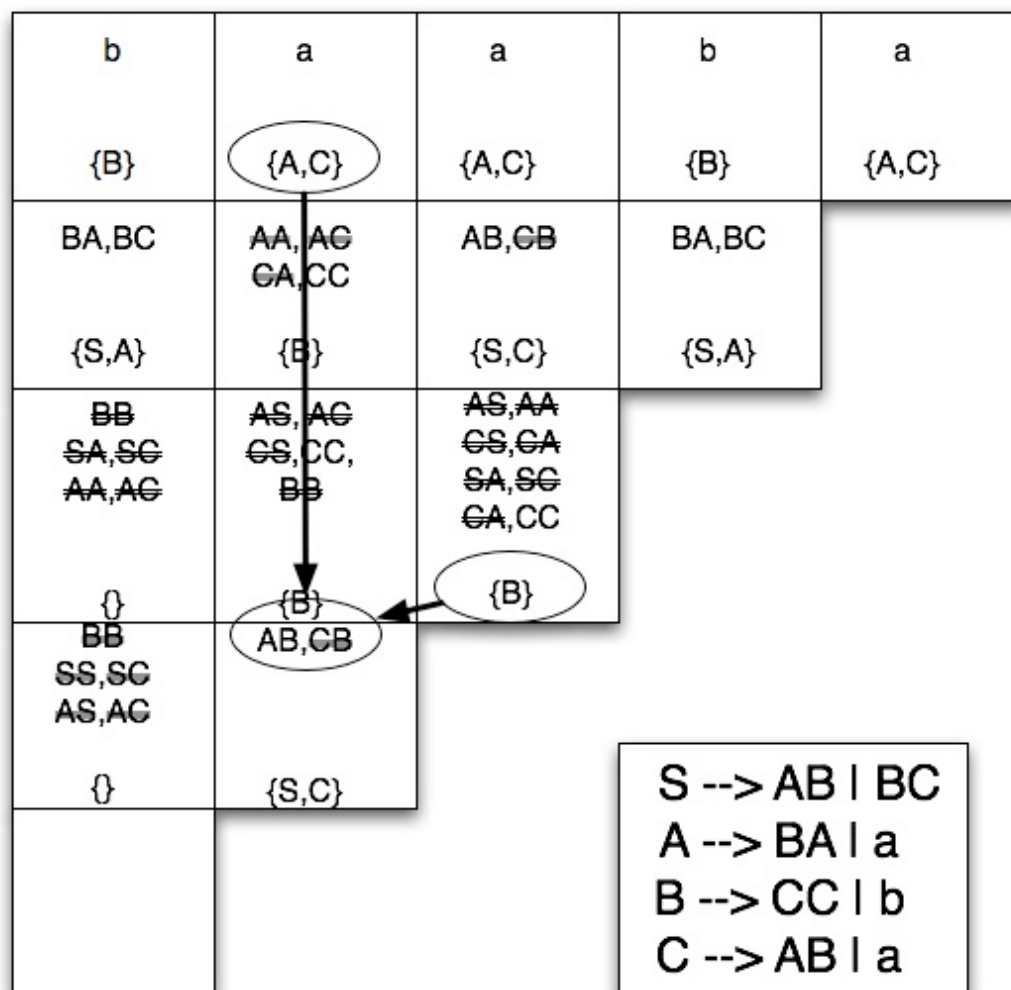
CYK Parser (Sequential Version)

b	a	a	b	a
{B}	{A,C}	{A,C}	{B}	{A,C}
BA,BC	AA, AG CA,CC	AB,CB	BA,BC	
{S,A}	{B}	{S,C}	{S,A}	
BB SA,SC AA,AG	AS, AG CS,CC, BB	AS,AA CS,CA SA,SC CA,CC		
{}	{B}	{B}		
BB SS,SC AS,AC				
{}				

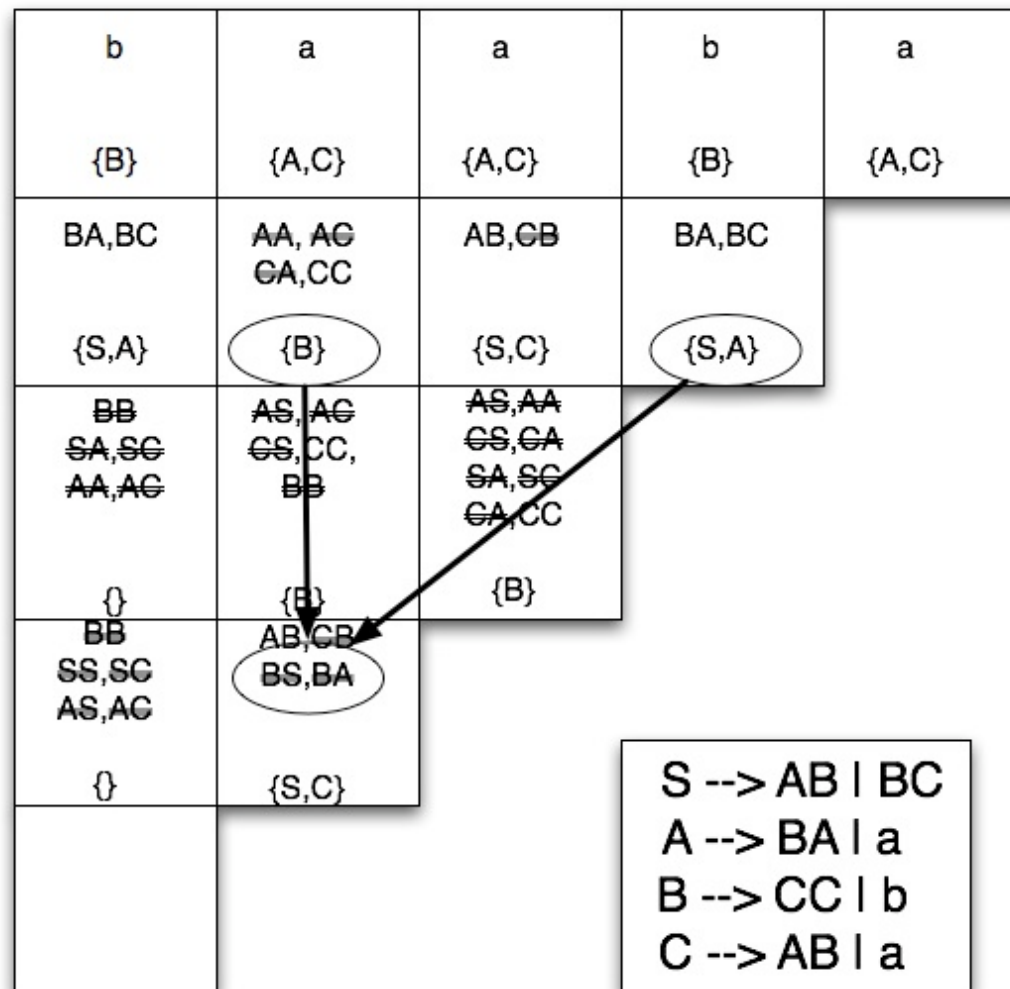
$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow AB \mid a$

Nothing Else Added

CYK Parser (Sequential Version)



CYK Parser (Sequential Version)

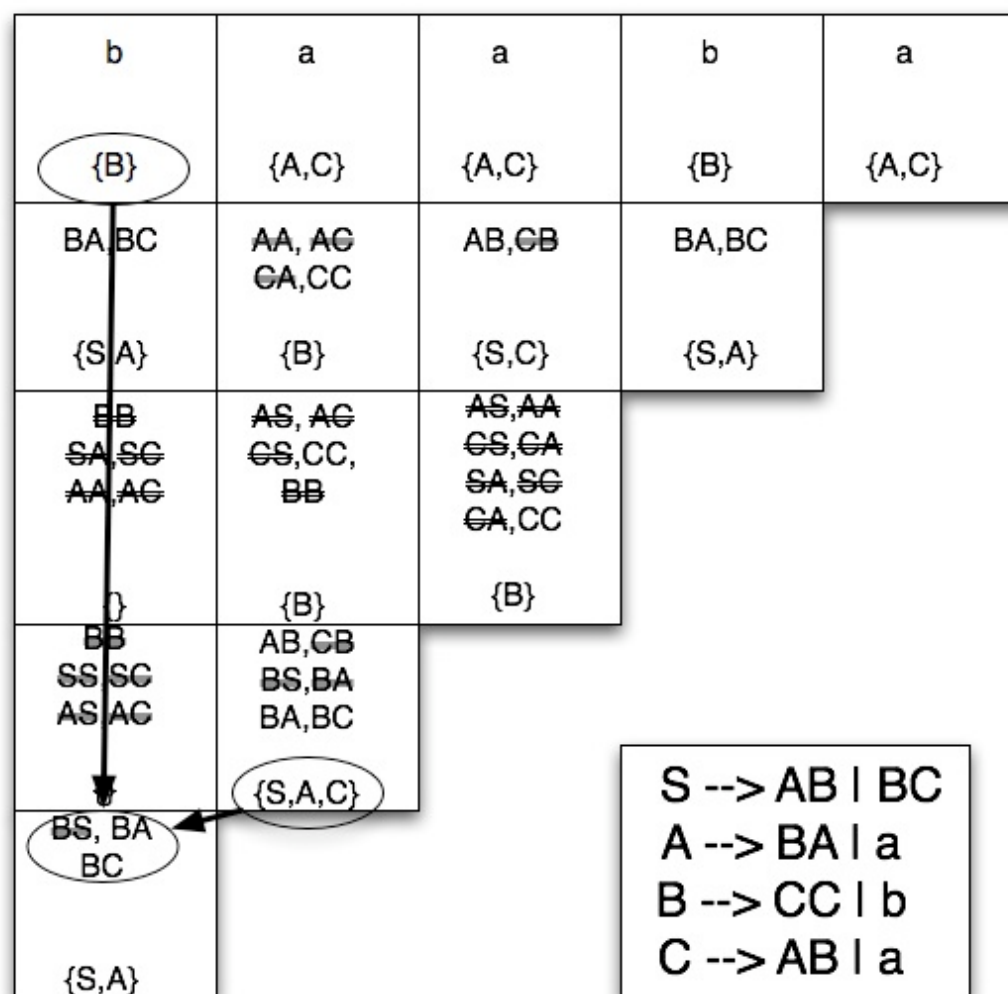


CYK Parser (Sequential Version)

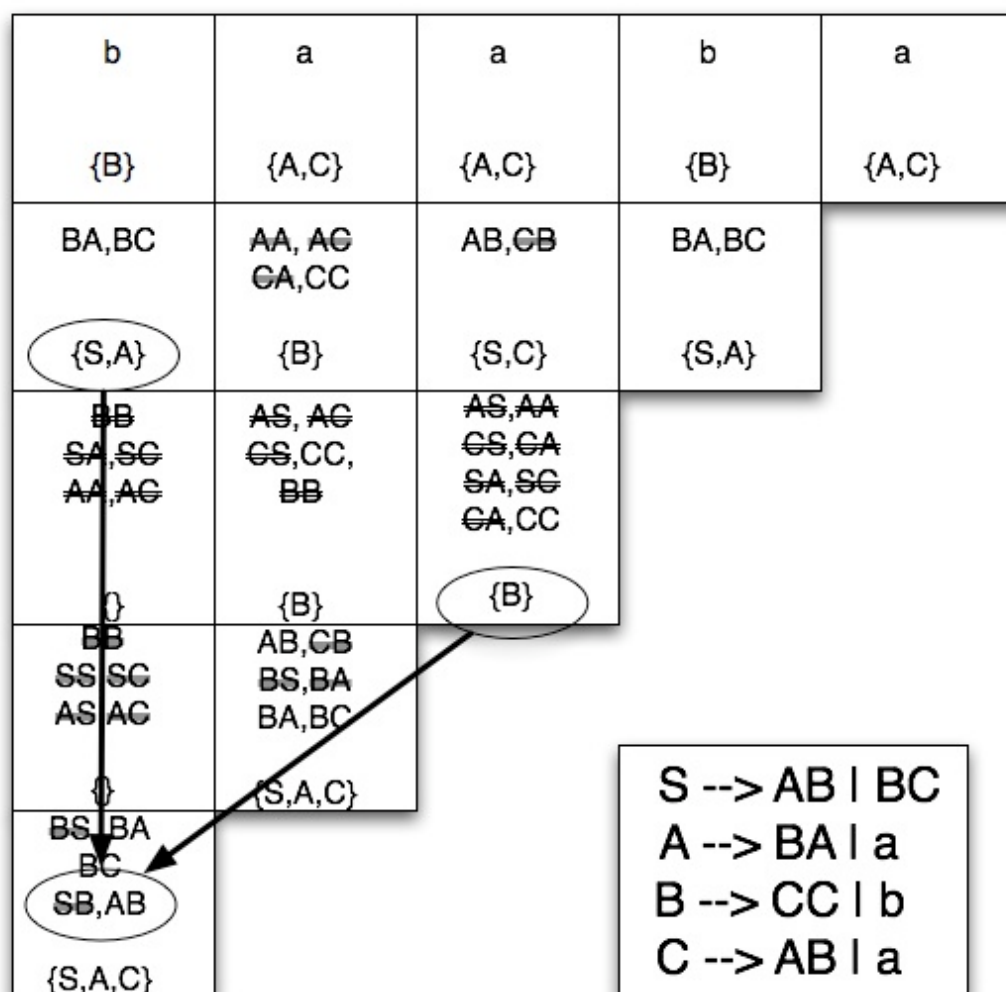
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{B}	{A,C}	{A,C}	{B}	{A,C}
BA,BC	AA, AG GA,CC	AB, CB	BA,BC	
{S,A}	{B}	{S,C}	{S,A}	
BB SA,SC AA,AG	AS, AG CS,CC, BB	AS,AA CS,CA SA,SC GA,CC		
{}	{B}	{B}		
BB SS,SC AS,AG	AB,CB BS,BA BA,BC			
{}	{S,A,C}			

$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
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 $C \rightarrow AB \mid a$

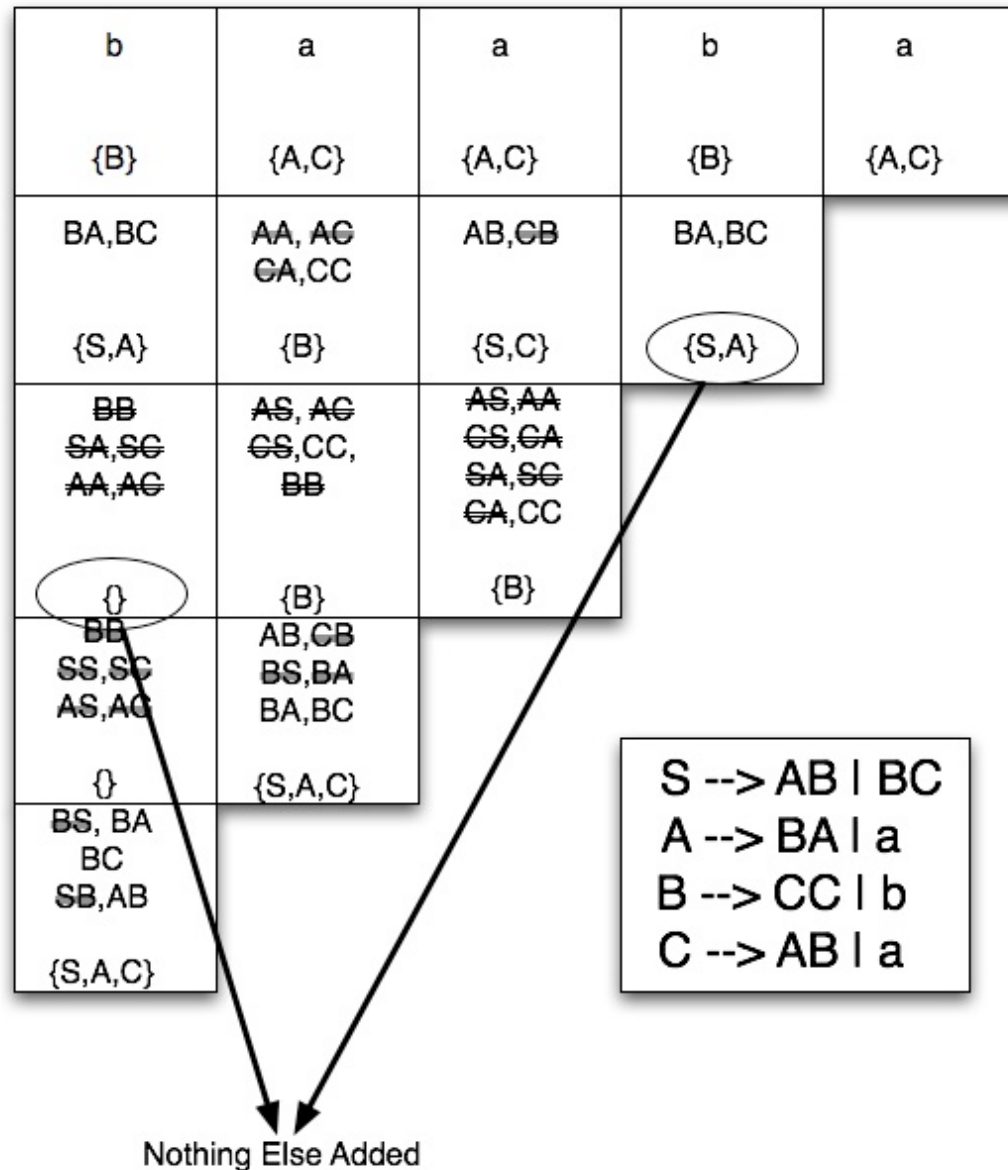
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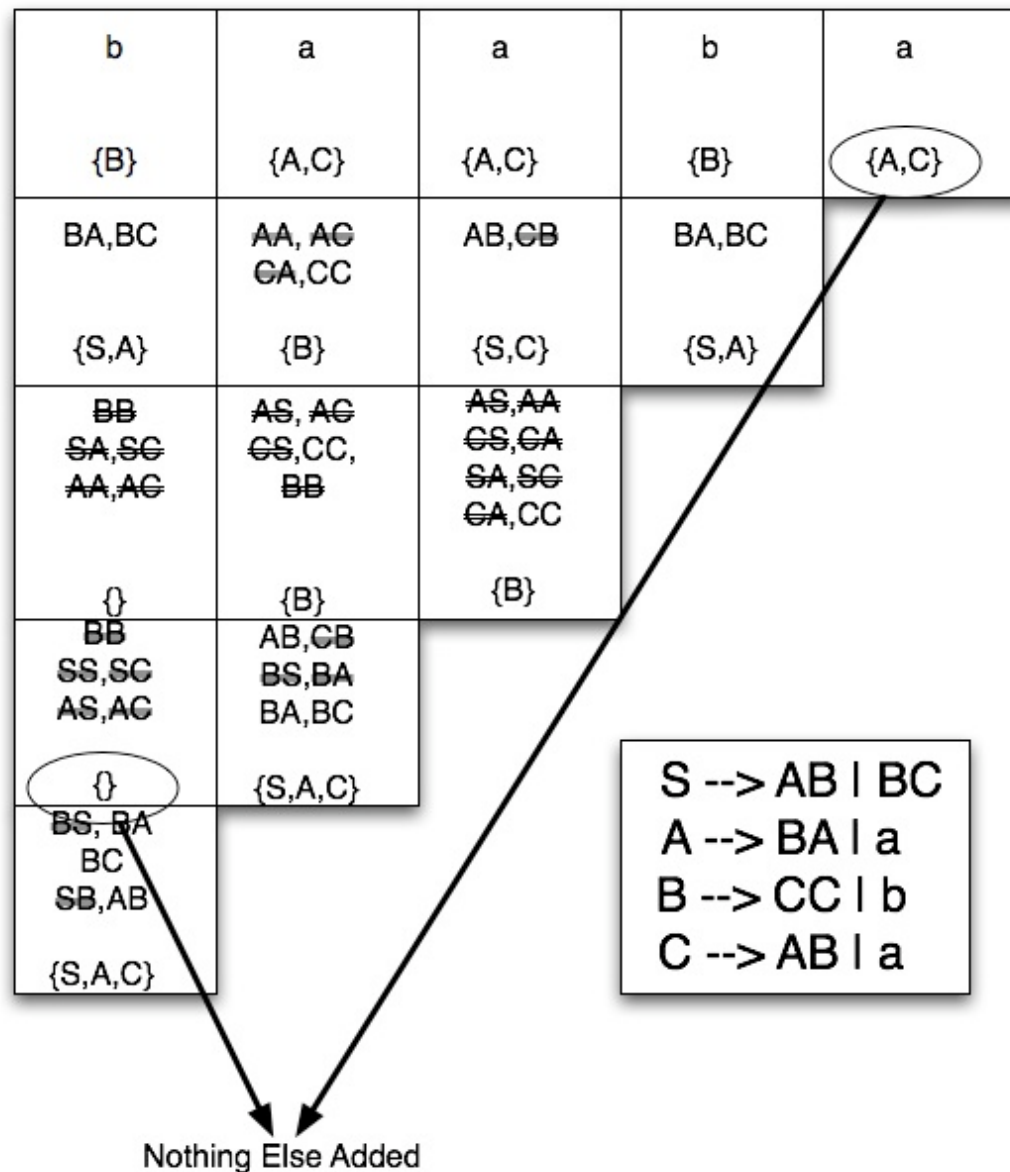
CYK Parser (Sequential Version)



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CYK Parser (Sequential Version)



CYK Parser (Sequential Version)

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BA,BC	AA, AC CA,CC	AB,CB	BA,BC	
{S,A}	{B}	{S,C}	{S,A}	
BB SA,SC AA,AC	AS, AC CS,CC, BB	AS,AA CS,CA SA,SC CA,CC		
{}	{B}	{B}		
BB SS,SC AS,AC	AB,CB BS,BA BA,BC			
{}	{S,A,C}			
BS, BA BC SB,AB				
{S,A,C}				

$S \rightarrow AB \mid BC$
 $A \rightarrow BA \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow AB \mid a$



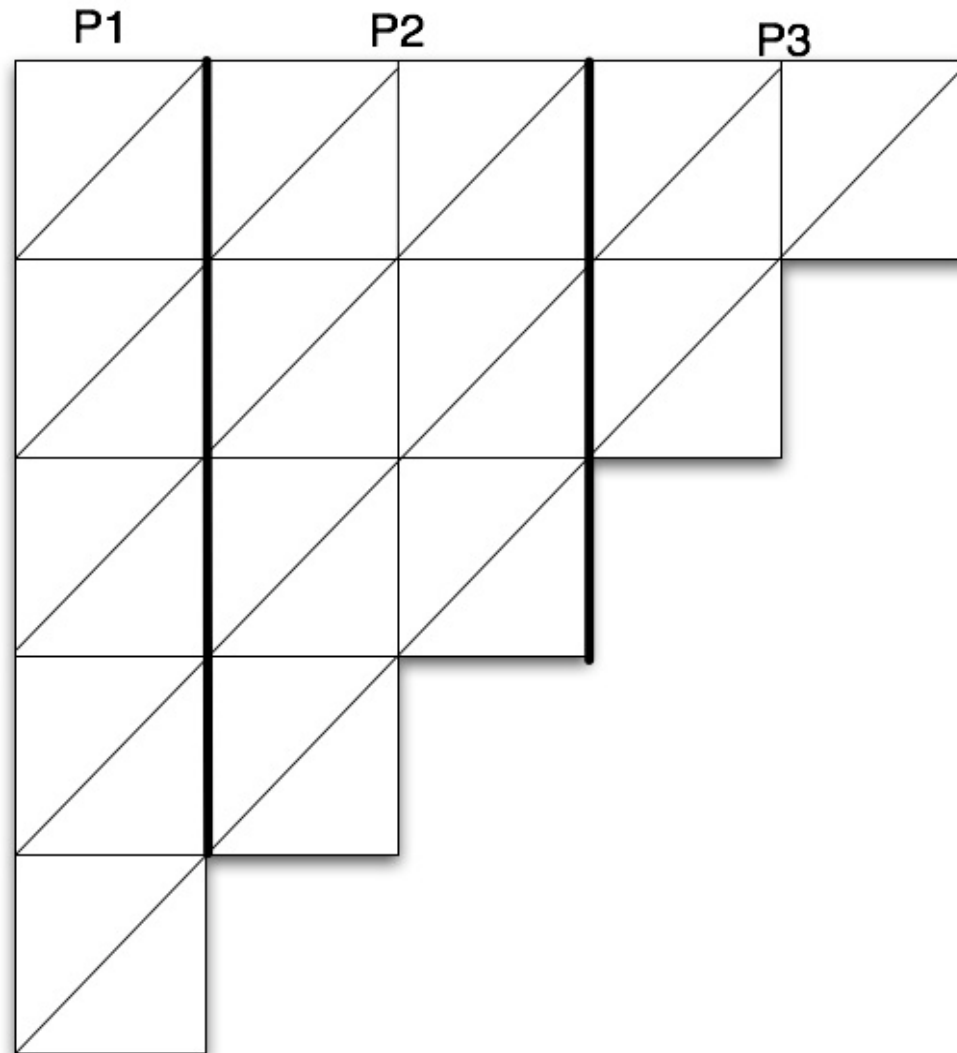
The CYK Parser Algorithm (Sequential Version)

```
(* for the first row *)
1) for i := 1 to n do
2)    $V_{i1} := \{ A \mid A \rightarrow a \text{ is a production rule and the } i\text{th symbol of } s \text{ is } a \}$ 

(* for subsequent rows *)
3) for j := 2 to n do
4)   for i := 1 to (n - j + 1) do
5)      $V_{ij} := \{ \}$ 
6)     for k := 1 to (j - 1) do
7)        $V_{ij} := V_{ij} \cup \{ A \mid A \rightarrow BC \text{ is a production rule, } B \text{ is in } V_{ik}, \text{ } C \text{ is in } V_{i+k, j-k} \}$ 
```

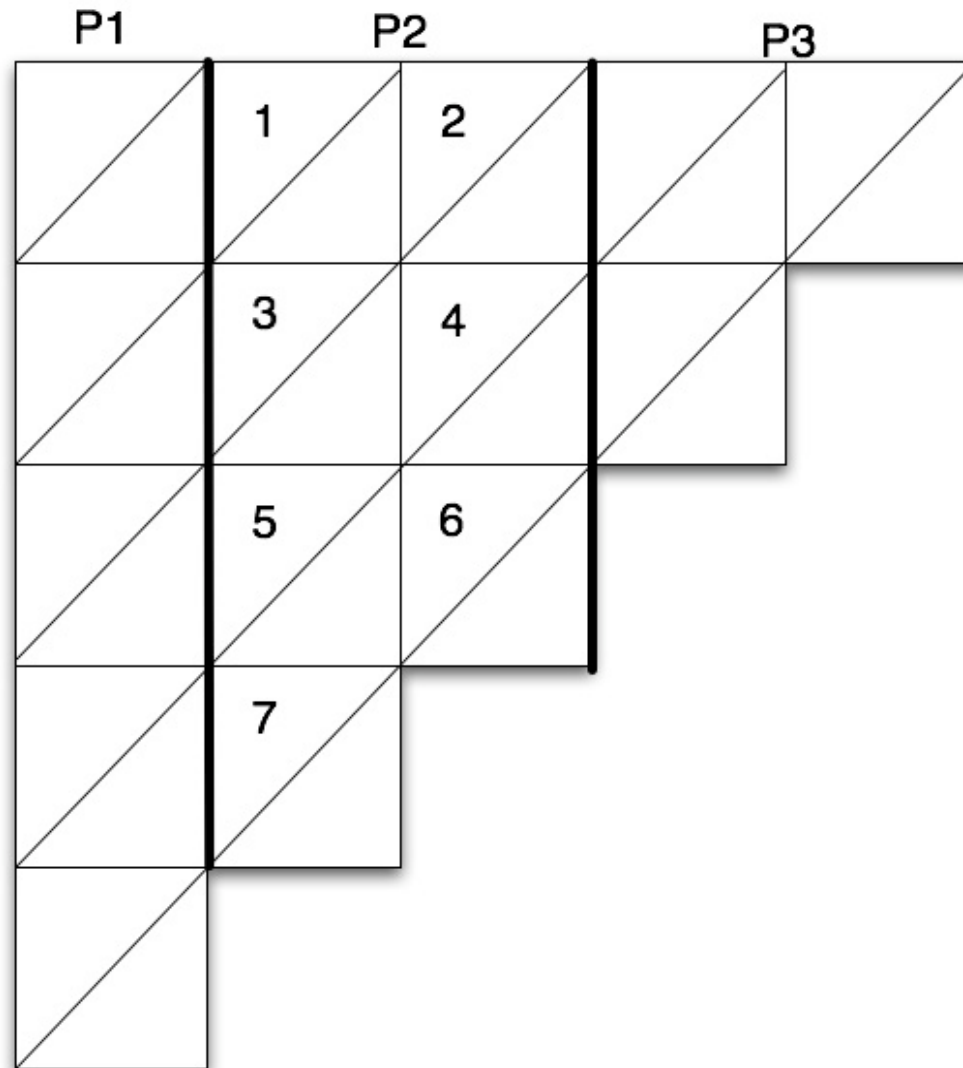
Figure3: Pseudo-code for the sequential CYK algorithm. Adapted from Hopcroft, Ullman, 1979, pp139-140.

CYK Parser (Parallel Version)



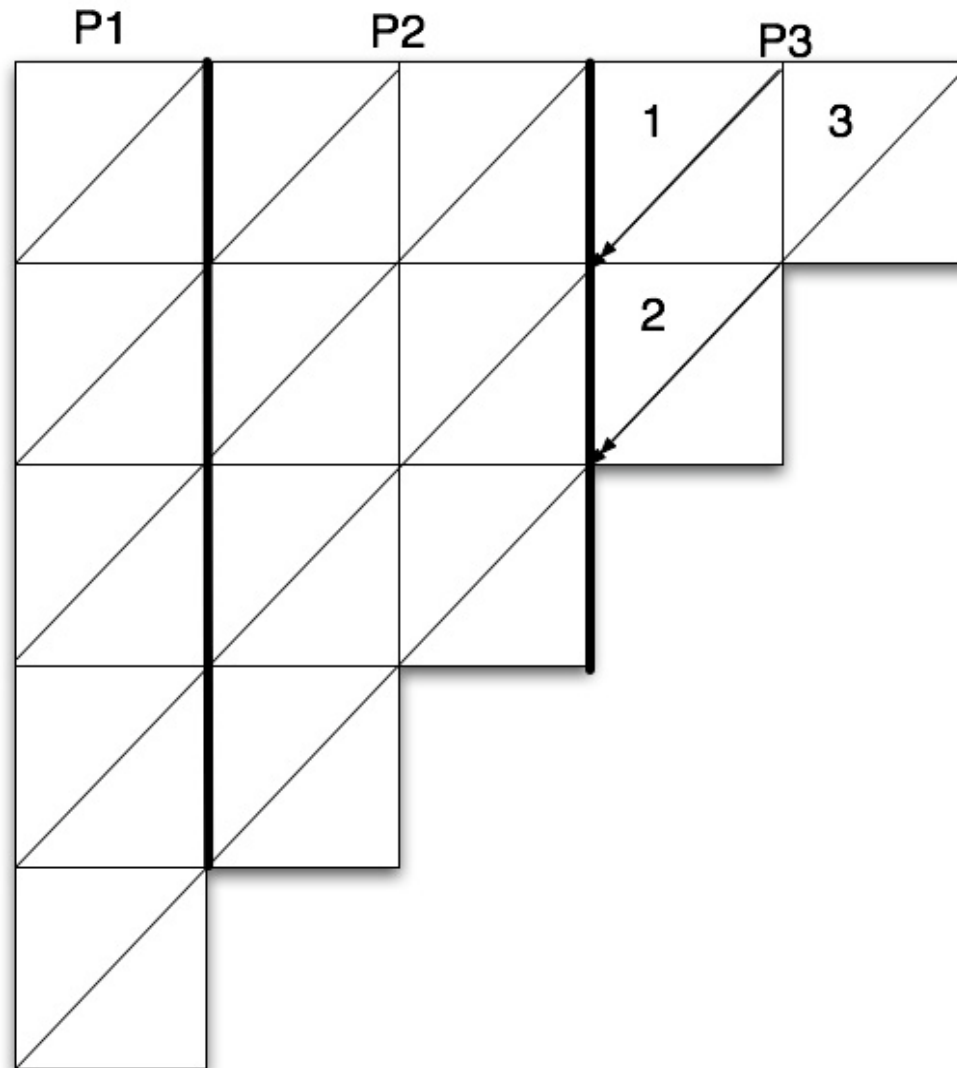
Matrix for a string of length 5 using 3 processors

CYK Parser (Parallel Version)



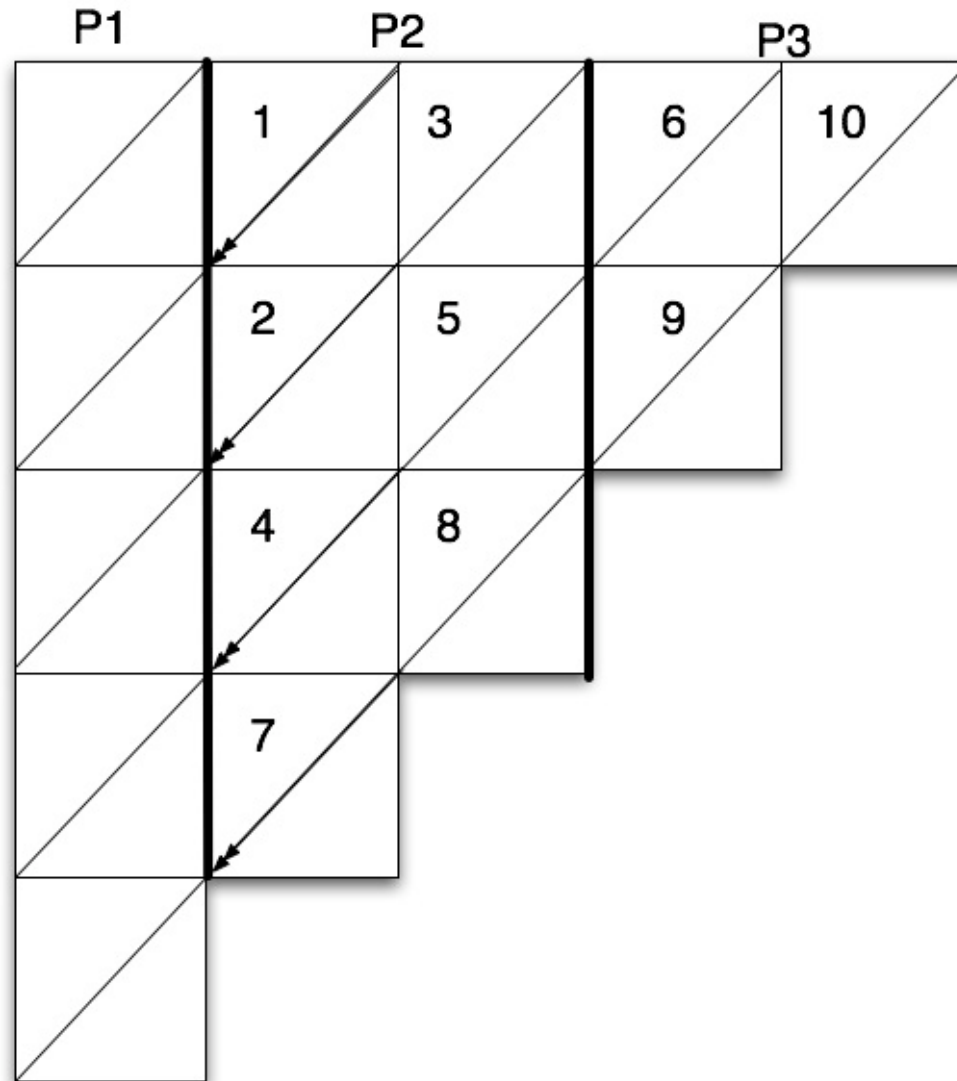
Order of calculation for processor P2. P2 calculates a diagonal at a time.

CYK Parser (Parallel Version)



Order of information received by P2. P2 receives a diagonal at a time.

CYK Parser (Parallel Version)



Order of information P2 sends to P1. P2 sends a diagonal at a time.



The CYK Parser Algorithm (Parallel Version)

if not last processor send all along to P_{i+1}

let $l = \sum_{q=1}^p$ length of substring for P_q

for $j := 1$ to l do

if necessary get diagonal from p_{i+1}

let $m =$ length of the diagonal within P_i

for $k := 1$ to m do

calculate $V_{j-k+1,k}$

if $i \neq 1$

then send back new diagonal to
 P_{i-1}

else send back $V_{1,n}$ to Host



Extra Slides Start Here



Example

(grammar & sets)

Simplified, right recursive expression grammar

$Goal \rightarrow Expr$
 $Expr \rightarrow Term - Expr$
 $Expr \rightarrow Term$
 $Term \rightarrow Factor * Term$
 $Term \rightarrow Factor$
 $Factor \rightarrow \underline{ident}$

Symbol	FIRST
<i>Goal</i>	{ <u>ident</u> }
<i>Expr</i>	{ <u>ident</u> }
<i>Term</i>	{ <u>ident</u> }
<i>Factor</i>	{ <u>ident</u> }
-	{ - }
*	{ * }
<u>ident</u>	{ <u>ident</u> }



Example

(building the collection)

Initialization Step

$$s_0 \leftarrow \text{closure}(\{ [Goal \rightarrow \cdot Expr, EOF] \})$$
$$\{ [Goal \rightarrow \cdot Expr, EOF], [Expr \rightarrow \cdot Term - Expr, EOF],$$
$$[Expr \rightarrow \cdot Term, EOF], [Term \rightarrow \cdot Factor * Term, EOF],$$
$$[Term \rightarrow \cdot Factor * Term, -], [Term \rightarrow \cdot Factor, EOF],$$
$$[Term \rightarrow \cdot Factor, -], [Factor \rightarrow \cdot \underline{ident}, EOF],$$
$$[Factor \rightarrow \cdot \underline{ident}, -], [Factor \rightarrow \cdot \underline{ident}, *] \}$$
$$S \leftarrow \{s_0\}$$



Example

(building the collection)

Iteration 1

$s_1 \leftarrow \text{goto}(s_0, \text{Expr})$

$s_2 \leftarrow \text{goto}(s_0, \text{Term})$

$s_3 \leftarrow \text{goto}(s_0, \text{Factor})$

$s_4 \leftarrow \text{goto}(s_0, \underline{\text{ident}})$

Iteration 2

$s_5 \leftarrow \text{goto}(s_2, -)$

$s_6 \leftarrow \text{goto}(s_3, *)$

Iteration 3

$s_7 \leftarrow \text{goto}(s_5, \text{Expr})$

$s_8 \leftarrow \text{goto}(s_6, \text{Term})$



Example

(Summary)

$S_0 : \{ [Goal \rightarrow \cdot Expr, EOF], [Expr \rightarrow \cdot Term - Expr, EOF],$
 $[Expr \rightarrow \cdot Term, EOF], [Term \rightarrow \cdot Factor * Term, EOF],$
 $[Term \rightarrow \cdot Factor * Term, -], [Term \rightarrow \cdot Factor, EOF],$
 $[Term \rightarrow \cdot Factor, -], [Factor \rightarrow \cdot \underline{ident}, EOF],$
 $[Factor \rightarrow \cdot \underline{ident}, -], [Factor \rightarrow \cdot \underline{ident}, *] \}$

$S_1 : \{ [Goal \rightarrow Expr \cdot, EOF] \}$

$S_2 : \{ [Expr \rightarrow Term \cdot - Expr, EOF], [Expr \rightarrow Term \cdot, EOF] \}$

$S_3 : \{ [Term \rightarrow Factor \cdot * Term, EOF], [Term \rightarrow Factor \cdot * Term, -],$
 $[Term \rightarrow Factor \cdot, EOF], [Term \rightarrow Factor \cdot, -] \}$

$S_4 : \{ [Factor \rightarrow \underline{ident} \cdot, EOF], [Factor \rightarrow \underline{ident} \cdot, -], [Factor \rightarrow \underline{ident} \cdot, *] \}$

$S_5 : \{ [Expr \rightarrow Term - \cdot Expr, EOF], [Expr \rightarrow \cdot Term - Expr, EOF],$
 $[Expr \rightarrow \cdot Term, EOF], [Term \rightarrow \cdot Factor * Term, -],$
 $[Term \rightarrow \cdot Factor, -], [Term \rightarrow \cdot Factor * Term, EOF],$
 $[Term \rightarrow \cdot Factor, EOF], [Factor \rightarrow \cdot \underline{ident}, *],$
 $[Factor \rightarrow \cdot \underline{ident}, -], [Factor \rightarrow \cdot \underline{ident}, EOF] \}$



Example

(Summary)

$S_6 : \{ [Term \rightarrow Factor * \cdot Term, EOF], [Term \rightarrow Factor * \cdot Term, -],$
 $[Term \rightarrow \cdot Factor * Term, EOF], [Term \rightarrow \cdot Factor * Term, -],$
 $[Term \rightarrow \cdot Factor, EOF], [Term \rightarrow \cdot Factor, -],$
 $[Factor \rightarrow \cdot \underline{ident}, EOF], [Factor \rightarrow \cdot \underline{ident}, -], [Factor \rightarrow \cdot \underline{ident}, *] \}$

$S_7 : \{ [Expr \rightarrow Term - Expr \cdot, EOF] \}$

$S_8 : \{ [Term \rightarrow Factor * Term \cdot, EOF], [Term \rightarrow Factor * Term \cdot, -] \}$



Example

(Summary)

The Goto Relationship (*from the construction*)

State	Expr	Term	Factor	-	*	<u>Ident</u>
0	1	2	3			4
1						
2				5		
3					6	
4						
5	7	2	3			4
6		8	3			4
7						
8						



Filling in the ACTION and GOTO Tables

The algorithm

x is the number of the state for s_x

```

 $\forall$  set  $s_x \in S$ 
   $\forall$  item  $i \in s_x$ 
    if  $i$  is  $[A \rightarrow \beta \cdot \underline{a}, \underline{b}]$  and  $\text{goto}(s_x, \underline{a}) = s_k, \underline{a} \in T$ 
      then  $\text{ACTION}[x, \underline{a}] \leftarrow \text{"shift } k\text{"}$ 
    else if  $i$  is  $[S' \rightarrow S \cdot, \text{EOF}]$ 
      then  $\text{ACTION}[x, \text{EOF}] \leftarrow \text{"accept"}$ 
    else if  $i$  is  $[A \rightarrow \beta \cdot, \underline{a}]$ 
      then  $\text{ACTION}[x, \underline{a}] \leftarrow \text{"reduce } A \rightarrow \beta\text{"}$ 
   $\forall n \in NT$ 
    if  $\text{goto}(s_x, n) = s_k$ 
      then  $\text{GOTO}[x, n] \leftarrow k$ 

```

Many items
generate no
table entry

e.g., $[A \rightarrow \beta \cdot B \alpha, \underline{a}]$
does not, but
closure ensures
that all the rhs'
for B are in s_x



Example

(Filling in the tables)

The algorithm produces the following table

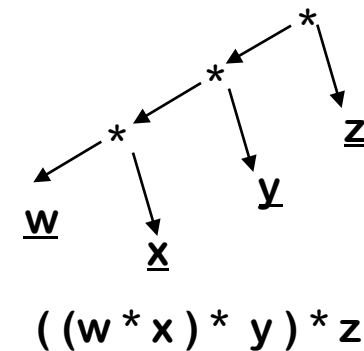
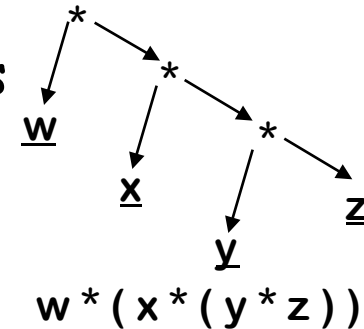
	ACTION				GOTO		
	<u>Ident</u>	-	*	EOF	<i>Expr</i>	<i>Term</i>	<i>Factor</i>
0	s 4				1	2	3
1				acc			
2		s 5		r 3			
3		r 5	s 6	r 5			
4		r 6	r 6	r 6			
5	s 4				7	2	3
6	s 4					8	3
7				r 2			
8		r 4		r 4			

Plugs into the skeleton LR(1) parser



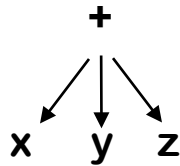
Left Recursion versus Right Recursion

- Right recursion
- Required for termination in top-down parsers
- Uses (on average) more stack space
- Produces right-associative operators
- Left recursion
- Works fine in bottom-up parsers
- Limits required stack space
- Produces left-associative operators
- Rule of thumb
- Left recursion for bottom-up parsers
- Right recursion for top-down parsers

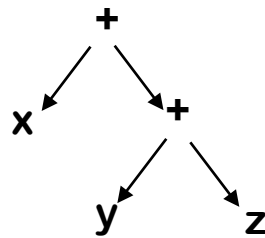


Associativity

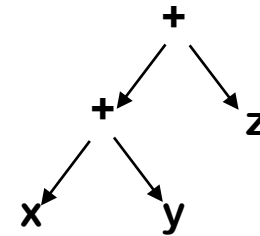
- What difference does it make?
- Can change answers in floating-point arithmetic
- Exposes a different set of common subexpressions
- Consider $x+y+z$



*Ideal
operator*



*Left
association*

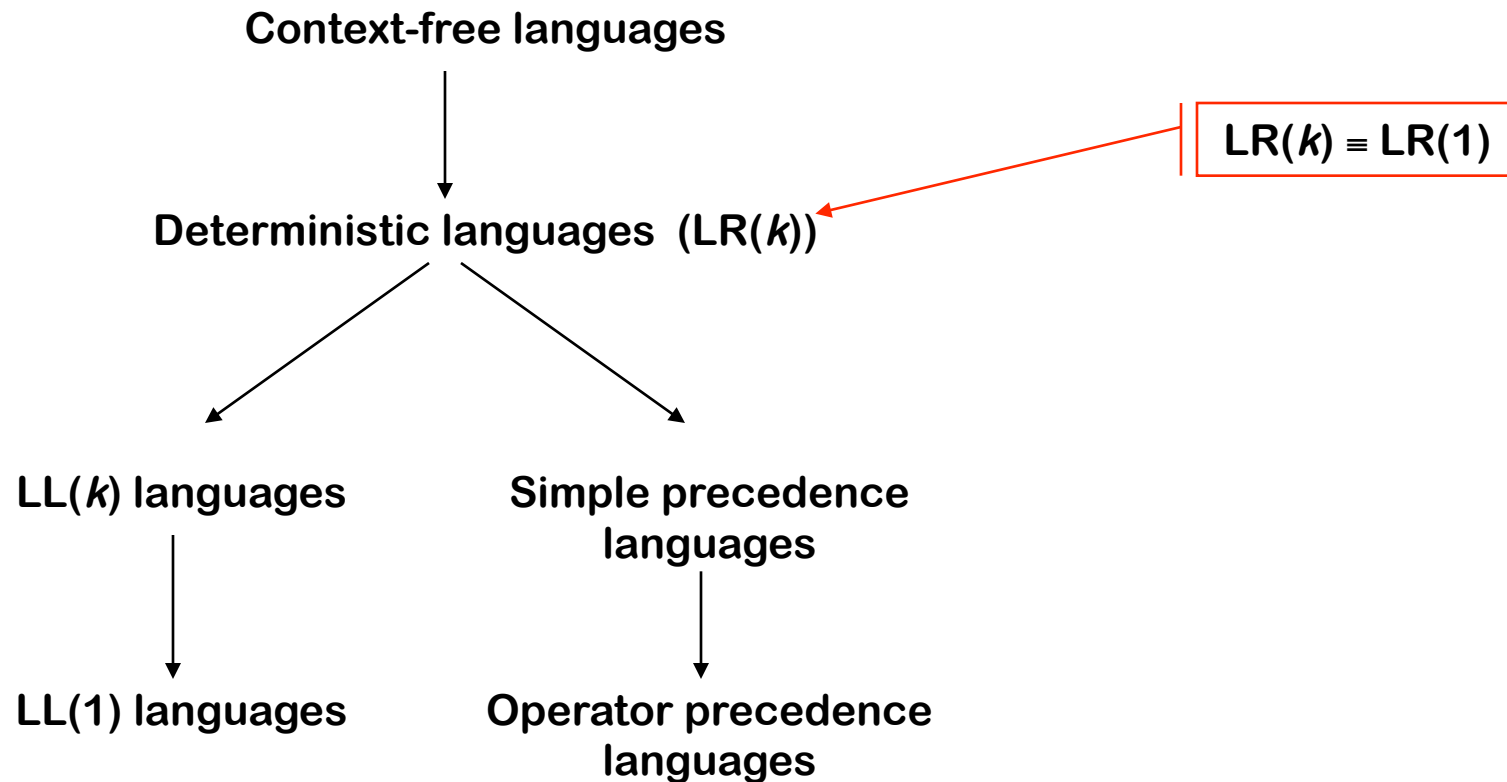


*Right
association*

- What if $y+z$ occurs elsewhere? Or $x+y$? or $x+z$?
- What if $x = 2$ & $z = 17$? Neither left nor right exposes 19.
- Best choice is function of surrounding context



Hierarchy of Context-Free Languages



*The inclusion hierarchy for
context-free languages*



Beyond Syntax

There is a level of correctness that is deeper than grammar

```
fie(a,b,c,d)
  int a, b, c, d;
  { ... }
fee() {
  int f[3],g[0],
    h, i, j, k;
  char *p;
  fie(h,i,"ab",j, k);
  k = f * i + j;
  h = g[17];
  printf("<%s,%s>.\n",
    p,q);
  p = 10;
}
```

What is wrong with this program?
(let me count the ways ...)

To generate code, we need to understand its meaning !



There is a level of correctness that is deeper than grammar

```
fie(a,b,c,d)
  int a, b, c, d;
{ ... }
fee() {
  int f[3],g[0],
    h, i, j, k;
  char *p;
  fie(h,i,"ab",j, k);
  k = f * i + j;
  h = g[17];
  printf("<%s,%s>.\n",
    p,q);
  p = 10;
}
```

What is wrong with this program?
(let me count the ways ...)

- declared g[0], used g[17]
- wrong number of args to fie()
- "ab" is not an int
- wrong dimension on use of f
- undeclared variable q
- 10 is not a character string

All of these are "deeper than syntax"



Beyond Syntax

To generate code, the compiler needs to answer many questions

- Is "x" a scalar, an array, or a function? Is "x" declared?
- Are there names that are not declared? Declared but not used?
- Which declaration of "x" does each use reference?
- Is the expression "x * y + z" type-consistent?
- In "a[i,j,k]", does a have three dimensions?
- Where can "z" be stored? (*register, local, global, heap, static*)
- In "f ← 15", how should 15 be represented?
- How many arguments does "fie()" take? What about "printf ()" ?
- Does "*p" reference the result of a "malloc()" ?
- Do "p" & "q" refer to the same memory location?
- Is "x" defined before it is used?

These cannot be expressed in a CFG



Beyond Syntax

These questions are part of context-sensitive analysis

- Answers depend on values, not parts of speech
- Questions & answers involve non-local information
- Answers may involve computation

How can we answer these questions?

- Use formal methods
 - Context-sensitive grammars?
 - Attribute grammars? *(attributed grammars?)*
- Use *ad-hoc* techniques
 - Symbol tables
 - *Ad-hoc* code *(action routines)*

In scanning & parsing, formalism won; different story here.



Beyond Syntax

Telling the story

- The attribute grammar formalism is important
 - Succinctly makes many points clear
 - Sets the stage for actual, *ad-hoc* practice
- The problems with attribute grammars motivate practice
 - Non-local computation
 - Need for centralized information
- Some folks in the community still argue for attribute grammars
 - Knowledge is power
 - Information is immunization

We will cover attribute grammars, then move on to *ad-hoc* ideas