Parsing III
Bottom-up Parsing

Parsing Techniques

Top-down parsers *(LL(1), recursive descent)*

- Start at the root of the parse tree and grow toward leaves
- Pick a production & try to match the input
- Bad "pick" ⇒ may need to backtrack
- Some grammars are backtrack-free *(predictive parsing)*

Bottom-up parsers *(LR(1), operator precedence)*

- Start at the leaves and grow toward root
- As input is consumed, encode possibilities in an internal state
- Start in a state valid for legal first tokens
- Bottom-up parsers handle a large class of grammars

Bottom-up Parsing (definitions)

The point of parsing is to construct a derivation

A derivation consists of a series of rewrite steps

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \dots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \textit{sentence}$$

- Each γ_i is a sentential form
  - If γ contains only terminal symbols, γ is a sentence in $L(G)$
  - If γ contains ≥ 1 non-terminals, γ is a sentential form
- To get γ_i from γ_{i-1}, expand some NT $A \in \gamma_{i-1}$ by using $A \rightarrow \beta$
  - Replace the occurrence of $A \in \gamma_{i-1}$ with β to get γ_i
  - In a leftmost derivation, it would be the first NT $A \in \gamma_{i-1}$

A left-sentential form occurs in a leftmost derivation.

A right-sentential form occurs in a rightmost derivation.

Bottom-up Parsing

A bottom-up parser builds a derivation by working from the input sentence back toward the start symbol S

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \textit{sentence}$$

bottom-up

To reduce γ_i to γ_{i-1} *(assuming the production $A \rightarrow \beta$)* match some *rhs* β against γ_i then replace β with its corresponding *lhs*, A.

In terms of the parse tree, this is working from leaves to root

- Nodes with no parent in a partial tree form its *frontier*
- Since each replacement of β with A shrinks the current frontier, we call it a *reduction*.

Finding Reductions

Consider the simple grammar

1	*Goal*	→	a *A* *B* e
2	*A*	→	*A* b c
3		|	b
4	*B*	→	d

And the input string abbcde

Sentential Form	*Next Reduction Prod'n*	*Next Reduction Pos'n*
abbcde	3	2
a *A* bcde	2	4
a *A* de	4	3
a *A* *B* e	1	4
Goal	—	—

The trick is scanning the input and finding the next reduction

The mechanism for doing this should be efficient

Finding Reductions (Handles)

The parser must find a substring β of the tree's frontier that

matches some production $A \rightarrow \beta$ that occurs as one step in the rightmost derivation

We call this substring β a *handle*

Formally,

A *handle* is a pair $\langle A \rightarrow \beta, k \rangle$ where $A \rightarrow \beta \in P$ and k is position in tree's current frontier of β's rightmost (last) symbol.

Replacing β at k with A in the bottom-up parse represents the next step in the reverse rightmost derivation.

Finding Reductions (Handles)

Critical Insight

> *If G is unambiguous, then every right-sentential form has a unique handle.*

If we can find those handles, we can build a derivation !

Sketch of Proof:

1 G is unambiguous $\Rightarrow$ rightmost derivation is unique

2 $\Rightarrow$ a unique production $A \rightarrow \beta$ applied to derive γ_i from γ_{i-1}

3 $\Rightarrow$ a unique position k at which $A \rightarrow \beta$ is applied

4 $\Rightarrow$ a unique handle $\langle A \rightarrow \beta, k \rangle$

This all follows from the definitions

Handle-pruning, Bottom-up Parsers

The process of discovering a handle & reducing it to the appropriate left-hand side is called *handle pruning*

Handle pruning forms the basis for a bottom-up parsing method

To construct a rightmost derivation

$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow$ sentence

Apply the following simple algorithm

for i ← n to 1 by –1

Find the handle $\langle A_i \rightarrow \beta_i , \boldsymbol{k}_i \rangle$ in γ_i

Replace β_i *with* A_i *to generate* γ_{i-1}

Handle-pruning, Bottom-up Parsers

One implementation technique is the *shift-reduce parser*

```
push INVALID
word ← NextWord( )
repeat until (top of stack = Goal and word = EOF)
    if a handle for A→β on top of the stack then
            // reduce β to A
            pop |β| symbols off the stack
            push A onto the stack
        else if (word ≠ EOF) then // shift
            push word
            word ← NextWord( )
        else    // either no handle or no input
            report an error
```

How do errors show up?

- **failure to find a handle**
- **hitting EOF & needing to shift (final else clause)**

Either generates an error

Figure 3.11 in EAC

Example

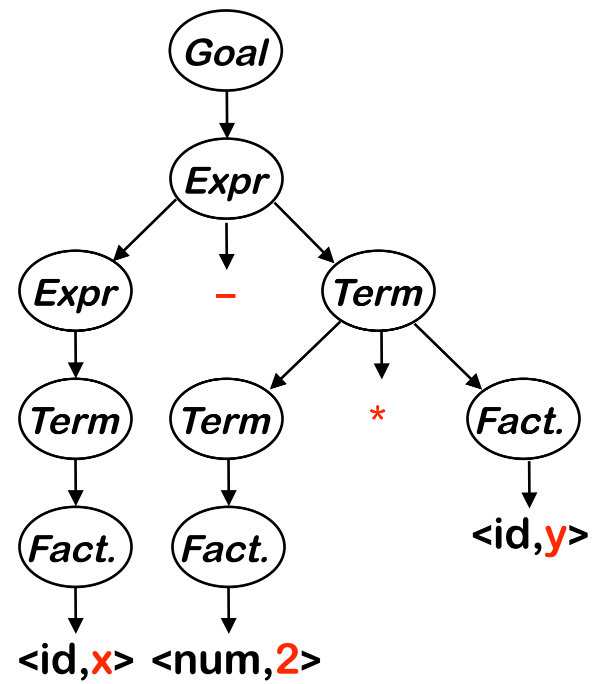
1	*Goal*	→	*Expr*
2	*Expr*	→	*Expr* + *Term*
3		|	*Expr* - *Term*
4		|	*Term*
5	*Term*	→	*Term* * *Factor*
6		|	*Term* / *Factor*
7		|	*Factor*
8	*Factor*	→	<u>number</u>
9		|	<u>id</u>
10		|	<u>(</u> *Expr* <u>)</u>

The expression grammar

<id,x> - <num,2> * <id,y>

Example <id,x> - <num,2> * <id,y>

Stack	Input	Action
$	id – num * id	shift
$ id	– num * id	red. 9
$ *Factor*	– num * id	red. 7
$ *Term*	– num * id	red. 4
$ *Expr*	– num * id	shift
$ *Expr* –	num * id	shift
$ *Expr* – num	* id	red. 8
$ *Expr* – *Factor*	* id	red. 7
$ *Expr* – *Term*	* id	shift
$ *Expr* – *Term* *	id	shift
$ *Expr* – *Term* * id		red. 9
$ *Expr* – *Term* * *Factor*		red. 5
$ *Expr* – *Term*		red. 3
$ *Expr*		red. 1
$ *Goal*		accept



Shift-reduce Parsing

Shift reduce parsers are easily built and easily understood

A shift-reduce parser has just four actions

- *Shift* — next word is shifted onto the stack
- *Reduce* — right end of handle is at top of stack
  - Locate left end of handle within the stack
  - Pop handle off stack & push appropriate *lhs*
- *Accept* — stop parsing & report success
- *Error* — call an error reporting/recovery routine

Accept & Error are simple

Shift is just a push and a call to the scanner

Reduce takes |*rhs*| pops & 1 push

Handle finding is key
- handle is on stack
- finite set of handles

$\Rightarrow$ use a DFA !

An Important Lesson about Handles

To be a handle, a substring of a sentential form γ must have two properties:

- It must match the right hand side β of some rule $A \rightarrow \beta$
- There must be some rightmost derivation from the goal symbol that produces the sentential form γ with $A \rightarrow \beta$ as the last production applied

- Simply looking for right hand sides that match strings is not good enough
- Critical Question: How can we know when we have found a handle without generating lots of different derivations?
  - Answer: we use look ahead in the grammar along with tables produced as the result of analyzing the grammar.
  - *LR(1)* parsers build a DFA that runs over the stack & finds them