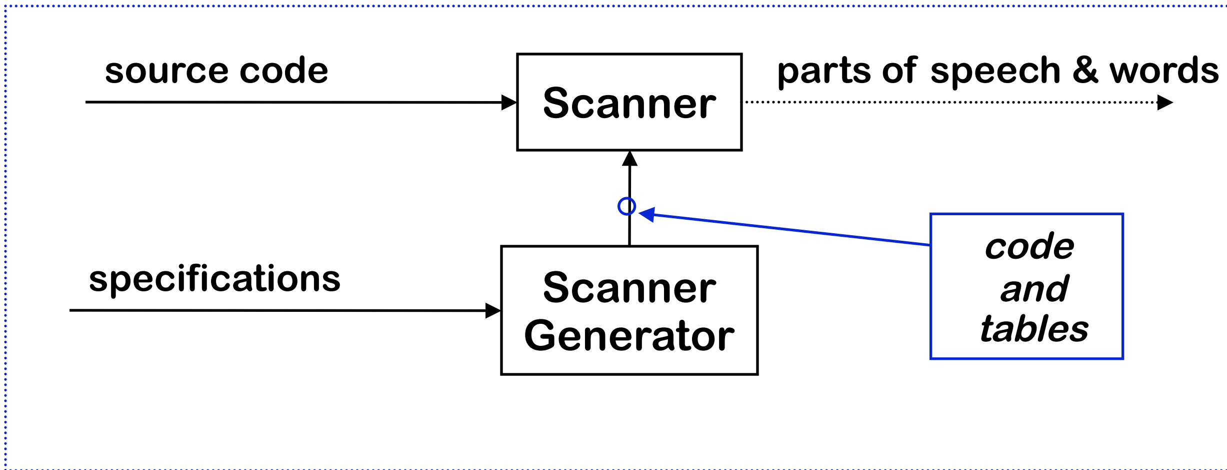
Lexical Analysis — Part II: Constructing a Scanner from Regular Expressions

Quick Review



Previous class:

- The scanner is the first stage in the front end
- Specifications can be expressed using regular expressions
- Build tables and code from a DFA

Goal

- We will show how to construct a finite state automaton to recognize any RE
- This Lecture
  - → Convert RE to an **nondeterministic finite automaton (NFA)**
    - Requires ε-**transitions** to combine regular subexpressions
  - → Convert an NFA to a **deterministic finite automaton (DFA)**
    - Use Subset construction

Next Lecture

- → Minimize the number of states
  - Hopcroft state minimization algorithm
- → Generate the scanner code
  - Additional code can be inserted

More Regular Expressions

- All strings of 1s and 0s ending in a 1

  (0 | 1)* 1

- All strings over lowercase letters where the vowels (a,e,i,o,u) occur exactly once, in ascending order

  Cons → (b|c|d|f|g|h|j|k|l|m|n|p|q|r|s|t|v|w|x|y|z)
  *Cons** a *Cons** e *Cons** i *Cons** o *Cons** u *Cons**

- All strings of 1s and 0s that do not contain three 0s in a row:

More Regular Expressions

- All strings of 1s and 0s ending in a 1

  $(\underline{0} \mid \underline{1})^* \underline{1}$

- All strings over lowercase letters where the vowels (a,e,i,o,u) occur exactly once, in ascending order

  $\mathit{Cons} \rightarrow (\underline{b}|\underline{c}|\underline{d}|\underline{f}|\underline{g}|\underline{h}|\underline{j}|\underline{k}|\underline{l}|\underline{m}|\underline{n}|\underline{p}|\underline{q}|\underline{r}|\underline{s}|\underline{t}|\underline{v}|\underline{w}|\underline{x}|\underline{y}|\underline{z})$

  $\mathit{Cons}^* \ \underline{a} \ \mathit{Cons}^* \ \underline{e} \ \mathit{Cons}^* \ \underline{i} \ \mathit{Cons}^* \ \underline{o} \ \mathit{Cons}^* \ \underline{u} \ \mathit{Cons}^*$

- All strings of 1s and 0s that do not contain three 0s in a row:

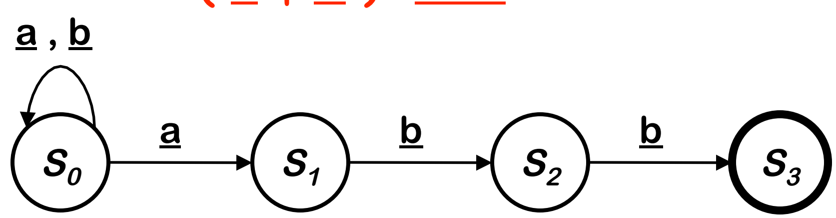
  $(\underline{1}^* (\varepsilon \mid \underline{01} \mid \underline{001}) \underline{1}^*)^* (\varepsilon \mid \underline{0} \mid \underline{00})$

Non-deterministic Finite Automata

Each RE corresponds to a *deterministic finite automaton* (DFA)

- May be hard to directly construct the right DFA

What about an RE such as (a | b)* abb?



This is a little different

- S_1 has two transitions on a

This is a *non-deterministic finite automaton* (NFA)

Non-deterministic Finite Automata

Each RE corresponds to a *deterministic finite automaton* (DFA)

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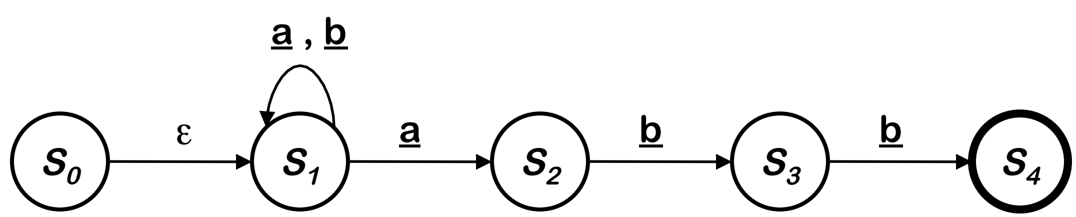
What about an RE such as (<u>a</u> | <u>b</u>)* <u>abb</u>?



This is a little different

- *S_1 has two transitions on <u>a</u>*
- *S_0* has a transition on ε

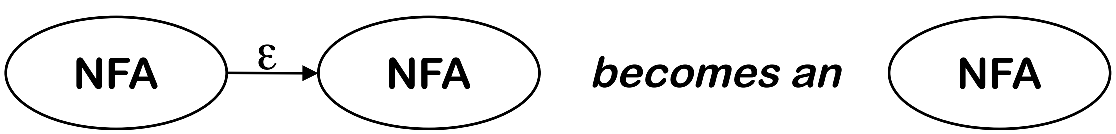
This is a *non-deterministic finite automaton* (NFA)

Nondeterministic Finite Automata

- An NFA accepts a string x iff $\exists$ a path though the transition graph from s_0 to a final state such that the edge labels spell x
- Transitions on ε consume no input
- To "run" the NFA, start in s_0 and *guess* the right transition at each choice point with multiple possibilities
  - → Always guess correctly
  - → If some sequence of correct guesses accepts x then accept

Why study NFAs?

- They are the key to automating the RE→DFA construction
- We can paste together NFAs with ε-transitions



Relationship between NFAs and DFAs

DFA is a special case of an NFA

- DFA has no ε transitions
- DFA's transition function is single-valued
- Same rules will work

DFA can be simulated with an NFA

→ *Obviously*

NFA can be simulated with a DFA *(less obvious)*

- Simulate sets of possible states
- Possible exponential blowup in the state space
- Still, one state per character in the input stream

Automating Scanner Construction

To convert a specification into code:

1. Write down the RE for the input language
2. Build a big NFA
3. Build the DFA that simulates the NFA
4. Systematically shrink the DFA
5. Turn it into code

Scanner generators

- Lex, Flex, and JLex work along these lines
- Algorithms are well-known and well-understood
- Key issue is interface to parser *(define all parts of speech)*

Automating Scanner Construction

RE→ NFA *(Thompson's construction)*

- Build an NFA for each term
- Combine them with ε-transitions

NFA → DFA *(subset construction)*

- Build the simulation

DFA → Minimal DFA

- Hopcroft's algorithm

DFA →RE *(Not part of the scanner construction)*

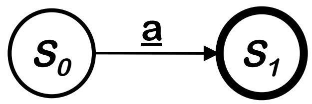
- All pairs, all paths problem
- Take the union of all paths from s_0 to an accepting state

The Cycle of Constructions

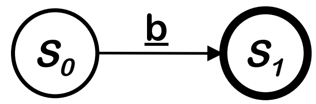
RE →NFA using Thompson's Construction

Key idea

- NFA pattern for each symbol & each operator
- Join them with ε transitions in precedence order

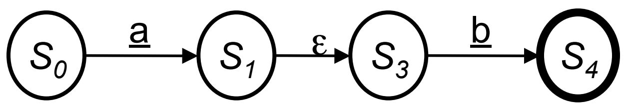


NFA for a



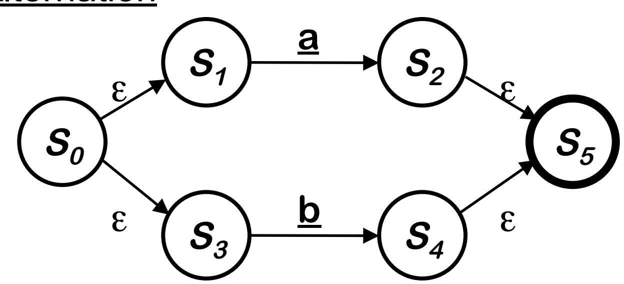
NFA for b

Concatenation



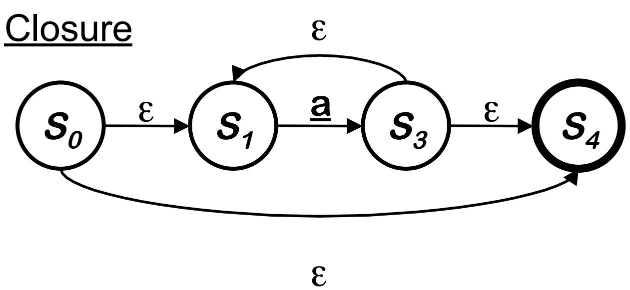
NFA for ab

Alternation



NFA for a | b

Closure



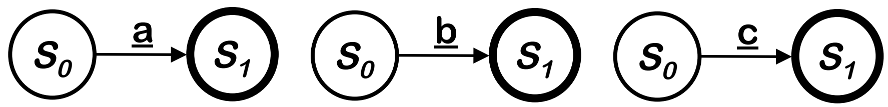
NFA for a^*

Ken Thompson, CACM, 1968

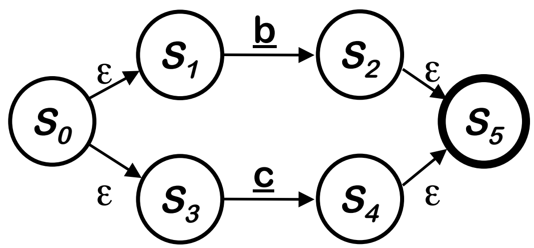
Example of Thompson's Construction

Let's try a (b | c)*

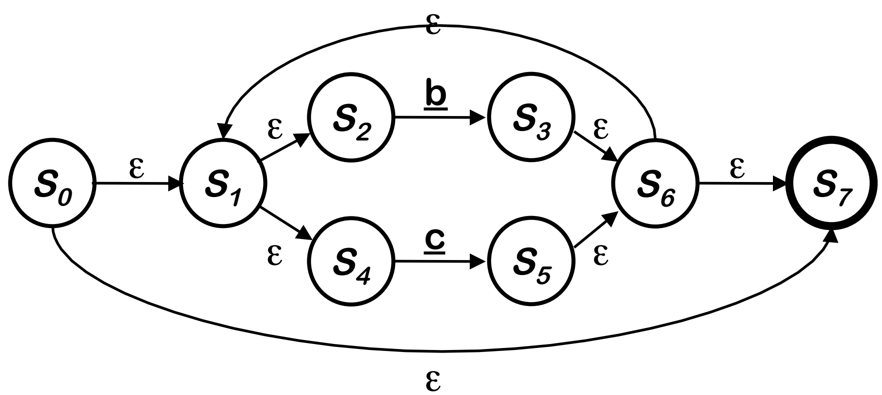
1. a, b, & c



2. b | c

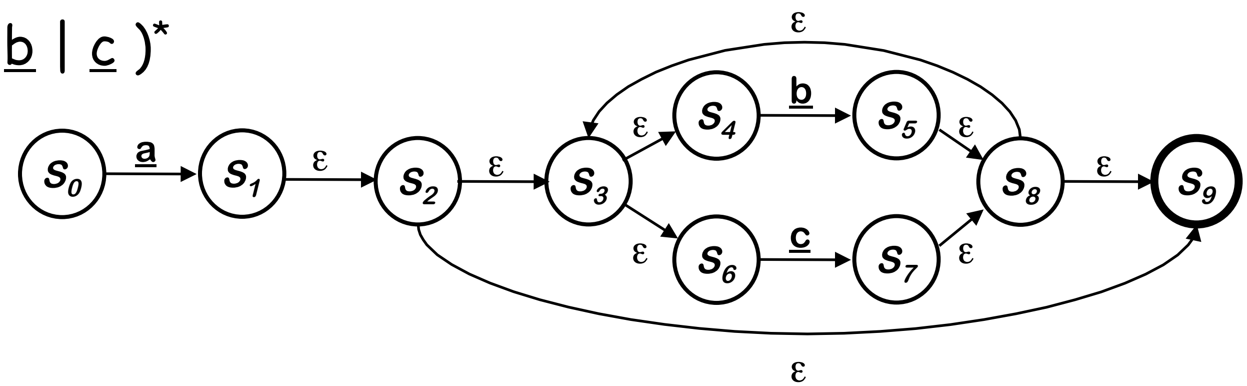


3. (b | c)*

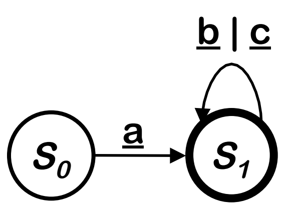


Example of Thompson's Construction *(cont'd)*

4. a (b | c)*



Of course, a human would design something simpler ...



But, we can automate production of the more complex one ...

NFA →DFA with Subset Construction

Need to build a simulation of the NFA

Two key functions

- *Delta*(q_i , $\underline{a}$) is set of states reachable from each state in q_i by $\underline{a}$
  - → Returns a set of states, for each $n \in q_i$ of δ_i (n, $\underline{a}$)
- *ε-closure*(s_i) is set of states reachable from s_i by ε transitions

The algorithm:

- Start state derived from n_0 of the NFA
- Take its ε-closure q_0 = ε-closure(n_0)
- Compute Delta(q, α) for each $\alpha \in \Sigma$, and take its ε-closure
- Iterate until no more states are added

Sounds more complex than it is…

NFA $\rightarrow$DFA with Subset Construction

The algorithm:

$q_0 \leftarrow \varepsilon$-closure($n_0$)

$Q \leftarrow \{q_0\}$

WorkList $\leftarrow \{q_0\}$

while (WorkList $\neq \phi$)
  remove q from WorkList
  for each $\alpha \in \Sigma$
    t$\leftarrow \varepsilon$-closure(Delta(q,α))
    T[q,α] $\leftarrow$ t
    if (t $\notin$ Q) then
      add t to Q and WorkList

Let's think about why this works

The algorithm halts:

1. *Q* contains no duplicates (test before adding)
2. 2^Q is finite
3. while loop adds to *Q*, but does not remove from Q *(monotone)*

$\Rightarrow$ the loop halts

Q contains all the reachable NFA states

It tries each character in each q.

It builds every possible NFA configuration.

$\Rightarrow$ *Q and T form the DFA*

NFA →DFA with Subset Construction

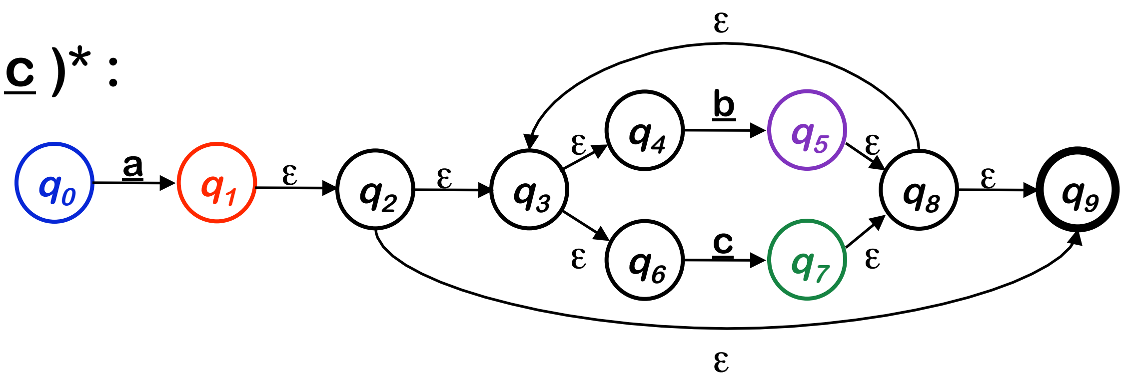
Example of a *fixed-point* computation

- Monotone construction of some finite set
- Halts when it stops adding to the set
- Proofs of halting & correctness are similar
- These computations arise in many contexts

We will see many more fixed-point computations

NFA →DFA with Subset Construction

a (b | c)* :



Applying the subset construction:

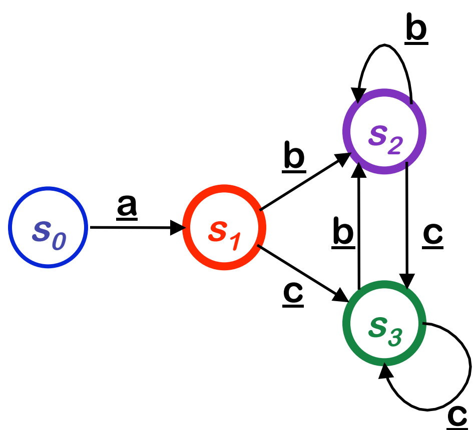
		ε-closure(Delta(q,*))		
	NFA states	a	b	c
s_0	q_0	$q_1, q_2, q_3, q_4, q_6, q_9$	none	none
s_1	$q_1, q_2, q_3, q_4, q_6, q_9$	none	$q_5, q_8, q_9, q_3, q_4, q_6$	$q_7, q_8, q_9, q_3, q_4, q_6$
s_2	$q_5, q_8, q_9, q_3, q_4, q_6$	none	s_2	s_3
s_3	$q_7, q_8, q_9, q_3, q_4, q_6$	none	s_2	s_3

Final states

NFA →DFA with Subset Construction

The DFA for a (b | c)*

- Ends up smaller than the NFA
- All transitions are deterministic
- Use same code skeleton as before



δ	a	b	c
s_0	s_1	-	-
s_1	-	s_2	s_3
s_2	-	s_2	s_3
s_3	-	s_2	s_3

Where are we? Why are we doing this?

RE → NFA *(Thompson's construction)* ✓

- Build an NFA for each term
- Combine them with ε-moves

NFA → DFA *(subset construction)* ✓

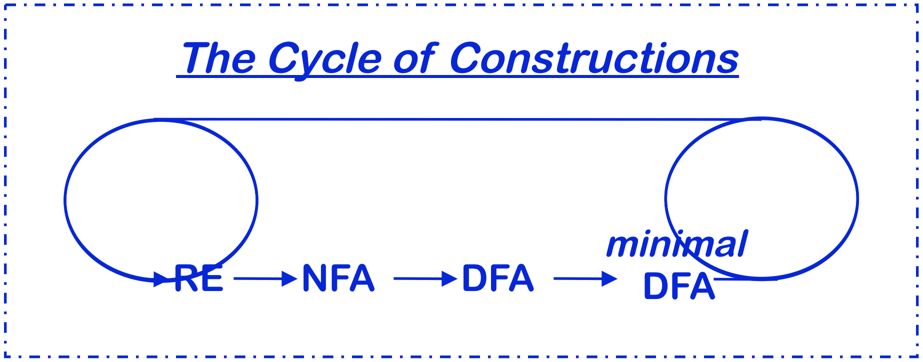
- Build the simulation

DFA → Minimal DFA

- Hopcroft's algorithm

DFA → RE

- All pairs, all paths problem
- Union together paths from s_0 to a final state



Enough theory for today

What we expect of the Scanner

- Report errors for lexicographically malformed inputs
  - → reject illegal characters, or meaningless character sequences
  - → E.g., '#' or "floop" in COOL
- Return an abstract representation of the code
  - → character sequences (e.g., "if" or "loop") turned into tokens.
- Resulting sequence of tokens will be used by the parser
- Makes the design of the parser a lot easier.

How to specify a scanner

- A scanner specification (e.g., for JLex), is list of (typically short) regular expressions.
- Each regular expressions has an action associated with it.
- Typically, an action is to return a token.
- On a given input string, the scanner will:
  - → find the longest prefix of the input string, that matches one of the regular expressions.
  - → will execute the action associated with the matching regular expression highest in the list.
- Scanner repeats this procedure for the remaining input.
- If no match can be found at some point, an error is reported.

Example of a Specification

- Consider the following scanner specification.
  1. aaa { return T1 }
  2. a*b { return T2 }
  3. b { return S }
- Given the following input string into the scanner

  aaabbaaa

  the scanner as specified above would output

  T2 T2 T1
- Note that the scanner will report an error for example on the string ‘aa’.

Special Return Tokens

- Sometimes one wants to extract information out of what prefix of the input was matched.
- Example:

  "[a-zA-Z0-9]*" { return STRING(yytext()) }

- Above RE matches every string that
  - → starts and ends with quotes, and
  - → has any number of alpha-numerical chars between them.
- Associated action returns a string token, which is the exact string that the RE matched.
- Note that yytext() will also include the quotes.
- Furthermore, note that this regular expression does not handle escaped characters.