

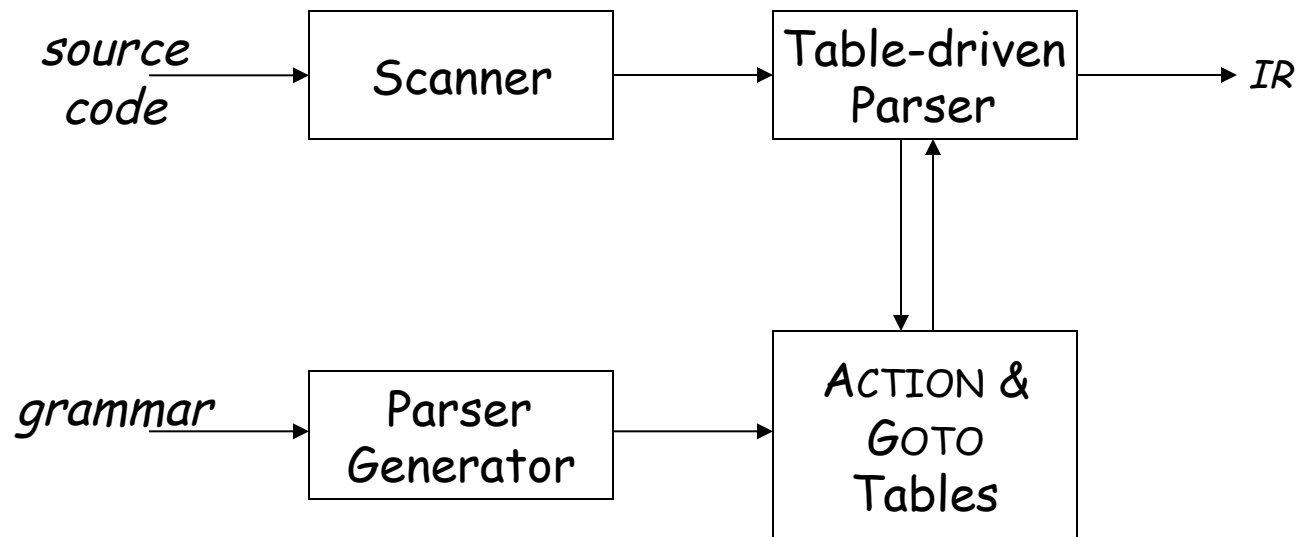


LR(1) Parsers



LR(1) Parsers

A table-driven LR(1) parser looks like



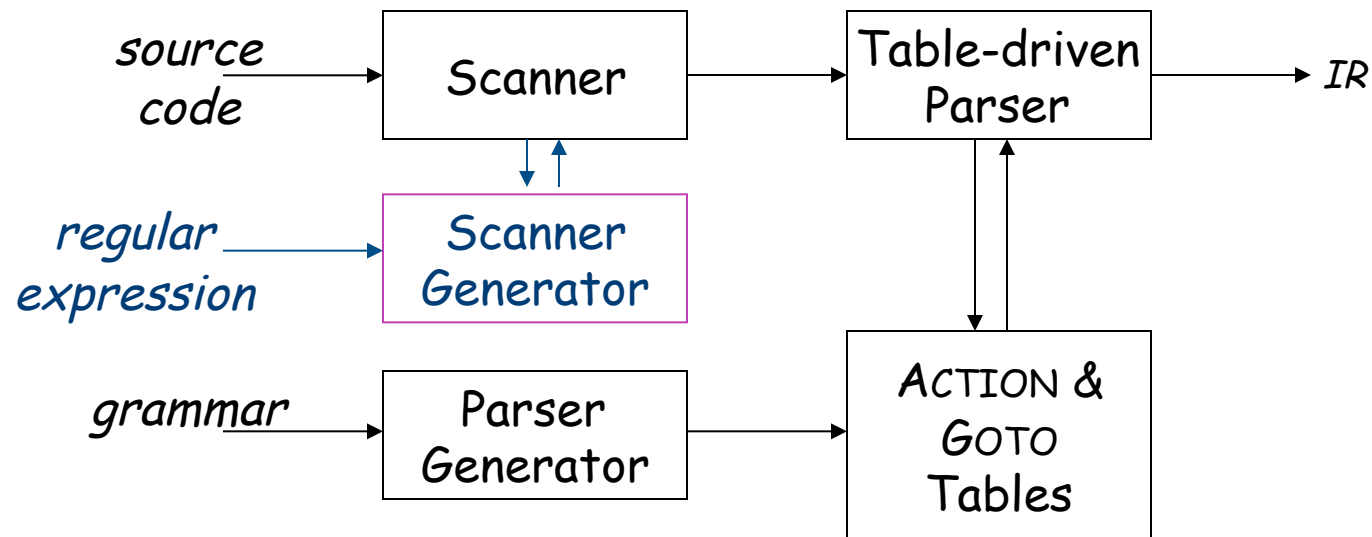
Tables can be built by hand

However, this is a perfect task to automate



LR(1) Parsers

A table-driven LR(1) parser looks like



Tables can be built by hand

However, this is a perfect task to automate

Just like automating construction of scanners ...

Except that compiler writers use parser generators ...



LR(1) Skeleton Parser

```
stack.push(INVALID);  
stack.push( $s_0$ ); // initial state  
token = scanner.next_token();  
loop forever {  
    s = stack.top();  
    if ( ACTION[s,token] == "reduce  $A \rightarrow \beta$ " ) then {  
        stack.popnum( $2 * |\beta|$ ); // pop  $2 * |\beta|$  symbols  
        s = stack.top();  
        stack.push(A); // push A  
        stack.push(GOTO[s,A]); // push next state  
    }  
    else if ( ACTION[s,token] == "shift  $s_i$ " ) then {  
        stack.push(token); stack.push( $s_i$ );  
        token ← scanner.next_token();  
    }  
    else if ( ACTION[s,token] == "accept"  
            & token == EOF )  
        then break;  
    else throw a syntax error;  
}  
report success;
```

The skeleton parser

- Relies on a stack



LR(1) Skeleton Parser

```
stack.push(INVALID);
stack.push( $s_0$ ); // initial state
token = scanner.next_token();
loop forever {
    s = stack.top();
    if ( ACTION[s,token] == "reduce  $A \rightarrow \beta$ " ) then {
        stack.popnum( $2 * |\beta|$ ); // pop  $2 * |\beta|$  symbols
        s = stack.top();
        stack.push(A); // push A
        stack.push(GOTO[s,A]); // push next state
    }
    else if ( ACTION[s,token] == "shift  $s_i$ " ) then {
        stack.push(token); stack.push( $s_i$ );
        token ← scanner.next_token();
    }
    else if ( ACTION[s,token] == "accept"
              & token == EOF )
        then break;
    else throw a syntax error;
}
report success;
```

The skeleton parser

- Relies on a scanner



LR(1) Skeleton Parser

```
stack.push(INVALID);
stack.push( $s_0$ ); // initial state
token = scanner.next_token();
loop forever {
    s = stack.top();
    if ( ACTION[s,token] == "reduce  $A \rightarrow \beta$ " ) then {
        stack.popnum( $2 * |\beta|$ ); // pop  $2 * |\beta|$  symbols
        s = stack.top();
        stack.push(A); // push A
        stack.push(GOTO[s,A]); // push next state
    }
    else if ( ACTION[s,token] == "shift  $s_i$ " ) then {
        stack.push(token); stack.push( $s_i$ );
        token ← scanner.next_token();
    }
    else if ( ACTION[s,token] == "accept"
              & token == EOF )
        then break;
    else throw a syntax error;
}
report success;
```

The skeleton parser

- Relies on two tables, called ACTION and GOTO



LR(1) Skeleton Parser

```
stack.push(INVALID);
stack.push( $s_0$ ); // initial state
token = scanner.next_token();
loop forever {
    s = stack.top();
    if ( ACTION[s,token] == "reduce  $A \rightarrow \beta$ " ) then {
        stack.popnum( $2 * |\beta|$ ); // pop  $2 * |\beta|$  symbols
        s = stack.top();
        stack.push(A); // push A
        stack.push(GOTO[s,A]); // push next state
    }
    else if ( ACTION[s,token] == "shift  $s_i$ " ) then {
        stack.push(token); stack.push( $s_i$ );
        token ← scanner.next_token();
    }
    else if ( ACTION[s,token] == "accept"
              & token == EOF )
        then break;
    else throw a syntax error;
}
report success;
```

The skeleton parser

- Accepts at most once
- Detects errors by failure of all three cases
- Follows the intuitive shift-reduce parser from last lecture



LR(1) Parsers

(*parse tables*)

To make a parser for $L(G)$, need a set of tables

The grammar

- 1 *Goal* $\rightarrow$ *SheepNoise*
- 2 *SheepNoise* $\rightarrow$ *SheepNoise* *baa*
- 3 | *baa*

The tables

ACTION Table		
State	EOF	<u><i>baa</i></u>
0	—	<i>shift 2</i>
1	<i>accept</i>	<i>shift 3</i>
2	<i>reduce 3</i>	<i>reduce 3</i>
3	<i>reduce 2</i>	<i>reduce 2</i>

GOTO Table	
State	<i>SheepNoise</i>
0	1
1	0
2	0
3	0



Example Parse 1

The string baa

Stack	Input	Action
\$ s_0	<u>baa</u> EOF	

1	<i>Goal</i>	→ <i>SheepNoise</i>
2	<i>SheepNoise</i>	→ <i>SheepNoise</i> <u>baa</u>
3		<u>baa</u>

s_0 is on top of stack
baa is lookahead

ACTION Table		
State	EOF	<u>baa</u>
0	—	<i>shift 2</i>
1	<i>accept</i>	<i>shift 3</i>
2	<i>reduce 3</i>	<i>reduce 3</i>
3	<i>reduce 2</i>	<i>reduce 2</i>

GOTO Table	
State	<i>SheepNoise</i>
0	1
1	0
2	0
3	0



LR(1) Skeleton Parser

```
stack.push(INVALID);
stack.push( $s_0$ ); // initial state
token = scanner.next_token();
loop forever {
    s = stack.top();
    if ( ACTION[s,token] == "reduce  $A \rightarrow \beta$ " ) then {
        stack.popnum( $2 * |\beta|$ ); // pop  $2 * |\beta|$  symbols
        s = stack.top();
        stack.push(A); // push A
        stack.push(GOTO[s,A]); // push next state
    }
    else if ( ACTION[s,token] == "shift  $s_i$ " ) then {
        stack.push(token); stack.push( $s_i$ );
        token ← scanner.next_token();
    }
    else if ( ACTION[s,token] == "accept"
              & token == EOF )
        then break;
    else throw a syntax error;
}
report success;
```

s_0 is on top of stack
baa is lookahead token
ACTION[0, baa]

ACTION Table		
State	EOF	<u>baa</u>
0	—	shift 2
1	accept	shift 3
2	reduce 3	reduce 3
3	reduce 2	reduce 2



LR(1) Skeleton Parser

```
stack.push(INVALID);
stack.push( $s_0$ ); // initial state
token = scanner.next_token();
loop forever {
    s = stack.top();
    if ( ACTION[s,token] == "reduce  $A \rightarrow \beta$ " ) then {
        stack.popnum( $2 * |\beta|$ ); // pop  $2 * |\beta|$  symbols
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        stack.push(A); // push A
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        stack.push(token); stack.push( $s_i$ );
        token ← scanner.next_token();
    }
    else if ( ACTION[s,token] == "accept"
              & token == EOF )
        then break;
    else throw a syntax error;
}
report success;
```

s_0 is on top of stack
baa is lookahead token
ACTION[0, baa]

ACTION Table		
State	EOF	<u>baa</u>
0	—	shift 2
1	accept	shift 3
2	reduce 3	reduce 3
3	reduce 2	reduce 2



Example Parse 1

The string baa

Stack	Input	Action
\$ s_0	baa EOF	<i>shift 2</i>
\$ s_0 <u>baa</u> s_2	<u>EOF</u>	

1	<i>Goal</i>	→ <i>SheepNoise</i>
2	<i>SheepNoise</i>	→ <i>SheepNoise</i> <u>baa</u>
3		<u>baa</u>

ACTION Table		
State	EOF	<u>baa</u>
0	—	<i>shift 2</i>
1	<i>accept</i>	<i>shift 3</i>
2	<i>reduce 3</i>	<i>reduce 3</i>
3	<i>reduce 2</i>	<i>reduce 2</i>

GOTO Table	
State	<i>SheepNoise</i>
0	1
1	0
2	0
3	0



LR(1) Skeleton Parser

```
stack.push(INVALID);
stack.push( $s_0$ ); // initial state
token = scanner.next_token();
loop forever {
    s = stack.top();
    if ( ACTION[s,token] == "reduce  $A \rightarrow \beta$ " ) then {
        stack.popnum( $2 * |\beta|$ ); // pop  $2 * |\beta|$  symbols
        s = stack.top();
        stack.push(A); // push A
        stack.push(GOTO[s,A]); // push next state
    }
    else if ( ACTION[s,token] == "shift  $s_i$ " ) then {
        stack.push(token); stack.push( $s_i$ );
        token ← scanner.next_token();
    }
    else if ( ACTION[s,token] == "accept"
              & token == EOF )
        then break;
    else throw a syntax error;
}
report success;
```

s_2 is on top of stack
EOF is lookahead token
ACTION[2, EOF]

ACTION Table		
State	EOF	<u>baa</u>
0	—	shift 2
1	accept	shift 3
2	reduce 3	reduce 3
3	reduce 2	reduce 2



LR(1) Skeleton Parser

```
stack.push(INVALID);
stack.push( $s_0$ ); // initial state
token = scanner.next_token();
loop forever {
  s = stack.top();
  if ( ACTION[s,token] == "reduce  $A \rightarrow \beta$ " ) then {
    stack.popnum( $2 * |\beta|$ ); // pop  $2 * |\beta|$  symbols
    s = stack.top();
    stack.push(A); // push A
    stack.push(GOTO[s,A]); // push next state
  }
  else if ( ACTION[s,token] == "shift  $s_i$ " ) then {
    stack.push(token); stack.push( $s_i$ );
    token ← scanner.next_token();
  }
  else if ( ACTION[s,token] == "accept"
            & token == EOF )
    then break;
  else throw a syntax error;
}
report success;
```

3 SheepNoise → baa

s_2 is on top of stack
EOF is lookahead token
ACTION[2, EOF]

ACTION Table		
State	EOF	<u>baa</u>
0	—	shift 2
1	accept	shift 3
2	reduce 3	reduce 3
3	reduce 2	reduce 2



LR(1) Skeleton Parser

```
stack.push(INVALID);
stack.push( $s_0$ ); // initial state
token = scanner.next_token();
loop forever {
    s = stack.top();
    if ( ACTION[s,token] == "reduce  $A \rightarrow \beta$ " ) then {
        stack.popnum( $2 * |\beta|$ ); // pop  $2 * |\beta|$  symbols
        s = stack.top();
        stack.push(A); // push A
        stack.push(GOTO[s,A]); // push next state
    }
    else if ( ACTION[s,token] == "shift  $s_i$ " ) then {
        stack.push(token); stack.push( $s_i$ );
        token ← scanner.next_token();
    }
    else if ( ACTION[s,token] == "accept"
        & token == EOF )
        then break;
    else throw a syntax error;
}
report success;
```

3 SheepNoise → baa

After 2 pops,
 s_0 is on top of stack

Stack
\$ s_0 <u>baa</u> s_2
\$ s_0

GOTO[0, SN]	
GOTO Table	
State	SheepNoise
0	1
1	0
2	0
3	0



Example Parse 1

The string baa

Stack	Input	Action
\$ s_0	<u>baa</u> EOF	<i>shift 2</i>
\$ s_0 <u>baa</u> s_2	EOF	<i>reduce 3</i>
\$ s_0 SN s_1	EOF	

1	<i>Goal</i>	→ <i>SheepNoise</i>
2	<i>SheepNoise</i>	→ <i>SheepNoise</i> <u>baa</u>
3		<u>baa</u>

ACTION Table		
State	EOF	<u>baa</u>
0	—	<i>shift 2</i>
1	<i>accept</i>	<i>shift 3</i>
2	<i>reduce 3</i>	<i>reduce 3</i>
3	<i>reduce 2</i>	<i>reduce 2</i>

GOTO Table	
State	<i>SheepNoise</i>
0	1
1	0
2	0
3	0



Example Parse 1

The string baa

Stack	Input	Action
\$ s_0	<u>baa</u> EOF	<i>shift 2</i>
\$ s_0 <u>baa</u> s_2	EOF	<i>reduce 3</i>
\$ s_0 SN s_1	EOF	<i>accept</i>

1	Goal	→	SheepNoise
2	SheepNoise	→	SheepNoise <u>baa</u>
3			<u>baa</u>

Notice that we never cleared the stack — the table construction moved *accept* earlier by one action

ACTION Table		
State	EOF	<u>baa</u>
0	—	<i>shift 2</i>
1	<i>accept</i>	<i>shift 3</i>
2	<i>reduce 3</i>	<i>reduce 3</i>
3	<i>reduce 2</i>	<i>reduce 2</i>

GOTO Table	
State	SheepNoise
0	1
1	0
2	0
3	0



Example Parse 2

The string baa baa

Stack	Input	Action
\$ s_0	<u>baa</u> <u>baa</u> EOF	
		1 <i>Goal</i> → <i>SheepNoise</i>
		2 <i>SheepNoise</i> → <i>SheepNoise</i> <u>baa</u>
		3 <u>baa</u>

ACTION Table		
State	EOF	<u>baa</u>
0	—	<i>shift 2</i>
1	<i>accept</i>	<i>shift 3</i>
2	<i>reduce 3</i>	<i>reduce 3</i>
3	<i>reduce 2</i>	<i>reduce 2</i>

GOTO Table	
State	<i>SheepNoise</i>
0	1
1	0
2	0
3	0



Example Parse 2

The string baa baa

Stack	Input	Action
\$ s_0	<u>baa</u> <u>baa</u> EOF	<i>shift 2</i>
\$ s_0 <u>baa</u> s_2	<u>baa</u> EOF	

1	<i>Goal</i>	→	<i>SheepNoise</i>
2	<i>SheepNoise</i>	→	<i>SheepNoise</i> <u>baa</u>
3			<u>baa</u>

ACTION Table		
State	EOF	<u>baa</u>
0	—	<i>shift 2</i>
1	<i>accept</i>	<i>shift 3</i>
2	<i>reduce 3</i>	<i>reduce 3</i>
3	<i>reduce 2</i>	<i>reduce 2</i>

GOTO Table	
State	<i>SheepNoise</i>
0	1
1	0
2	0
3	0



Example Parse 2

The string baa baa

Stack	Input	Action
\$ s_0	<u>baa</u> <u>baa</u> EOF	<i>shift 2</i>
\$ s_0 <u>baa</u> s_2	<u>baa</u> EOF	

1	<i>Goal</i>	→	<i>SheepNoise</i>
2	<i>SheepNoise</i>	→	<i>SheepNoise</i> <u>baa</u>
3			<u>baa</u>

ACTION Table	
State	EOF
0	—
1	<i>accept</i>
2	<i>reduce 3</i>
3	<i>reduce 2</i>

```

...
if ( ACTION[s,token] == "reduce A→β" ) then {
    stack.popnum(2*|β|);    // pop 2*|β| symbols
    s = stack.top();
    stack.push(A);          // push A
    stack.push(GOTO[s,A]); // push next state
}
...

```

2	0
3	0



Example Parse 2

The string baa baa

Stack	Input	Action
\$ s_0	<u>baa</u> <u>baa</u> EOF	<i>shift 2</i>
\$ s_0 <u>baa</u> s_2	<u>baa</u> EOF	<i>reduce 3</i>
\$ s_0 SN s_1	<u>baa</u> EOF	

1	<i>Goal</i>	→	<i>SheepNoise</i>
2	<i>SheepNoise</i>	→	<i>SheepNoise</i> <u>baa</u>
3			<u>baa</u>

Last example, we faced EOF and we accepted. With baa, we shift ...

ACTION Table		
State	EOF	<u>baa</u>
0	—	<i>shift 2</i>
1	<i>accept</i>	<i>shift 3</i>
2	<i>reduce 3</i>	<i>reduce 3</i>
3	<i>reduce 2</i>	<i>reduce 2</i>

GOTO Table	
State	<i>SheepNoise</i>
0	1
1	0
2	0
3	0



Example Parse 2

The string baa baa

Stack	Input	Action
\$ s_0	<u>baa</u> <u>baa</u> EOF	<i>shift 2</i>
\$ s_0 <u>baa</u> s_2	<u>baa</u> EOF	<i>reduce 3</i>
\$ s_0 SN s_1	<u>baa</u> EOF	

1	<i>Goal</i>	→	<i>SheepNoise</i>
2	<i>SheepNoise</i>	→	<i>SheepNoise</i> <u>baa</u>
3			<u>baa</u>

ACTION Table	
State	EOF
0	—
1	<i>accept</i>
2	<i>reduce 3</i>
3	<i>reduce 2</i>

```

...
else if ( ACTION[s,token] == "shift  $s_i$ " ) then {
    stack.push(token); stack.push( $s_i$ );
    token ← scanner.next_token();
}
...

```

shift 3

1	0
2	0
3	0



Example Parse 2

The string baa baa

Stack	Input	Action
\$ s_0	<u>baa</u> <u>baa</u> EOF	<i>shift 2</i>
\$ s_0 <u>baa</u> s_2	<u>baa</u> EOF	<i>reduce 3</i>
\$ s_0 SN s_1	<u>baa</u> EOF	<i>shift 3</i>
\$ s_0 SN s_1 <u>baa</u> s_3	EOF	

1	<i>Goal</i>	→	<i>SheepNoise</i>
2	<i>SheepNoise</i>	→	<i>SheepNoise</i> <u>baa</u>
3			<u>baa</u>

ACTION Table		
State	EOF	<u>baa</u>
0	—	<i>shift 2</i>
1	<i>accept</i>	<i>shift 3</i>
2	<i>reduce 3</i>	<i>reduce 3</i>
3	<i>reduce 2</i>	<i>reduce 2</i>

GOTO Table	
State	<i>SheepNoise</i>
0	1
1	0
2	0
3	0



Example Parse 2

The string baa baa

Stack	Input	Action
\$ s_0	<u>baa</u> <u>baa</u> EOF	<i>shift 2</i>
\$ s_0 <u>baa</u> s_2	<u>baa</u> EOF	<i>reduce 3</i>
\$ s_0 SN s_1	<u>baa</u> EOF	<i>shift 3</i>
\$ s_0 SN s_1 <u>baa</u> s_3	EOF	

1	Goal	→	SheepNoise
2	SheepNoise	→	SheepNoise <u>baa</u>
3			<u>baa</u>

ACTION Table	
State	EOF
0	—
1	<i>accept</i>
2	<i>reduce 3</i>
3	<u><i>reduce 2</i></u>

```

...
if ( ACTION[s,token] == "reduce A→β" ) then {
    stack.popnum(2*|β|);    // pop 2*|β| symbols
    s = stack.top();
    stack.push(A);         // push A
    stack.push(GOTO[s,A]); // push next state
}
...

```

reduce 2

3

0



Example Parse 2

The string baa baa

Stack	Input	Action
\$ s_0	<u>baa</u> <u>baa</u> EOF	<i>shift 2</i>
\$ s_0 <u>baa</u> s_2	<u>baa</u> EOF	<i>reduce 3</i>
\$ s_0 SN s_1	<u>baa</u> EOF	<i>shift 3</i>
\$ s_0 SN s_1 <u>baa</u> s_3	EOF	<i>reduce 2</i>
\$ s_0 SN s_1	EOF	

1	<i>Goal</i>	→	<i>SheepNoise</i>
2	<i>SheepNoise</i>	→	<i>SheepNoise</i> <u>baa</u>
3			<u>baa</u>

Now, we accept

ACTION Table		
State	EOF	<u>baa</u>
0	—	<i>shift 2</i>
1	<i>accept</i>	<i>shift 3</i>
2	<i>reduce 3</i>	<i>reduce 3</i>
3	<i>reduce 2</i>	<i>reduce 2</i>

GOTO Table	
State	<i>SheepNoise</i>
0	1
1	0
2	0
3	0



Example Parse 2

The string baa baa

Stack	Input	Action
\$ s_0	<u>baa</u> <u>baa</u> EOF	<i>shift 2</i>
\$ s_0 <u>baa</u> s_2	<u>baa</u> EOF	<i>reduce 3</i>
\$ s_0 SN s_1	<u>baa</u> EOF	<i>shift 3</i>
\$ s_0 SN s_1 <u>baa</u> s_3	EOF	<i>reduce 2</i>
\$ s_0 SN s_1	EOF	<i>accept</i>

1	<i>Goal</i>	→	<i>SheepNoise</i>
2	<i>SheepNoise</i>	→	<i>SheepNoise</i> <u>baa</u>
3			<u>baa</u>

ACTION Table		
State	EOF	<u>baa</u>
0	—	<i>shift 2</i>
1	<i>accept</i>	<i>shift 3</i>
2	<i>reduce 3</i>	<i>reduce 3</i>
3	<i>reduce 2</i>	<i>reduce 2</i>

GOTO Table	
State	<i>SheepNoise</i>
0	1
1	0
2	0
3	0

Building LR(1) Tables : ACTION and GOTO



How do we build the parse tables for an LR(1) grammar?

- Use the grammar to build a model of the Control DFA
- Encode actions & transitions into the ACTION & GOTO tables
- If construction succeeds, the grammar is LR(1)
 - "Succeeds" means defines each table entry uniquely



Building LR(1) Tables: The Big Picture

- Model the state of the parser with "LR(1) items"
- Use two functions $goto(s, X)$ and $closure(s)$
 - $goto()$ is analogous to $move()$ in the subset construction
 - $closure()$ adds information to round out a state
- Build up the states and transition functions of the DFA ← This is a fixed-point algorithm
- Use this information to fill in the ACTION and GOTO tables



LR(1) Items

We represent valid configuration of LR(1) parser with a data structure called an LR(1) item

An LR(1) item is a pair $[P, \delta]$, where

P is a production $A \rightarrow \beta$ with a $\cdot$ at some position in the *rhs*

δ is a lookahead string (*word or EOF*)

The $\cdot$ in an item indicates the position of the top of the stack