**Homework 8: 50 points**
Due Friday, Oct. 25, 2002

1. (10 points) For each of the following pairs of expressions, find the substitution that is the most general unifier (mgu) or explain why they cannot be unified:

   a) \[ \begin{align*} &P(x, f(y), g(y, c)) \\ &P(z, f(c), z) \end{align*} \]

   b) \[ \begin{align*} &P(f(a, c), w, f(w, y)) \\ &P(x, g(x, f(a, c)), f(a, c)) \end{align*} \]

   c) \[ \begin{align*} &P(w, c, f(w, b, w), x) \\ &P(y, z, y, g(b)) \end{align*} \]

2. (10 points) Use resolution to prove that \( \neg T(a, c) \) is a consequence of the following set \( K \) of formulas. (Be sure to show your work.)

\[
\begin{align*}
&\forall x \forall y \forall z [\neg Q(x, y, z) \rightarrow (\neg T(y, c) \land M(a))] \\
&\forall x \forall y \forall z [S(x, y, z) \rightarrow (\neg R(y, c) \lor \neg Q(x, y, c))] \\
&\forall w \forall u [\neg S(w, a, u) \lor \neg R(a, c) \rightarrow \neg P(b)] \\
&P(b) \land M(c)
\end{align*}
\]

3. (10 points) Use resolution to prove that the following is a tautology:

\[
[(\forall x)(\neg P(x, b) \rightarrow Q(x, a)) \land (\forall y)\neg P(c, y)] \rightarrow Q(c, a)
\]

4. (10 points) Assume the following facts:

- No members of the football team are history or geography or political science majors.
- Some members of the football team are accounting majors.
- All members of the basketball team are history or geography or political science majors.
- All members of the football team and all members of the basketball team are athletes.
- If an athlete is a history major or a geography major or a political science major, then President Roselle has met with them.
- Joe is a member of the basketball team.
- Ben is a member of the football team.

Using the predicates **Athlete**, **On-FB-Team**, **On-BB-Team**, **Majors-in**, and **Met-with**, use resolution to answer the question, \textit{“Who has President Roselle met with?”}. Be sure to show your work.
5. (10 points) Assume the following facts:

- \((\forall x)(\forall w)(\forall y)(\forall z)[(\text{Monument}(y) \land \text{In-City}(y,z) \land \text{In-State}(z,x) \land \text{Governs}(w,x)) \rightarrow \text{Decides-Finances}(w,y)]\)
- \(\text{Monument(Caesar-Rodney-Statue)}\)
- \(\text{Monument(Gateway-Arch)}\)
- \(\text{Monument(Minuteman-Statue)}\)
- \(\text{City(Wilmington)}\)
- \(\text{City(Concord)}\)
- \(\text{In-City(Caesar-Rodney-Statue, Wilmington)}\)
- \(\text{In-City(Minuteman-Statue, Concord)}\)
- \(\text{In-State(Wilmington, Delaware)}\)
- \(\text{In-State(Concord, Massachusetts)}\)
- \(\text{State(Delaware)}\)
- \(\text{State(Massachusetts)}\)

Use resolution to answer the following question:

(a) Who decides the finances for the Minuteman Statue?

6. (Bonus: 5 points) Assuming the same facts as in the previous problem, answer the following question: \text{Who decides the finances for the Gateway-Arch?}