1. (5 pts.) Using resolution, show that $Q$ is a consequence of the following clause set:

$$\{\{-P_3, P_4\}, \{-P_1, P_4\}, \{P_3, Q, P_1\}, \{-P_4\}\}$$

2. (10 pts.) Using resolution, show that $(X \lor (\neg Y \land Z))$ is a consequence of the following formulae:

$$Y \rightarrow (S \lor U)$$

$$(S \lor U) \rightarrow (W \land Q)$$

$$(\neg P \land \neg R) \rightarrow Z$$

$$\neg S \rightarrow (\neg P \land \neg R)$$

$$\neg W$$

3. (10 pts.) Using resolution, show that the following is a tautology:

$$((P \rightarrow Q) \lor (P \rightarrow R)) \rightarrow (P \rightarrow ((Q \lor S) \lor R))$$

Do **NOT** use the unsatisfiability laws to simplify. You **MUST** use resolution.

4. (10 pts.) Suppose that we have the following propositions:

- $P$: x has a mortgage
- $Q$: x owns a boat
- $R$: x owns property
- $S$: x pays interest on a loan
- $U$: x votes on bond issues
- $W$: x pays property taxes
- $Y$: x holds a drivers license
- $Z$: x has passed the drivers exam

Consider the following statements:

- Those who own property pay property taxes and vote on bond issues.
- Only those who have passed the drivers exam hold a drivers license.
- Anyone who has a mortgage also owns property and pays interest on a loan.

(a) Translate each of these statements into formulas in the propositional calculus.
(b) Convert each of the formulas to clause form.
(c) Use resolution to show that anyone who has a mortgage and holds a drivers license either owns a boat or both has passed the drivers exam and votes on bond issues.

5. (10 pts.) Assume that we have the following propositions:
Consider the following statements:

- If John has inherited a fortune, then he is wealthy and he gets calls from investment advisors.
- If John can afford to purchase an airplane, then he can afford a BMW.
- Unless John can buy a castle and afford a BMW, he is not wealthy.
- Only if John holds a job does he earn money.
- John either has inherited a fortune or he earns money.

(a) Translate each of these statements into formulas in the propositional calculus. (Hint: the last statement above is not translated as an implication.)

(b) Convert each of the formulas to clause form

(c) Use resolution to show that either John holds a job or he can afford a BMW or he can afford to purchase an airplane.