To appear in *Handbook in Wireless Mesh Networking.* McGraw-Hill

On Maximizing Capacity in Fixed Mesh Networks with MIMO Links

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Abstract: Multiple antennas can dramatically improve the performance communication. It is a design choice as to how the antennas are used and how they improve performance. For example, by transmitting different data over the different antennas, the bit-rate can be increased. On the other hand, the same data can be transmitted over the antennas but the antennas can be used to mitigate interference, i.e., beamforming. Due to the added capabilities of multi-antenna links, capacity maximization is potentially considerable more complicated when links have multiple antennas. This chapter shows that in general, MIMO links greatly increase the computational complexity of capacity maximization, however, in the high SNR setting that is typical of infrastructure mesh networks, capacity maximization can be achieved without any additional computational complexity.

1 Introduction

There has been extensive work on scheduling transmissions to optimize the capacity of the infrastructure part[1] of wireless networks (e.g., [Bohacek 2007], [Chen 2005], [Lin 2006]). This work has primarily focused on communication with single-input single-output (SISO) links. However, multiple-input multiple-output (MIMO) links are able to provide far higher data rates than SISO links. For example, IEEE 802.11n will support MIMO and provides data rates that are four times that of IEEE 802.11g.

Multiple antennas can be used in different ways. For example, it is possible to transmit different

[1]We distinguish the fixed or infrastructure part of the mesh network from the links between the mobile clients and the infrastructure.

data streams over different antennas. In some settings, this results in the data rate increasing linearly with the number of antennas. Alternatively, multiple antennas can also be used for beamforming. The ability to beamform means that MIMO has the capability to mitigate interference. Specifically, it is possible that two links that significantly interfere may be able simultaneously transmit at relatively high data rates. Consequently, when maximizing the capacity of a network composed of MIMO links, not only must schedules and transmission powers be determined, but also, beamforming must be considered. This added degree of freedom provided by MIMO has the potential to greatly complicate capacity optimization. This chapter examines whether and when this added complexity provides a substantial improvement in performance. This chapter examines some aspects of capacity maximization of mesh networks with MIMO links.

The key findings of this chapter are that while achieving optimal capacity with MIMO links is potentially computationally complex, in high SNR settings (which are typical for the infrastructure part of mesh networks), simple computational approaches will perform nearly optimally. Furthermore, in a high SNR environment, the behavior of MIMO is similar to the behavior of single antenna systems, and hence the techniques and insights used to maximize network capacity with SISO links can be easily extended to networks with MIMO links. On the other hand, in low SNR settings, the full capabilities of MIMO are required and hence the computational complexity of maximizing the capacity of networks with MIMO links is considerably greater than the computational complexity of capacity maximization of networks with SISO links.

This chapter proceeds as follows. In the next section an overview of capacity maximization of MIMO links with interference is provided. Section 3 provides an example of the normalized capacity region achieved by different capacity optimization techniques. Section 3 provides a basis and reference for the more detailed analysis of the normalized capacity region in Section 4. Finally, concluding remarks are given in Section 5.

Throughout this paper it is assumed that all channel gains are known. Such an assumption is reasonable in the case of fixed mesh networks. Also, it is assumed that the transmitter and receiver both have N antennas. Finally, due to space limitations, this chapter only examines the two link case. However, the results and insights can be applied to the multi-link setting.

2. Optimization Techniques for MIMO Links

The availability of multiple antennas allows different signals to be applied to each antenna, where the signals may represent different data or the same data. We denote with $\vec{x}_l$ the vector of signals transmitted from the transmitters of link l. The correlation matrix of this vector is $Q_l = E\left(\vec{x}_l \vec{x}_l^{\dagger}\right)$, where $Q_l \in \mathcal{H}^{+,0}$ and $\mathcal{H}^{+,0}$ is the space of nonnegative definite Hermitian matrices. The correlation matrix, Q_l, is a design parameter. The total transmission power over link l is $trace(Q_l)$. It is assumed that the channel gains are normalized so that the maximum allowable transmission power corresponds to $trace(Q_l) = 1$. Given a set of correlation matrices, Q_1, Q_2 it can be shown [Cover 1991] that the maximum data rate over link l is

$$B_l(Q_1, Q_2) = \log_2\left(\det\left(R_l + H_{l,l} Q_l H_{l,l}^{\dagger}\right)\right) - \log_2\left(\det\left(R_l\right)\right),$$

where $R_l = I + \sum_{k \neq l} H_{k,l} Q_k H_{k,l}^{\dagger}$ and where $H_{k,l}$ is the channel gain matrix from the transmit antennas of link k to the receive antennas of link l. Thus, $H_{l,l}$ is the matrix of channel gains across link l and $H_{k,l}$ are the channel gains from the transmit antennas of link k to the receive antennas of link l. Hence, R_l represents the noise plus interference. Note that the channel gains are normalized to that the noise power is one.

As mentioned above, MIMO can be used to both maximize the data rate and reduce interference. However, in general, the highest data rate across one link will result in interference with other links. Hence, it is not possible to simultaneously maximize the data rate across each link. Instead, a trade-off between data rate and beamforming must be achieved. For example, given weights λ_1 and λ_2, the following weighted capacity problem can be solved

$$\begin{aligned} &\max_{Q_1, Q_2 \in \mathcal{H}^{+,0}} \lambda_1 B_1(Q_1, Q_2) + \lambda_2 B_2(Q_1, Q_2) \\ &\text{subject to} : \text{trace}(Q_1) \leq 1 \\ &\qquad\qquad\quad\;\; \text{trace}(Q_2) \leq 1. \end{aligned}$$

In [Ye 2003], this optimization problem was investigated. It was found that for high SNR, the optimization is convex and can be solved with the projected gradient approach [Bertsekas 1999]. It was also found that this technique works well when the SNR is not high.

Since optimizing the data rate over $Q_1 \in \mathcal{H}^{+,0}$ with $trace(Q_1) \leq 1$ is able to take advantage of all the capabilities of MIMO communication, we refer to such optimization as *full optimization*. Full optimization is capable of high data rates and/or can employ beamforming to reduce interference. However, since the space $\{Q_1 \in \mathcal{H}^{+,0} : trace(Q_1) \leq 1\}$ is quite large, full optimization is computationally complex. This computational complexity increases with the number of links, quickly making capacity optimization beyond the range of today's computers. Thus, we consider a scheme based on *eigenchannels*.

Letting *N* be the number of antennas, the MIMO link can be divided into *N* eigenchannels as follows[2]. Let x' denote the vectors of signal transmitted across N eigenchannels. Given x', the vector of signals transmitted by the antennas is $x = V_l x'$, where V_l is from the singular value decomposition of $H_{l,l}$, i.e., $U_l \Lambda_l V^{\dagger}_l = H_{l,l}$. Let y be the vector of signals received by the receive antennas, hence $y = H_{l,l} x$. The received signal across the eigenchannel is denote with y' and is given by $y' = U^{\dagger}_l y = U^{\dagger}_l H_{l,l} V_l x' = \Lambda_l x'$, where the Λ_l is diagonal matrix of the singular values of $H_{l,l}$, which are denoted with $h_{l,l,i,i}$. Hence, $y_i' = h_{l,l,i,i} x_i'$. The total data rate achievable across this MIMO link is $\sum_{i=1}^{N} \log_2\left(1 + h^2_{l,l,i,i} P_{l,i}\right)$, where $P_{l,i}$ is the transmission power allocated to the ith eigenchannel of the lth link.

Based on this eigenchannel representation, a MIMO link is a collection of parallel, noninterfering channels. However, eigenchannels from other links will cause interference. Specifically, the gain across the interfering channel from the transmitter of the ith eigenchannel of the kth link to the receiver of the jth eigenchannel of the lth link is $h_{k,l,I,j} = u^{\dagger}_{l,j} H^{\dagger}_{k,l} v_{k,i}$, where $u_{l,j}$ and $v_{k,i}$ are the jth column of U_l and the ith column of V_k respectively, and U_l and V_k are from the singular value decomposition of the channel gain matrices $H_{l,l}$ and $H_{k,k}$ respectively. Given P, a vector of transmit power assigned to different eigenchannels, the data rate across the lth link is

$$E_l(P) := \sum_{i=1}^{N} \log_2\left(1 + \frac{h_{l,l,i,i} P_{l,i}}{1 + \sum_{k \neq l} h_{k,l,j,i} P_{k,j}}\right).$$

If the eigenchannel approach is used, then weighted capacity problem is replaced with

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[2] See pages 291-293 in [Tse 2005] for further explanation of eigenchannels.

$$\max_{\vec{p}} \lambda_1 E_1(P) + \lambda_2 E_2(P)$$
$$\text{subject to} : \sum_{i=1}^{N} P_{l,i} \le 1 \text{ for all } l.$$

This optimization is the same as the ones arise in the optimization of networks with SISO links, except that instead of a limit on the power transmitted over each channel, here there is a constraint on the *total* power allocated to eigenchannels of a link.

When either $\lambda_1 = 0$ or $\lambda_2 = 0$, then the optimal solution to the above weighted capacity problems is given by water-filling. Specifically, the powers allocated to the eigenchannels are $P_{l,i}^* = \frac{1}{\mu_l} - \frac{1}{h_{l,l,i,i}}$, where μ_l is the largest number such that $\sum_{i=1}^{N} P_{l,i}^* = 1$. (See page 293 in [Tse2005] for details).

To further decrease the computational complexity, we consider treating the MIMO link as a single channel with the total power allocated to the link denoted by p_l, where p_l scales the power allocated to each eigenchannel, that is, the power allocated to the ith eigenchannel of the lth link is allocated $P^*_{l,i}\, p_l$, where $P^*_{l,i}$ is the water-filling capacity for the ith eigenchannel of the lth link. In this case, the weighted capacity problem becomes

$$\begin{aligned} &\max \lambda_1 E_1(P) + \lambda_2 E_2(P) \\ &\text{subject to} : P_{l,i} = P_{l,i}^* p_l \text{ for all } l \text{ and } i \\ &\qquad 0 \le p_l \le 1. \end{aligned}$$

Since this scheme models the MIMO link as a single channel, we refer to this approach as the *Single Channel* approach.

The following points should be emphasized.

- When only considering a single link transmitting, these three approaches achieve the same data rate as given by water-filling. We denote the water-filling data rates as $\overline{BR1}$ and $\overline{BR2}$ respectively. This paper focuses on data rates relative to these maximum data rates. Hence, when water-filling is applied, the link achieves a normalized data rate of one.

- When considering multiple links transmitting (and interfering), the single channel approach has fewer degrees of freedom than the eigenchannel approach, and the eigenchannel approach has fewer degrees of freedom than full optimization. Hence, we expect that the single channel approach is the least computational complex, but yields to worst performance. Whereas full optimization is the most computationally complex, but should provide the best results. The next sections examine the performance of these schemes.

3. Capacity of Simultaneously Transmitting Links

When sending data across two links, there are two options, transmissions can occur across both links simultaneously or TDM can be used and links transmit at different times. Which of these approaches yields the highest capacity depends on the channels. The *simultaneously transmitting normalized capacity region* can be defined by the function *nBR2(BR1)* which is the maximum normalized data rate across link 2 given the normalized data rate across link 1. In the case of full optimization, we consider

$$
\begin{aligned}
nBR2_{Full}(BR1) := \max \frac{B_2(Q_1, Q_2)}{\overline{BR2}} \\
\text{subject to}: \frac{B_1(Q_{1,}Q_{,2})}{\overline{BR1}} \geq BR1 \\
Q_i \in H_{l,l}^{\dagger} \\
trace(Q_i) = 1.
\end{aligned}
$$

In the eigenchannel approach, we consider

$$nBR2_{\text{EigenChannel}}(BR1) := \max \frac{E_2(P)}{\overline{\overline{BR2}}}$$

$$\text{subject to}: \frac{E_1(P)}{\overline{\overline{BR1}}} \geq BR_1$$

$$\sum_{i=1}^{N} P_{1,i} \leq 1 \text{ for } i = 1, 2.$$

Examples of these capacity regions are shown in Figure 1 for several different values of SNR and for the SIR fixed at 10 dB. The upper left frame shows that when the SNR is considerably lower than the SIR, the interference can be ignored, and hence both links can transmit at their maximum data rate (i.e., a normalized data rate of one). This is true both for full optimization and the eigenchannel approach. However, as the SNR becomes comparable with the SIR, the system becomes interference limited and hence interference reduces the data rate. In the eigenchannel approach, when the SNR is high, if both links are transmitting, the data rate for each link is relatively small. However, if full optimization is used, then relatively high data rate across both links is still possible at high SNR. This is due to the ability of MIMO to mitigate interference as well as transmit at relatively high data rates.

4. Multiplexing versus Simultaneous Transmissions

While the lower right frame of Figure 1 seems to indicate that the eigenchannel approach performs considerably worst than the full optimization, the difference is more subtle than Figure 1 indicates. Specifically, Figure 1 shows the data rate when both links transmit simultaneously. It is also possible to use TDM and have each link transmit individually. The capacity region when TDM is allowed is the convex hull of the capacity region when links transmit simultaneously.

The lower right-hand frame of Figure 1 shows the normalized capacity region for the eigenchannel approach when TDM is allowed. Consequently, when the SNR is high or low, the eigenchannel approach with TDM achieves nearly the same capacity as full optimization. For moderate values of SNR, the full optimization provides higher capacity than the eigenchannel approach. This section closely examines the performance when multiplexing is permitted.

In order to determine whether multiplexing or simultaneously transmissions provide higher capacity we consider the *normalized capacities*

$$\begin{aligned}
&\max \frac{B_1(Q_1,Q_2)}{\overline{BR1}} + \frac{B_2(Q_1,Q_2)}{\overline{BR2}} \\
&\text{subject to}: \; Q_1 \in H_{l,l}^{\dagger}, \quad Q_2 \in H_{l,l}^{\dagger} \\
&\qquad\qquad\quad \operatorname{trace}(Q_1) \le 1, \operatorname{trace}(Q_2) \le 1,
\end{aligned}$$

$$\begin{aligned}
&\max \frac{E_1(P)}{\overline{BR1}} + \frac{E_2(P)}{\overline{BR2}} \\
&\text{subject to}: \sum_{i=1}^{N} P_{1,i} \le 1 \text{ for } l = 1,2 \text{ and } i = 1,..,N,
\end{aligned}$$

and

$$\begin{aligned}
&\max \frac{E_1(P)}{\overline{BR1}} + \frac{E_2(P)}{\overline{BR2}} \\
&\text{subject to}: P_{l,i} = p_l P_{l,i}^{*} \text{ for } l = 1,2 \text{ and } i = 1,..,N \\
&\qquad\qquad 0 \le p_l \le 1 \text{ for } l = 1,2.
\end{aligned}$$

Note that these maximums are never less than one and never greater than two. For example, if only one link transmits at its maximum rate, $\overline{BRl}$, then the weighted capacity is one. Furthermore, this same capacity is achieved by multiplexing. Hence, when the solutions to these problems are equal to one, then multiplexing achieves a weighted capacity that is no worst than simultaneous transmissions. The lower right frame of Figure 1 is an example where this ratio would be one for the eigenchannel approach and slightly larger than one for full optimization. On the other hand, when the maximum is two, then both links can simultaneously achieve their maximum data rate. The upper left frame of Figure 1 is an example where the solution to these problems is two. For intermediate values between one and two, the maximum weighted capacity region is given by both links transmitting simultaneously. However, since interference cannot be neglected, optimization is required in order to achieve the maximum capacity.

Figure 2 shows the average values of the above normalized capacities for several values of SNR and as a function of SIR. Here the channel gains are

$$H_{l,l} = \left(R_{l,l} + \sqrt{-1}I_{l,l}\right)\times 10^{SNR/10}$$
$$H_{l,k} = \left(R_{l,k} + \sqrt{-1}I_{l,k}\right)\times 10^{(SNR-SIR)/10},$$

where $R_{l,l}$, $I_{l,l}$, $R_{l,k}$, and $I_{l,k}$ are $N\times N$ normally distributed random matrices with zero mean and unit variance. Figure 2 was made by averaging 100 trials for each combination of SNR and SIR. Figure 2 shows the results for N=1, ..., 4. Clearly, in the single antenna case, these approaches are the same, hence these lines are indistinguishable.

Several conclusions can be drawn from Figure 2. First, the eigenchannel approach has a similar *normalized* capacity region to the single antenna case and the eigenchannel approach is the same as the single channel approach. Hence, while the eigenchannel approach has more degrees of freedom than the single channel approach, it has no ability to reduce interference. Therefore, we conclude that the eigenchannel approach should not be used. On the other hand, in some cases, due to MIMO's interference mitigating abilities, full optimization is able to reduce interference and hence provides a larger normalized capacity region than the single channel case. For example, if the SNR is low (e.g., 0 dB) and the SIR is even lower (e.g., less than -10 dB), then full optimization is able to achieve a considerably larger normalized capacity region than SISO links. However, in the case of a planned mesh network, it is unlikely that links would have such low SNRs. In the case of high SNR (e.g., greater than 20 dB), the impact of MIMO's ability to reduce interference is limited.

These conclusions have important implications: in high SNR environments, full optimization is not needed, the single channel approach is sufficient. Furthermore, full optimization is only needed when SIR is considerably lower than the SNR. Based on these (and other) results, we find that full optimization only provides significant improvement in capacity when $SNR < 10$ dB and $SIR < 2.5 - SNR/2$ dB. Otherwise, the single channel approach is sufficient, and, of course, considerably less computationally complex. Since mesh networks often have high SNR, we conclude that network capacity optimization with MIMO links can be easily accomplished. However, if the links have low SNR, then capacity maximization requires solving computationally intensive optimization problems.

5. Conclusions

MIMO not only permits high data rates, but also allows interference mitigation through beamforming. Hence, MIMO provides new degrees of freedom when maximizing network capacity. On the other hand, these added degrees of freedom increase the computational complexity. This chapter investigated the improvement in the normalize capacity offered by MIMO's beamforming capacities. It was found that beamforming only provides performance improvement when the SNR is relatively low (specifically, $SNR < 10$ dB) and the interference is also low (specifically, $SIR < 2.5 - SNR/2$ dB). Otherwise, the beamforming capacities of MIMO can be neglected, the MIMO link can be treated as a single link, and the traditional techniques for network capacity can be applied.

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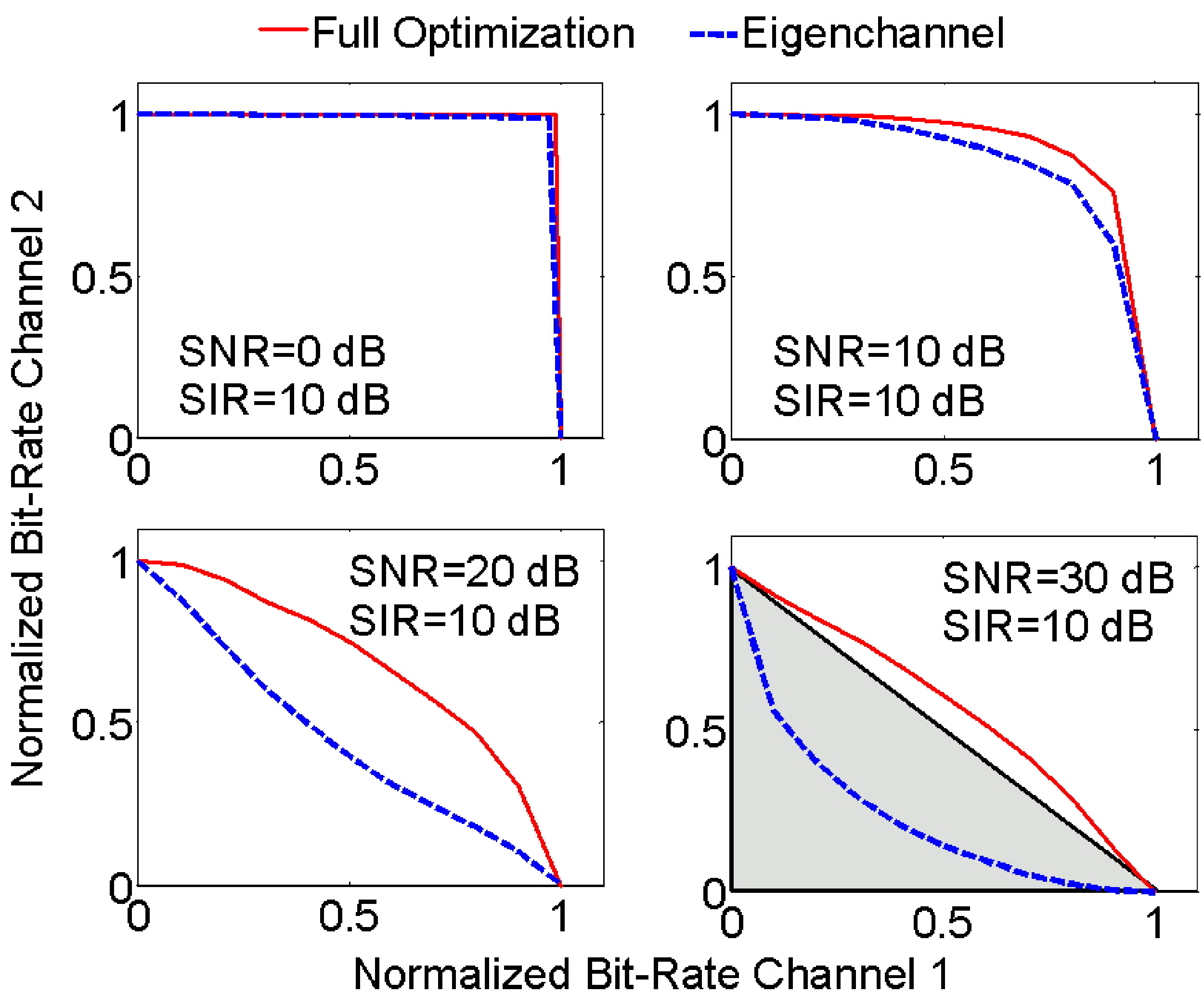

Figure 1, Normalized capacity region for different values of SNR and SIR. The gray area in the lower right frame is the capacity region for the eigenchannel approach when multiplexing is also allowed.

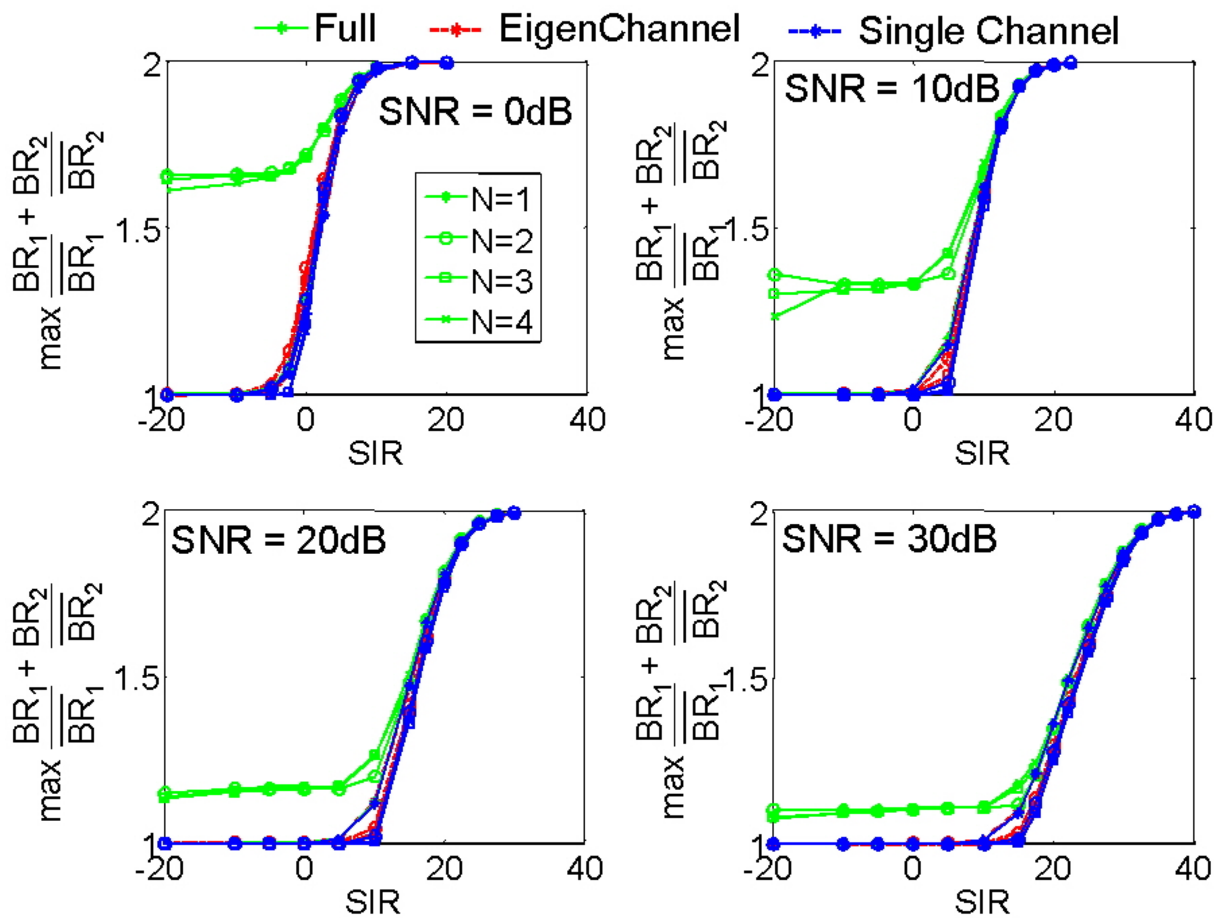

Figure 2. Average normalized weighted capacity as a function of the SIR for different values of the SNR, different numbers of antennas (N=1,..,4), and for different optimization schemes. The full optimization scheme with N=2,3,4 achieve nearly the same