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Performance Improvements Provided by Route Diversity in Multihop Wireless Networks

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Abstract

In multihop wireless networks, the variability of channels results in some paths providing better performance than other paths. While it is well known that some paths are better than others, a significant number of routing protocols do not focus on utilizing optimal paths. However, cooperative diversity, an area of recent interest, provides techniques to efficiently exploit path and channel diversity. This paper examines the potential performance improvements offered by path diversity. Three settings are examined, namely, where the path loss and channel correlation are neglected, where path loss is considered, but channel correlation is neglected, and where path loss and channel correlation are both accounted for. It is shown that by exploiting path diversity, dramatic improvements in the considered route metric may be achieved. Furthermore, in some settings, if the link statistics are held constant, then when path diversity is exploited, the route metric improves with path length. This implies that if links statistics are fixed and if sufficient path diversity exists, then paths with more hops tend to support higher bit-rates than paths with fewer hops. It is shown that such behavior occurs when a particular map has a non-zero fixed point.

Keywords: Diversity, Multihop Wireless Networks, Wireless Network Modeling, Channel Loss

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I. Introduction

Channel variability is one of the most important characteristics of wireless networks. In wireless networking, great pains are taken to mitigate the impacts of channel variability. While all layers must cope with the effects of time-varying channels, there has been extensive research focus at the network layer. For example, there are a number of techniques that seek to find precomputed backup paths (e.g., [1]-[11]). Thus, when the primary path fails, a new route search is not required. However, time-varying channels do not only imply that links may break, it also implies that some links are better than others. Indeed, in the context of communication theory, channel diversity means that there may be some channels between the same transmitter and receiver (but with different antennas) that have better performance than other channels. This diversity is closely related to the stochastic nature of channels. For example, popular models for the channel gain[1] include the lognormal distribution, exponential distribution, and Nakagami. If multiple transmit and/or receive antennas results in a set of channels that can be modeled as independent random variables, then the larger the set of channels, the higher the probability that a good channel can be found. While communication theory provides a clear picture of how diversity can be used to improve performance (e.g., Chapter 11 of [12]), diversity in the setting of multihop wireless networks is poorly understood.

This paper examines the performance improvement that results when path diversity in multihop networks is exploited as oppose to using an arbitrary or a geographically optimal route. This investigation is performed in several topologies and under several propagation models. One important result of this paper is that, in terms of the route metric considered, large improvements are possible when diversity is exploited. Several insights of path diversity are developed. For example, it is shown that in some settings when diversity is fully exploited, if the link statistics

[1]The received signal power is proportional to the transmitted power multiplied by the channel gain.

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are fixed, then longer paths provide a higher average throughput than shorter paths. This behavior contrasts the single path case where if the link statistics are fixed, then shorter paths provide a higher average throughput than longer paths. It will be shown that the conditions when longer paths provide a higher throughput than shorter paths are related to the existence of a fixed point of a particular one-dimensional map. This paper also develops efficient techniques and approximations for computing the performance improvement offered by diversity.

This paper does not address the practical aspects of exploiting diversity. However, cooperative diversity is an active area of research that focuses on developing efficient ways to exploit diversity. Section II provides some references of work in this area. The findings of this paper motivate continued research of cooperative diversity. This paper also examines the performance improvement offered by diversity as a function of node density, which, for example, has implications for the design of sensor networks. Recently, there has been interest in robot-based communication. In this case, nodes can move into locations that will provide good propagation and good paths. While this paper explores the performance improvements that could be achieved by such an approach, it does not address how such a search is performed.

The paper proceeds as follows. In the next section a brief overview of related work is presented. In Section III, the problem definition is provided along with several basic assumptions. The next three sections explore the impact of diversity in increasing complicated environments. First Section IV examines the performance when path loss and channel correlation can be neglected. This case is amenable to analysis. Section V then examines diversity when path loss is included. Finally, Section VI examines diversity when both path loss and channel correlation is included. Section VII provides some concluding remarks.

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II. Related work

As mentioned above, communication theory provides a good understanding of the impact of exploiting diversity when the transmitter and receiver have several antennas. However, recently, there has been interest in examining the impact of allowing nearby nodes assist in a transmission [13]-[19]. Several strategies were investigated in [18] and [19]. However, in most cases, it is often assumed that the source can directly communicate with the destination. Furthermore, it is assumed that one or more intermediate nodes are available to increase reliability. In [15], several techniques for communicating over such networks were investigated. One technique, known as *selection relaying*, utilizes channel measurements to determine which nodes should transmit. While selection relaying does not require feedback, another method, *incremental relaying,* makes use of feedback to determine which and whether nodes should transmit. This strategy proved to yield good performance. While [15] focuses on the setting with one relay, [17] examines the case where a large number of nodes available to act as the relay. There it was found that a technique referred to as *best-select* provided the best performance of the techniques investigated. In best-select, the node with the best channel is selected to transmit. This best-select approach is similar to the ideas pursued here. Specifically, in best-select, the best path is found among the two-hop paths. This paper analyzes the impact of utilizing the best path among multihop paths.

Route diversity is a well known concept in multihop networks. However, previous studies of route diversity focus on the performance of a specific routing protocol [1]-[11]. Here the focus is on the performance improvement offered by exploiting diversity in general.

III. Problem Definition

There are several ways to select a route between a source and a destination. A common approach is to select a path based on a route metric. In this paper, the route metric of interest is

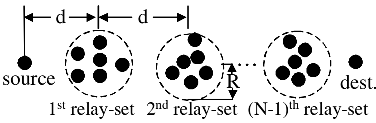

Fig. 1. Cluster Topology.

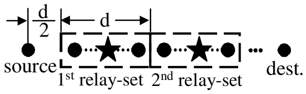

Fig. 2. Horizontal Row Topology. The nodes marked with stars are part of the nominal path.

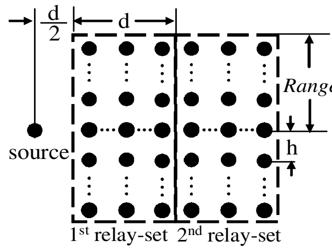

Fig. 3. Vertical Column Topology. Each relay-set is made of one or more vertical columns of nodes. Columns are uniformly spaced across a region of width d. If there is only one column in each relay-set, then it is located in the center of the region. Such a topology is referred to as the single vertical column topology. The nodes marked with stars are part of the nominal path.

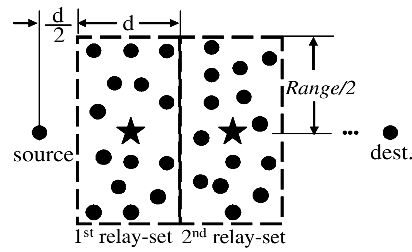

Fig. 4. Random Column Topology. Each relay-set is made of nodes randomly distributed over the region of dimension $d \times Range$. However, a node is always located in the center of the relay-set. These nodees are marked with stars and form the nominal path.

the *maximum channel loss* (MxCL) along the route, which is defined as follows. Let $P_{\text{transmitted}}$ and P_{received} be the transmitted power and received power, in dBm, over the nth hop, respectively. And let H_n be the channel loss, in dB, over the nth hop, i.e., $P_{\text{received}} = P_{\text{transmitted}} - H_n$. Then,

$$\text{MxCL} := \max_{n=1,2,\ldots N} H_n,$$

where there are N hops from the source to the destination. To put it another way, the route metric is the channel loss of the worst link along the route. To explore the impact of diversity provided by multiple paths, this paper compares the average MxCL provided by nominal routes to the average of the minimum of the MxCL when the minimization is performed over all available paths. To make the presentation more concise, the phrase *the improvement of the MxCL* is used to mean the improvement of the average MxCL when diversity is exploited (i.e., by minimizing the MxCL over all paths) as compared to the MxCL over the nominal path, which is defined below.

While the MxCL does not cover all aspects of performance, this route metric is related to the end-to-end bit-rate and, in some settings, the transmission power. To see the relationship between the MxCL and end-to-end bit-rate, consider that most of today's physical layers support several bit-rates. For example, the bit-rates supported by 802.11g range from 1 Mbps to 54 Mbps. However, in order to achieve a higher bit-rate, a higher received signal strength is required. For example, in the case of 802.11g, while 1 Mbps requires only -95 dBm received signal strength, 54 Mbps requires -72 dBm received signal strength [20]. Similarly, Shannon's Theorem provides the value of the maximum achievable bit-rate for a given SNR. Hence, the maximum end-to-end bit-rate is controlled by the signal strength over the weakest link.

If the bit-rate is fixed and the transmission power is varied so that the signal is received with a target received signal strength (i.e., $P_{\text{transmitted}} = P_{\text{target}} + H_n$, where P_{target} is the target received signal strength), then a lower channel loss results in a lower transmission power. Hence, minimizing the MxCL results in minimizing the maximum transmission power. Note that if all nodes have the same battery power, then minimizing the maximum transmission power is related to maximizing the battery lifetime of the nodes along the connection [2].

The performance improvement provided by diversity is closely related to the variability of the channels. Hence, the channel model plays an important role in the analysis of diversity. It is common to decompose the channel loss into two parts, namely, a deterministic part that depends on the distance between the nodes and a stochastic part. In particular, the channel loss, in dB, is modeled as

$$\text{Channel loss [dB]} = C + X + \alpha \times 10 \log_{10}(d) \tag{1}$$

where X accounts for the stochastic part, d is the distance between the transmitter and receiver, α is the path loss exponent, and C is a constant that is independent of the channel. While α has been found to be as small as 1.7 and as large as 6, for sake of concreteness, in many cases we use $\alpha = 2.7$ following the findings presented in [22]. However, in order to investigate the

[2]Since the transmitted power is only one part of the energy used to relay a packet, the importance of transmission power and battery lifetime is dependent on the implementation of the physical layer and the transmission power [21]

impact of α, in some cases we use $\alpha = 6$. Frequently, the deterministic part of the channel (i.e., $C + \alpha \times 10 \log_{10}(d)$) is referred to as the *path loss*. There are many possible distributions of X. The techniques developed in Sections IV and V are applicable to any distribution. In Section IV-B, the impact of different distributions is investigated. However, in Section VI it is assumed that X models the effect of lognormal shadowing. One motivation for focusing on a stochastic channel loss that only accounts for shadowing and not multipath fading is that when wide bandwidth communication physical layers are employed, the impact of multipath fading is mitigated (see, for example, page 330 in [22]). Further discussion on the impact of multipath fading on diversity can be found in Sections IV-B.

If the transmitter and receiver of one channel are near to the transmitter and receiver of another channel, then these channels are correlated[23], [24]. This spatial correlation of channel is another important aspect of the channel model and is studied in Section VI.

The average value of the minimum of the MxCL depends on the topology. This paper focuses on the topologies shown in Figures 1-4. In all cases, the nodes are grouped into *relay-sets*. It is assumed that packets are transmitted from a node in one relay-set to a node in the neighboring relay-set. The set of nodes that are n hops from the source are referred to as the nth relay-set. The distance between relay-sets is denoted with d, and the number of nodes in each relay-set is denoted with M. In the case of the vertical column topology, if there is only a single vertical column in each relay-set, then $M = \lfloor Range/h \rfloor$, where $\lfloor Range/h \rfloor$ is the largest integer that is no greater than $Range/h$. The nodes within the nth relay-set are labeled $(n, 1)$ to (n, M). Sometimes it is convenient to think of the source as a node in the zeroth relay-set, i.e., $(0, \lfloor M/2 \rfloor)$. Similarly, the destination may be denoted with $(N, \lfloor M/2 \rfloor)$, where there are N hops between the source and the destination.

As mentioned above, this paper compares the average of the minimum MxCL and the average

MxCL along a nominal path. In the case of the Cluster Topology (Figure 1), since nodes within a cluster are near to each other, the nominal path is an arbitrary path (See Section IV for more discussion). In the case of the Random Topology, the Vertical Column Topologies, and the Horizontal Row Topology, the nominal path is composed of the nodes marked with stars in Figure 2 and 3. Hence, in all cases, all links along the nominal path are the same length. Obviously, if there is no channel variability (i.e., $X \equiv 0$), then the nominal path gives the minimum MxCL. In the case of the Single Vertical Column Topologies, the nominal path is also the geographically shortest path.

Network topology and the channel model are important parts of the study of diversity. Thus, in order to understand and "probe" the impact of the topology and the channel model on diversity, a wide range of topologies are investigated. The Single Vertical Column Topology is of interest since the channel correlation and path loss should both impact the performance. Specifically, in order to include a large number of nodes (and hence include a large number of alternative paths), either nodes are tightly spaced, in which case the channel correlation will reduce diversity, or the nodes are be placed far from the nominal path, in which case the path loss will limit the diversity. In contrast to the Single Vertical Column Topology, in the Random Topology, nodes are placed throughout a large region, and hence channel correlation has less of an impact, but path loss may have a large impact. The Multiple Vertical Columns Topologies and the Horizontal Row Topology are used in Section V-D to demonstrate the case where the topology has a minor effect on diversity. As explained next, the Cluster Topology is used to denote that path loss and channel correlation are ignored. The improvement of the MxCL for the Cluster Topology is an upper bound on the improvement of the MxCL and hence, the improvement of the MxCL serves as a baseline for the improvement of the other topologies.

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IV. Performance Without Path Loss and Without Channel Correlation

A. Analytic Model

This section develops analytic models for topologies where path loss and correlation can be neglected. Thus, instead of (1), the channel loss is modeled as

$$\text{Channel loss } [dB] = X,$$

where X is an i.i.d. random number. The justification for neglecting the path loss is as follows. The main focus of this paper is to investigate the impact of exploiting diversity. This impact is measured by the difference (in dB) between the average minimum value of the MxCL (i.e., when diversity is exploited) and the average value of the MxCL along a nominal path (i.e., when diversity is not exploited). If the path loss is the same for the route that optimizes the MxCL and the nominal route, then this deterministic part of the channel will not impact the difference between the MxCLs. For example, if the nodes are in the cluster topology shown in Figure 1, then the path loss over all routes is approximately the same, and hence path loss will have little impact. In Section V-D, this issue is discussed in more detail and it is found that the cluster topology provides a good estimate of the impact of diversity for a broad range of topologies.

If the transmitter and receiver of one channel are nearby the transmitter and receiver of another channel, then the stochastic part of the channel losses will be correlated. While such correlation is accounted for in Section VI, it is neglected here. Such an approximation is justified if the clusters are not too small (i.e., R in Figure 1 is large). Note that path loss and channel correlation decrease the impact of diversity. Hence, this section provides an estimate of the impact of diversity in some settings and an upper bound on the impact in general. In the sequel, the cluster topology denotes that path loss and correlation are neglected.

In this setting, it is possible to develop an analytic model of the impact of diversity. Specifically, we seek the average of the minimum of the MxCL and the average of the MxCL as functions of the size of relay-sets and the number of hops. In order to compute such averages, we require the cumulative distribution function (CDF) of the minimum value of the MxCL, i.e., $P\left(\text{the minimum of the MxCL is less than } r\right)$. To this end, we utilize the terminology of percolation theory. Fixing the channel loss threshold at r, we say that a link is *open* if the channel loss is less than r and is *closed* otherwise. Furthermore, we say that a node is *occupied* if there is a sequence of open links from the source to the node. Letting F be the CDF of the channel loss, then the probability that a link is open is $F\left(r\right)$. This probability is denoted with p and the probability the link is closed is denoted with $q := 1-p$. Moreover, the source is always occupied and the destination is occupied if there is an end-to-end route with MxCL less than r. The probability that the destination is occupied is denoted with $T_{M,N}\left(p\right)$, where there are M nodes in each relay-set and N hops between the source and destination. Since

$$P\left(\text{the minimum of the MxCL is less than } r\right) = T_{M,N}\left(F\left(r\right)\right), \tag{2}$$

the desired CDF can be easily computed once T is known. T can be computed as follows.

Proposition 1: With the definitions given above,

$$T_{M,N}\left(p\right) = \mathbf{Y}_{1-p,M} \times \mathbf{\Theta}_{1-p,M}^{N-2} \times \mathbf{V}_{1-p,M}, \tag{3}$$

where $\Theta_{q,M}$ is a $(M+1) \times (M+1)$ matrix, Y is a $1 \times (M+1)$ vector, and $\mathbf{V}_{q,M}$ is the $(M+1) \times 1$ with

$$\mathbf{\Theta}_{q,M}\left(i,j\right) := \binom{M}{j}\left(1-q^{i}\right)^{j} q^{i(M-j)},$$
$$\mathbf{Y}_{q,M}\left(j\right) := \binom{M}{j}\left(1-q\right)^{j} q^{(M-j)},$$
$$\mathbf{V}_{q,M}\left(i\right) := \left(1-q^{i}\right)$$

Note that the indexes i and j range from 0 to M.

Proof: To determine T, the number of occupied nodes is represented as a Markov chain as follows. Suppose that i nodes are occupied in the nth relay-set. Then, the probability that a particular node in the $(n+1)$th relay-set is *not* occupied is the same as the probability that none of the links from the i occupied nodes in the nth relay-set are open. This probability is q^i. And hence, the number of occupied nodes in the $(n+1)$th relay-set is a binomial random variable with parameters $(1-q^i)$ and M. Thus, $\Theta_{q,M}(i,j)$ is the probability that j nodes are occupied in the $(n+1)$th relay-set that is $n+1$ given that i nodes are occupied in the nth relay-set. Since the source is always occupied, there is one node occupied in the relay-set that is zero hops from the source. Therefore, $\mathbf{Y}_{q,M}$ is the probability distribution of the number of occupied nodes in the first relay-set. In general, setting $\mathbf{W} = \mathbf{Y}_{q,M} \times \Theta_{q,M}^{n-1}$, we see that $\mathbf{W}(j)$ is the probability that exactly j nodes are occupied within the nth relay-set. Finally, the probability that the destination is occupied given that i nodes within the last relay-set are occupied is $1-q^i$. Therefore, the right-hand side of (3) is the probability the destination is occupied. ■

B. Examples of Performance Gains Due to Diversity in the Cluster Topology - Neglecting Path Loss and Channel Correlation

In order to gauge the performance improvement due to diversity, we examine the difference between the average MxCL over an arbitrary route and the average minimum MxCL, where the minimization is over all available routes. Thus, an improvement of 0 dB implies no improvement, and a positive improvement implies a better MxCL.

Figures 5 (a)-(c) show the improvement provided by diversity when the channel loss is lognormally distributed with σ, the standard deviation, set to 4, 8, and 11 dB. Each curve shows the variation of the improvement as a function of the number of nodes in each relay-set. As

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indicated, the curves that shows the least improvement corresponds to the case where there are two hops from the source to the destination, while the largest improvement shown corresponds to the cases where there are 14 hops. Note each link is lognormally distributed, regardless of the path length. As expected, when there is only one node in each relay-set, the improvement is 0 dB. This is the case where diversity cannot be exploited and only the nominal route is used. On the other hand, as the number of nodes in each relay-set increases, there are more paths to choose from, and hence the performance gains due to diversity increase as M increases.

Figure 5 shows that diversity improves the MxCL by more than 23 dB when $\sigma = 11$ dB and there are seven nodes in each relay-set and five hops from the source to destination. This implies, for example, that if the nominal path has a MxCL of 95 dBm, then by exploiting diversity, the average MxCL can be reduced to 72 dBm. In the case of 802.11g [20], this would imply that by exploiting diversity, it is possible to increase the bit-rate from 1 Mbps to 54 Mbps. Similarly, by exploiting diversity, on average, the maximum transmission power over all nodes along the path can be reduced by a factor of 200 without impacting the worst SNR along the path. Of course, in general, the impact on the bit-rate and the transmission power depends on the exact scenario and the capabilities of the physical layer.

Lognormal shadowing is parameterized by σ, the standard deviation in dB. Figures 5 (a)-(c) show the improvement of the MxCL for $\sigma = 4, 8,$ and 11 dB ([22] presents experimental results that indicate that $\sigma = 11$ dB.). Figure 5 (d) shows the improvement of the MxCL as a function of σ. As the standard deviation increases, the variability of the channel loss increases and hence the improvement of the MxCL provided by diversity increases.

In the case of narrow bandwidth communication, the stochastic part of the channel loss is well modeled as a combination of shadowing and multipath fading where the shadowing is modeled by

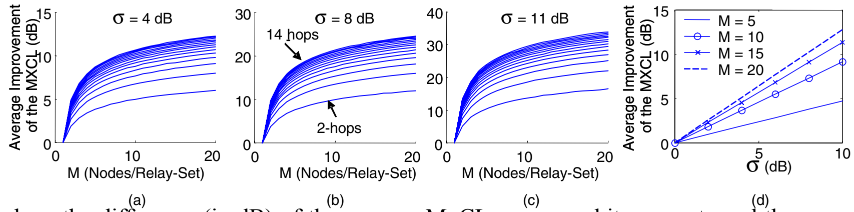

Fig. 5. (a), (b), (c) show the difference (in dB) of the average MxCL over an arbitrary route and the average minimum MxCL, where the minimization is over all available routes. This difference is the improvement achieved by exploiting diversity and depends on the relay-set size and the number of hops. The number of hops varies from 2 hops (the lower curves) to 14 hops (the upper curves). Regardless of the path length, it is assumed that the channel is impaired by lognormal shadowing with standard deviation σ. (a), (b), (c) show the performance for different σ. (d) shows the improvement offered by diversity over a three hop network as a function of σ and for relay-sets sizes ranging from 5 to 20.

a lognormal random variable and the multipath fading is modeled by an exponentially distributed[3] random variable. More specifically, the stochastic part of the channel gain (in dB) is $Y+Z$ where Y is normally distributed with standard deviation σ and $10^{Z/10}$ is exponentially distributed.

Since the addition of multipath fading provides further channel variability, diversity offers more benefits when the channel is subject to both shadowing and multipath fading as compared to when the channel is subject to shadowing alone. Figure 6 demonstrates such improvements by examining the difference between the improvement of the MxCL when the channel is subject to shadowing and multipath fading and the improvement of the MxCL when the channel is only subject to shadowing. Figure 6 shows that the multipath fading provides a significant improvement when the standard deviation of the lognormal distribution is small (e.g., $\sigma = 4$ dB), but provides less improvement of the MxCL when the shadowing has large variability (e.g., $\sigma = 11$ dB). This behavior is expected; the standard deviation (in dB) of exponentially distributed multipath fading is around 5 dB, which is large compared to the variability due to shadowing when $\sigma = 4$ dB, but small when $\sigma = 11$ dB.

[3]The amplitude of the signal is Rayleigh distributed, so the signal power is exponentially distributed.

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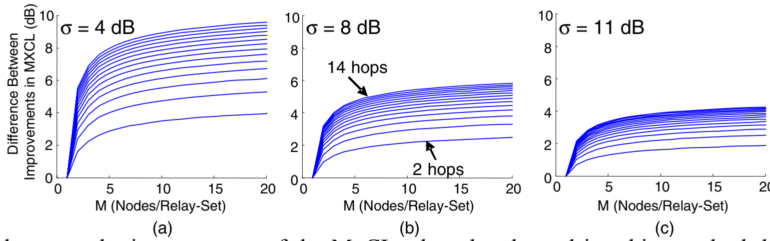

Fig. 6. The difference between the improvement of the MxCL when the channel is subject to both lognormal shadowing and multipath fading and the improvement in the MxCL when the channel is subject to lognormal shadowing only. The multipath fading is modeled as an exponential random variable (not in dB) and the standard deviation of the shadowing is σ. The channel model for each link is the same regardless of the number of hops and the number of nodes in each relay-set.

C. The Impact of Path Length

1) Overview: The computational experiments above showed that when diversity is exploited, longer paths provide more improvement of the MxCL than shorter paths. This can be seen in Figure 5 where the improvement of the MxCL when the path has 2 hops is significantly less than the improvement when the path has 14 hops. The main reason for this behavior is that, as a function of the path length, the average value of the MxCL increases faster when diversity is not exploited than when diversity is exploited. Another view of this behavior is provided by Figure 7, which shows the average minimum MxCL. This figure confirms that as the path length increases, the average MxCL increases faster if diversity is not used (i.e., $M = 1$) than if diversity is exploited. Figure 7 shows that not only does the MxCL increase faster when diversity is not exploited, but that if M is large enough, then the minimum MxCL decreases with path length, rather than increase. Thus, in terms of the MxCL, if diversity is exploited, then a longer path provides better performance than a shorter path.

To understand the implications of this relationship between the minimum MxCL and the path length, consider the simplified example where the communication over a link can occur at a high bit-rate if the channel loss is less than $Thresh$ and occurs at a low bit-rate otherwise. Furthermore, suppose that the propagation environment is such that links have a channel loss less than $Thresh$ with probability p. Now, if a single path is selected at random (hence, diversity

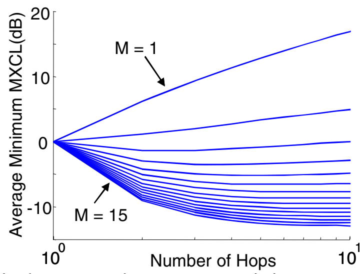

Fig. 7. The above plot shows the relationship between the average minimum MxCL and the path length where the number of nodes in the relay-sets ranges from $M = 1$ to $M = 15$. Note that the average channel loss over a single link is 0 dB. In this case, the random part of the channel gain is lognormally distributed with $\sigma = 11$ dB.

is not exploited), then the probability of all links along an N-hop path providing high bit-rate is p^N, which goes to zero exponentially fast with the path length. However, if diversity is exploited and M is large enough, then the probability of finding a path such that all links along the path support high bit-rate increases with the path length. In this way, the performance of longer paths may exceed the performance of shorter paths.

It is important to note that as the path length is increased, the statistics of the links are held fixed. For example, the distance d in Figures 1-4 is held constant. Furthermore, while the MxCL may decrease with path length, there are other metrics that might increase with path length. Therefore, the fact that MxCL might decrease with path length does not imply that longer paths are preferable in all respects. See [25] for a discussion of some advantages of shorter paths.

The MxCL does not always improve with path length. For example, if M is small, then there is not enough diversity and so the MxCL increases with path length. Figure 7 shows that if $M \geq 2$, then MxCL increases with path length, and if $M = 3$, then MxCL is initially nearly constant, but then increases with path length. Also, for any M, if the path is long enough, then the MxCL increases with the path length. However, for large M, this increase occurs slowly and only occurs for large path lengths. Another way in which the MxCL increases with path length is that the nodes are not in clusters. This scenario is examined in Section V-D.

The next section provides an analytic explanation for why the MxCL improves with path length. However, some insight is possible by considering how the number of available paths increases with the path length. Specifically, besides the last hop, selecting a next hop is a selection of one out of M. Thus, when there are N hops, then there are M^{N-1} different paths between the source and destination. Therefore, on the one hand, the MxCL over an *arbitrary* path increases with path length. While on the other hand, the number of paths increases with the path length, which tends to decrease the minimum MxCL over all paths. Which of these factors is more significant dictates whether the MxCL increases or decreases with the path length. This problem is complicated by the fact that the M^{N-1} paths are not completely distinct, but share some common links.

2) The Impact of Long Paths - A Mean Field Analysis: In this section, analysis is used to gain intuition into the conditions under which the MxCL decreases with the number of hops. The central idea developed in this section is as follows. If the mean number of occupied nodes at the nth hop increases with n, then the probability that a "good path" exists between the source and destination increases with the path length. There are two steps to formalizing this idea. First, we find the mean number of nodes occupied at the $(n+1)$th hop as a function of the mean number of occupied nodes in the nth hop. Second, we examine the conditions under which this function implies that the mean number of occupied nodes increases with the number of hops. As it turns out, the mean number of occupied nodes converges. Specifically, there are two possibilities, either the mean number of occupied nodes increases toward a non-zero limit point, or decreases and converges to zero nodes occupied. We begin by finding the desired function.

In the proof of Theorem 1, it is shown that if i nodes are occupied in the nth relay-set, then the mean number of nodes occupied in the $(n+1)$th relay-set is $(1-q^i)\,M$, where q is the probability that a link is closed. Employing a type of mean field analysis, we assume that if there are i nodes occupied within the nth relay-set, then there are *exactly* $(1-q^i)\,M$ nodes

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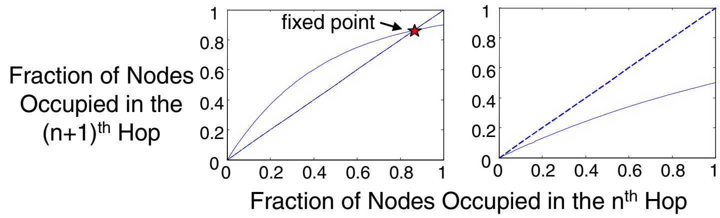

Fig. 8. The above plots show two possibilities of the mapping from the mean fraction of nodes occupied in the nth hop to the mean fraction of nodes occupied in the $(n+1)$th hop. For reference, the 45° line is also shown. On the left, if the mean fraction of nodes occupied in the nth hop is less than 0.85, then the fraction of nodes occupied increases with each hop and converges to the non-zero fixed point of 0.85. On the right, the fraction of nodes occupied decreases with each hop and converges to zero. Hence, the existence of the non-zero fixed point is equivalent to the average MxCL improving as the path length is increased. The conditions under which there is a non-zero fixed point are given in Proposition 2.

occupied within the $(n+1)$th relay-set. Figure 9 (a) compares this mean field model to the exact computation. Under the mean field model, define $\overline{\overline{u}}_n$ to be the mean number of nodes occupied in the nth relay-set. It follows that $\overline{\overline{u}}_1 = (1-q)\,M$, and $\overline{\overline{u}}_{n+1} = \left(1 - q^{\overline{\overline{u}}_n}\right) M$. This one-dimensional dynamical system can be normalized by defining $\tilde{u} := \overline{\overline{u}}/M$ and $\tilde{q} := q^M$, yielding

$$\tilde{u}_{n+1} = 1 - \tilde{q}^{\tilde{u}_n} \tag{4}$$

$$\tilde{u}_0 = 1/M.$$

Figure 8 shows examples of this mapping for two different values of $\tilde{q}$.

It is clear that if $\tilde{q}$ is very small, then $\tilde{u}_n \approx 1$, i.e., nearly all the nodes in the relay-set will be occupied. However, more detailed analysis is possible by using standard tools for analyzing one-dimensional maps of iteration (e.g., Chap. 2 of [26]). In particular, a key question is whether (4) has a non-zero stable equilibrium. That is, we seek to determine when there exists a $\tilde{u}^*$ such that $\tilde{u}^* = \lim_{n\to\infty} \tilde{u}_n$ and $\tilde{u}^* > \tilde{u}_0$.

Proposition 2: The map (4) has a non-zero attracting equilibrium, $\tilde{u}^*$, if $q^M < 1/e$. If $q^M \geq 1/e$, then $\tilde{u} = 0$ is the only equilibrium and is attracting. If $\tilde{u}_0 < \tilde{u}^*$ ($\tilde{u}_0 > \tilde{u}^*$), then the sequence $\tilde{u}_n$ for $n = 0, 1, \cdots$ is monotonically increasing (decreasing).

Proof: We first find the conditions under which $\tilde{u}_n$ is increasing. Define $g\left(\tilde{u}_n\right) = \tilde{u}_{n+1} -$

$\tilde{u}_n = \left(1 - \tilde{q}^{\tilde{u}_n}\right) - \tilde{u}_n$. Then $g'(\tilde{u}) = -\tilde{q}^{\tilde{u}} \ln(\tilde{q}) - 1$. Note that $g(0) = 0$, implies that $\tilde{u} = 0$ is a fixed point. Furthermore, $g'(0) > 0$ if and only if $-\ln(\tilde{q}) > 1$ or

$$\tilde{q} < 1/e, \tag{5}$$

or, equivalently, $q^M < 1/e$. In this case, $\tilde{u}_{n+1} - \tilde{u}_n = g(\tilde{u}_n) > 0$ for $\tilde{u}_n \approx 0$ and positive. Thus, $\tilde{u} = 0$ is a non-attracting fixed point if and only if (5) holds.

Now, suppose that (5) holds. Since $g(1) = -\tilde{q} < 0$ and $g(\tilde{u}) > 0$ for $\tilde{u}$ small and positive, the Intermediate Value Theorem implies that there must be a point $\tilde{u}^*$ between 0 and 1 such that $g(\tilde{u}^*) = 0$. Thus, if (5) holds, then g has at least two zeros (with $\tilde{u} = 0$ one of the zeros). To see that there are exactly two zeros, note that $g''(\tilde{u}) = -\tilde{q}^{\tilde{u}} (\ln(q))^2 < 0$.

Next we show that $\tilde{u}_n$ is monotonic. If (5) holds, then $g'(0) > 0$. And since $g(0) = 0$, we have $g(\tilde{u}) > 0$ for $\tilde{u} \in (0, \tilde{u}^*)$, or, equivalently, $\tilde{u}_{n+1} > \tilde{u}_n$ for $\tilde{u} \in (0, \tilde{u}^*)$. Similarly, since $g(\tilde{u}^*) = 0$ and $g'(0) > 0$, we have $g'(\tilde{u}^*) < 0$ and hence $g(\tilde{u}) < 0$ for $\tilde{u} > \tilde{u}^*$, or equivalently, $\tilde{u}_{n+1} < \tilde{u}_n$. On the other hand, $\frac{d}{d\tilde{u}}\left(1 - \tilde{q}^{\tilde{u}}\right) = -\tilde{q}^{\tilde{u}} \ln(\tilde{q}) \geq 0$. This implies that $1 - \tilde{q}^u < 1 - \tilde{q}^w$ if $u < w$. Thus, if $\tilde{u}_n < \tilde{u}^*$, then $\tilde{u}_{n+1} = 1 - \tilde{q}^{\tilde{u}_n} < 1 - \tilde{q}^{\tilde{u}^*} = \tilde{u}^*$ and if $\tilde{u}_n > \tilde{u}^*$, then $\tilde{u}_{n+1} > \tilde{u}^*$. In summary, if $\tilde{u}_0 < \tilde{u}^*$, then $\tilde{u}_n < \tilde{u}^*$ and $\tilde{u}_{n+1} > \tilde{u}_n$; in other words, u_n is monotonically increasing. Similarly, if $\tilde{u}_0 > \tilde{u}^*$, then $\tilde{u}_n$ is monotonically decreasing. And in both cases, $\lim_{n \to \infty} \tilde{u}_n = \tilde{u}^*$. In a similar fashion it can be shown that $\tilde{u}_n$ is monotonically decreasing when $\tilde{u}_0 > 0$ and (5) does not hold. ■

A straightforward application of this proposition is the following.

Corollary 3: Let $F(r)$ be the probability that a link has a channel loss less than r. If $1 - F(r) < (1/e)^{1/M}$, then, based on the mean field model, the probability that there exists a path with MxCL less than r will increase with the number of hops.

Hence, by this mean field analysis, for very long paths, the median value of the minimum MxCL is the value $r_{Diversity}$ such that $1 - F(r_{Diversity}) = (1/e)^{1/M}$. Wherever F is invertible,

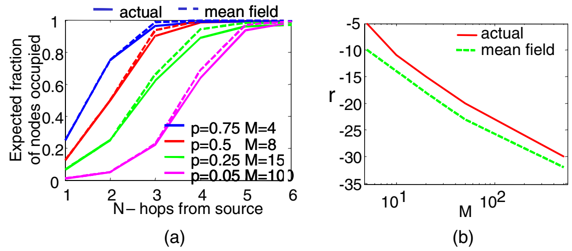

Fig. 9. (a) Mean field estimate of the mean number of occupied nodes and the actual number of occupied nodes as a function of the number of hops from the source. (b) From Proposition 2, the mean value of the MxCL for large N is the r such that $r = F^{-1}\left(1 - 1/e^{1/M}\right)$. The above shows this value of r as well as the actual value of r found from detailed computations using Theorem 1. In this casee, F is lognormal with $\sigma = 11$ dB.

we have $r_{Diversity} = F^{-1}\left(1 - (1/e)^{1/M}\right)$. Figure 9 (b) shows the case where F is lognormal with $\sigma = 11$ dB. For reference, this figure also shows the actual mean value of r. Since the mean field analysis only provides an approximation, the actual value differs from the value found from mean field analysis. However, for M large, the quality of the approximation improves.

The above expression for $r_{Diversity}$ can be compared to the single path version. Specifically, the median value the MxCL when diversity is not used is $r_{NoDiversity} = F^{-1}\left(0.5^{1/N}\right)$, where there are N hops from the source to destination. One can consider, for example, the case extreme case where $N \rightarrow \infty$ and $M \rightarrow \infty$, in which case, $r_{Diversity} \rightarrow F^{-1}(0)$ and $r_{NoDiversity} \rightarrow F^{-1}(1)$.

V. The Impact of Path Loss and Dependent Node Occupancy

While the analysis above provides a clear view of the improvement of the MxCL that can be achieved by exploiting diversity, it neglects path loss and hence, is only directly applicable to the cluster topology shown in Figure 1 where the channels are uncorrelated. In this section, the impact of path loss is considered. Thus,

$$\text{Channel loss [dB]} = X + \alpha \times 10 \log_{10}(d),$$

where X is a random variable that is independent for each link and d is the distance between the transmitter and receiver. The independence of channels is only applicable if the node density is sufficiently low. Section VI includes channel correlation.

The next subsection examines the actual improvement of the MxCL. This section is applicable to any topology. In Subsection V-B, some examples of the performance of the Single Vertical Column Topology are given. Section V-C presents a computationally efficient approximation of the improvement of the MxCL. While the techniques developed in Section V-C are applicable to any topology, the examples given focus on the Single Vertical Column Topology. Section V-D applies this approximation in order to compare the impact of diversity in different topologies. The main results of this section are that the dependency of node occupancy can be neglected and that for topologies where $d \approx Range$, the impact of exploiting diversity can be gauged without considering path loss (see Figure 1-4 for the definition of d).

A. Exact Model

As discussed in Section III, the channel loss depends on the path loss which depends on the distance between the transmitter and receiver. More specifically, letting $D(i,j)$ denote the distance between node (n,i) and node $(n+1,j)$, and letting $g_{Thresh}(i,j)$ denote the probability that the channel loss is less than $Thresh$, then

$$g_{Thresh}(i,j) = P\left(X + 2.7 \times 10\log_{10}\left(D(i,j)\right) < Thresh\right),$$

where X is a random variable that represents the stochastic part of the channel loss. In the specific example of the single vertical column topology shown in Figure 3, we have

$$g_{Thresh}(i,j) = P\left(X + 2.7 \times 10\log_{10}\left(\sqrt{d^2 + h^2 (i-j)^2}\right) < Thresh\right).$$

As was done in the previous section, we can represent node occupancy with a Markov chain. However, while in the previous section the state of the Markov chain was the number of occupied nodes in the cluster, here, the state of the Markov chain is a vector of zeros and ones indicating which nodes are occupied. Thus, a particular element of this state can be represented as an integer between 0 and $2^M - 1$. Specifically, let $a_i = 1$ if the ith node is occupied and $a_i = 0$ otherwise. Then, this state is represented by $A \in \left[0, 2^M - 1\right]$ where $A = \sum_{i=0}^{M-1} a_i 2^i$. Thus, we can define Q_{Thresh} to be the probability transition matrix via

$$Q_{Thresh}\left(A,B\right) := P\left(\text{moving from state } A \text{ to state } B\right) \tag{6}$$
$$= \prod_{\{j:b_j=1\}} \left(1 - \prod_{\{i:a_i=1\}} \left(1 - g_{Thresh}\left(i,j\right)\right)\right) \prod_{\{j:b_j=0\}} \left(\prod_{\{i:a_i=1\}} \left(1 - g_{Thresh}\left(i,j\right)\right)\right).$$

To understand (6), note that the probability of node $(n+1, j)$ not being occupied is the probability that each link from every occupied node in the previous relay-set is closed, which is $\prod_{\{i:a_i=1\}} \left(1 - g_{Thresh}\left(i,j\right)\right)$ where $\{i : a_i = 1\}$ is the set of nodes in the nth relay-set that are occupied. Furthermore, the set of nodes in the $(n+1)$th relay-set that are occupied in state B is $\{j : b_j = 1\}$. Thus, the probability of being in state B is the probability that nodes $\{j : b_j = 1\}$ are occupied and node $\{j : b_j = 0\}$ are not occupied.

Since the source is occupied, we have $P\left(\text{node } \left(0, \lfloor M/2 \rfloor\right) \text{ is occupied}\right) = 1$. Similarly, $P\left(\min MxCL < Thresh\right) = P\left(\text{node } \left(N, \lfloor M/2 \rfloor\right) \text{ is occupied}\right)$. Thus, (6) can be used to compute the average minimum of the MxCL and the average MxCL over the nominal path.

B. Examples of the Exact Impact of Path Loss

As an example of the impact of path loss, consider the Single Vertical Column Topology shown in Figure 3 with the stochastic part of the channel modeled as lognormal shadowing with $\sigma = 11$ dB. Figure 10 (a) shows the average of the minimum MxCL as a function of the number of hops from source to destination for $M = 1, 2, 4, ...14$. Here the columns are 100 m apart (i.e., $d = 100$ m) and the distance between neighboring nodes within a relay-set is $h = 20m$ and $h = 60m$ (see Figure 3 for the definition of h). Note that when $M = 1$, there is only one path from the source to the destination. Thus, as above, the $M = 1$ case corresponds to the case where the diversity is not exploited. The impact of path loss can be detected in Figure 10 (a). When the node spacing is large (e.g., $h = 60$ m), the routes that reach from the source, to

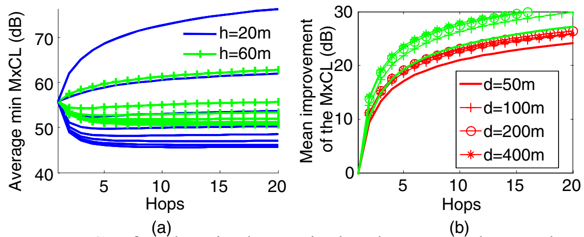

Fig. 10. (a) The average minimum MxCL for the single vertical column topology. The solid curves are for the case when $h = 20m$ and the marked curves are for $h = 60m$. The upper most curve is when there is only one node in each relay-set, i.e., when diversity is not exploited. The curve is the same regardless of the value of h. This figure shows the performance for relay-set sizes of 1, 2, 4, 6, $\cdots$, 14. The best performance is indicated by the lowest curve and corresponds to the case where $M = 14$. (b) The performance improvement (as compare to the geographically shortest path) for different spacings between relay-sets. Two sets of curves are shown. The upper set is for relay-sets with 12 nodes and the lower set is for relay-sets with 6 nodes. Each set of curves consists of four curves corresponding to $d = 50$m to $d = 400$m. The lowest curve in each set is for $d = 50$m. Here $h = 20$ m.

the ends of the column, and then back to the destination must have links with higher path loss than the geographically shorter paths. As a result, such routes are rarely optimal, and hence do not impact the average minimum MxCL. Therefore, the improvement of the MxCL provided by diversity when $h = 60$ m is less than it is when $h = 20$ m.

When the columns in the Single Vertical Column Topology are short compared to the distance between columns, the column topology resembles the cluster topology shown in Figure 1. Hence, in this case, the analysis in Section IV should provide a good approximation. Figure 10 (b) illustrates that the improvement of the MxCL converges as the distance between the columns increases. Figure 10 (b) shows the behavior when there are 6 and 12 nodes in each column and $h = 20$ m. Since a column with 12 nodes is longer than a column with 6 nodes, a larger value of d is required before the improvement of the MxCL converges and ceases to depend on the path loss.

C. An Approximation

In Figure 10, the largest relay-set size is $M = 14$. In this case, there are 2^{14} elements in the state-space and the state transition matrix has 2^{28} elements[4]. With current processors, this

[4]It is possible to compute the probability distributions without computing the entire state transition matrix.

was the largest topology that could be investigated. By assuming that the occupancy of a node is independent of the occupancy of nodes within the same relay-set, a dramatic improvement in computational efficiency can be achieved. Under this assumption, we only need to compute the M dimensional vector of the probability that each node is occupied. It will be shown that this independence assumption provides an upper bound on the MxCL. Furthermore, it will be observed that for large relay-sets, this approximation results in an error in the average minimum MxCL that is less than 1 dB.

First we compute the probability of occupancy under the independence assumption. Let $P_{n,Thresh}(i)$ be the probability that the ith node in the nth relay-set is occupied. Then the probability that node jth in the $(n+1)$th relay-set is occupied is

$$P_{n+1,Thresh}(j) = 1 - \prod_{i=1}^{M} \left((1 - P_{n,Thresh}(i)) + (1 - g_{Thresh}(i,j)) P_{n,Thresh}(i)\right). \tag{7}$$

As in the previous Section, the fact that the source is occupied is expressed by $P_{0,Thresh}(i) = 1$ for $i = \lfloor M/2 \rfloor$ and 0 otherwise. The probability that there exists a path from the source to the destination such that $MxCL < Thresh$ is $P_{N,Thresh}(\lfloor M/2 \rfloor)$.

This approximation yields relationships that are quite similar to those shown in Figure 10. Figure 11 shows the difference between the average minimum MxCL found using (7) and using (6) for the single vertical column topology with $h = 20$ m and $d = 100$ m with lognormal shadowing and $\sigma = 11$ dB. Note that for large networks the error is less than 1 dB and converges as the path length grows. For smaller networks, the error is larger, but for small relay-set sizes, an approximation is not needed since the actual MxCL can be easily computed. Figure 10 also indicates that the error is always positive implying that the approximated average minimum MxCL is always smaller than the actual value. This is always the case as explained next.

Proposition 4: The assumption that the event that a node is occupied is independent of the

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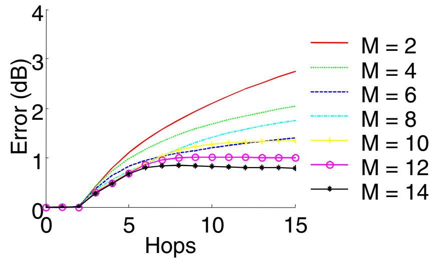

Fig. 11. The difference between the exact value of the average minimum MxCL and the approximated average minimum MxCL, where the approximation assumes that the occupancy of a node in a relay-set is independent of the occupancy of other nodes within the same relay-set. This figure shows this error for the single vertical column topology with 2 - 14 nodes per relay-set and for networks with 2 to 15 hops. Note that an error of 0 dB means that there is no error and an error of 3 dB means that the approximation suggests a performance that is 3 dB better than the actual performance. The results here are for $d = 100m$, $h = 20m$, and with the stochastic part of the channel modeled as lognormal with $\sigma = 11$ dB.

event that other nodes within the same relay-set are occupied results in an upper bound on the probability that a node is occupied.

Proof: Let U_i be the event that node (n, i) is occupied and node i has and open link to node $(n+1, k)$. Then the probability that node $(n+1, k)$ is occupied is $P\left(\bigcup_i U_i\right)$ and

$$P\left(\bigcup_i U_i\right) = \sum_i P\left(U_i\right) - \sum_{i \neq j} P\left(U_i \cap U_j\right).$$

Under the assumption that the events U_i are independent

$$\tilde{P}\left(\bigcup_i U_i\right) = \sum_i P\left(U_i\right) - \sum_{i \neq j} P\left(U_i\right) P\left(U_j\right),$$

where $\tilde{P}$ denotes the probability under the independence assumption. However, since the events U_i are increasing,

$$P\left(U_i \cap U_j\right) \geq P\left(U_i\right) P\left(U_j\right). \tag{8}$$

Thus $\tilde{P}\left(\bigcup_i U_i\right) \geq P\left(\bigcup_i U_i\right)$. For further discussion on increasing events see Chapter 2 of [27]. ∎

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D. Diversity in Different Topologies - Including Path Loss and Neglecting Channel Correlation

By using the approximation (7), it is possible to explore the improvement of the MxCL when diversity is exploited in different topologies. Here, the path loss is included, but the correlation between channels is neglected. Figure 12 shows the relationship between the improvement of the average value of the MxCL when diversity is exploited as compared to the average MxCL over the nominal path (See Section III for the definition of the nominal path). This figure shows the improvement of the MxCL for the Cluster Topology (Figure 1), the Random Topology (Figure 4), the Horizontal Row Topology (Figure 2), and the Vertical Column Topologies (Figure 3) with 1, 3, and 5 columns per relay-set. In all cases, $d = 100$ m and $Range = 100$ m (See Figures 1-4 for the definitions of d and $Range$). In the Vertical Column Topologies, the nodes are uniformly spaced along each column with one node always in the center of each column. Finally, the stochastic part of the channel is modeled with lognormal shadowing with $\sigma = 11$ dB and the path loss exponent, α, is set to 2.7.

Figure 12 shows the performance for $d = Range = 100$. However, the performance is unchanged by scaling. To see this, consider a channel that spans between nodes that are r apart. If the node locations are scaled by s, then the channel under consideration spans between nodes that are $s \times r$ apart. Hence, the path loss becomes $\alpha 10 \log_{10}(s \times r) = \alpha 10 \log_{10}(r) + \alpha 10 \log_{10}(s)$. Thus, scaling the node locations by s results in the constant $\alpha 10 \log_{10}(s)$ being added to the channel loss of each link. However, our focus is on the *improvement* of the MxCL, which is not affected by a constant factor added to all channel losses.

Figure 12 shows that the improvement of the MxCL for the various topologies coincide within 1 dB. Recall that the cluster topology neglects path loss. Figure 12 indicates that for a broad range of topologies, the path loss does not have a significant impact on the improvement of the MxCL, and hence, the behavior of the Cluster Topology provides a good estimate of the improvement of the MxCL.

To gain insight into this behavior, consider the Single Vertical Column Topology and the Cluster Topology. In the Single Column Topology with $Range = d$, the path loss is $\alpha 10 \log_{10}\left(\sqrt{d^2 + s^2 d^2}\right) = \alpha 10 \log_{10}(d) + \alpha 10 \log_{10}\left(\sqrt{1+s^2}\right)$ where $s \in [-1, 1]$. Typically, $\alpha 10 \log_{10}\left(\sqrt{1+s^2}\right)$ is small compared to the random part of the channel. For example, when $\alpha = 2.7$, we have $\alpha 10 \log_{10}\left(\sqrt{1+s^2}\right) \in [0 \text{ dB}, 4.1 \text{ dB}]$, which is small compared to the random part of the channel, which has $\sigma = 11$ dB. Similar comparisons between the Cluster Topology and other topologies show that when $Range = d$, the impact of the topology on the channel is considerably smaller than the random part of the channel, and hence, topology does not have a significant impact on the improvement of the MxCL.

It is important to note that Figure 12 does not imply that the topology never plays a role in the improvement of the MxCL when diversity is exploited. For example, if the nodes within a relay-set are dispersed over a large area (i.e., $Range \gg d$), then the path loss to distant nodes is so large that the diversity will not offer substantial benefits. To examine this behavior more closely, consider Figure 13, which shows the improvement of the MxCL when the ratio $Range/d$ varies from 0.25 to 100 (Note that in Figure 12 $Range/d = 1$.). Figure 13 also shows the behavior for the path loss exponent set to $\alpha = 2.7$ and $\alpha = 6$. Figure 13 shows the case when $d = 100$, however, as explained above, other values of d give the same results.

As expected, when $Range/d$ is large, the improvement of the MxCL is decreased. This decrease is amplified when the path loss exponent is large. Moreover, Figure 13 shows a clear impact of topology, with the random topology resulting in less improvement of the MxCL than the single vertical column topology in all cases except for when $\alpha = 2.7$ and $Range/d \leq 1.5$. Figure 13 also shows that when there are relatively few nodes, then the impact of topology is minor whereas the path loss exponent plays an important role. On the other hand, when there

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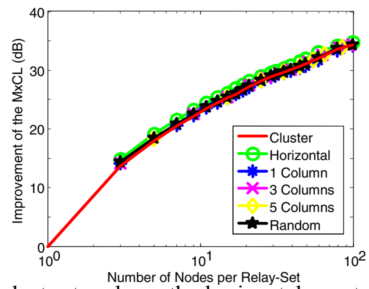

Fig. 12. Improvement of the MxCL for the cluster topology, the horizontal row topology, the random topology, and the vertical column topology with 1, 3 and 5 columns in each relay-set. Here $d = Range = 100$ m and all topologies have 5 hops.

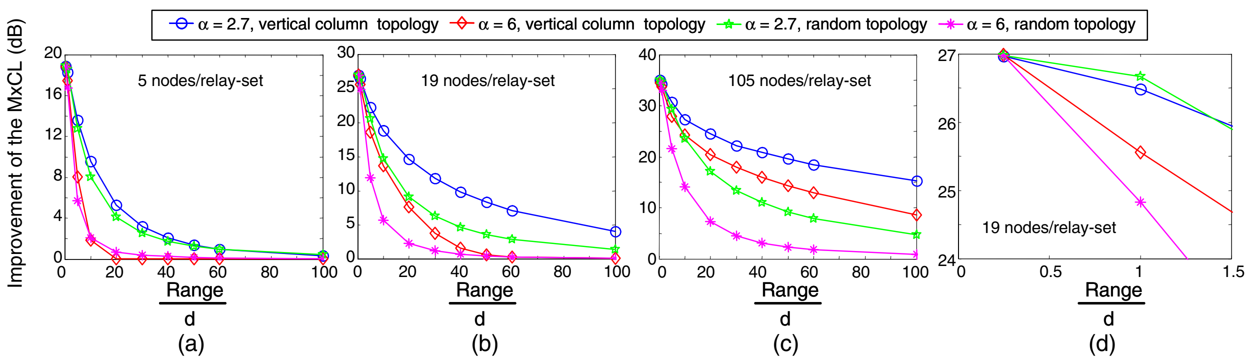

Fig. 13. The impact of path loss. In the above figures, path loss is included and channel correlation is neglected. The Single Vertical Column Topology and the Random Topology are shown. The x-axis is the ratio $Range/d$. See Figures 4 and 3 for a definition of these parameters.

are a very large number of nodes, changing the path loss exponent from 2.7 to 6 has less of an impact than changing from the vertical column topology to the random topology. On the other hand, $Range/d$ has less of an impact on the improvement of the MxCL when there are 105 nodes/relay-set than when there are 5 nodes/relay-set. In conclusion, while the topology and the path loss exponent do have an impact on the improvement of the MxCL, if $Range \approx d$, then this impact is small compared to the total improvement of the MxCL.

VI. Correlated Channels

If the transmitter and receiver of one channel are nearby the transmitter and receiver of another channel, then these channels are correlated. This correlation reduces the diversity of channels, and hence limits the benefits of diversity. The techniques developed above neglect the correlation between channels. Thus, this analysis is only directly applicable when the node density is low.

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However, as shown above, the impact of diversity is greatest when the node density is high. The objective of this section is to determine the degree to which channel correlation limits the impact of diversity. The next section describes the channel model that includes correlation. Then, Section VI-B develops computationally efficient techniques for determining the MxCL when the node density is high. The techniques developed also account for path loss. And finally, Section VI-C explores the impact of channel correlation on diversity.

A. Channel Model

The spatial correlation of channels has been measured in practice and is well modeled [23], [24]. While correlation models have been developed for multipath fading [28], this investigation only focuses on lognormal shadowing. Specifically, letting $L(a,b)$ denote the part of the channel loss, in dB, due to shadowing from a transmitter at location a to a receiver at location b, we have $L(a,b) \sim N(0,\sigma)$. Then, following the approach of [23], [24], the correlation between two links is modeled as

$$E\left(L(a,b)\,L(c,d)\right) = \sigma^2 \exp\left(-\frac{(dist\,(a,c) + dist\,(b,d))}{\rho}\right), \tag{9}$$

where $dist\,(a,c)$ is the distance between location a and c, and ρ is the referred to as the *correlation distance*. Experiments have found that $\rho \approx 10$ m[29].

B. A Reduced Density Approximation

When M is large, simulating correlated channels is computationally complex. Specifically, in order to generate a realization of M^2 channels between two relay-sets, it is necessary to perform Cholesky factorization of the correlation matrix, which has complexity $O(M^6)$. Thus, it is not possible to simulate a network with correlated channels for large values of M. In order to estimate the benefits from diversity, a network with a large number of correlated channels can be approximated by a network with a smaller number of uncorrelated channels. The justification for this approach is as follows.

It is known from Extreme Value Theory that, when normalized correctly, the maximum of a sequence of correlated Gaussian random variables converges to a random variable with Gumbel distribution [30]. Furthermore, it is also known that, when normalized correctly, the maximum of a sequence of uncorrelated Gaussian random variables also converges to a random variable with Gumbel distribution [31]. That is, if the channels are modeled by an independent set of Gaussian random variables or if the channels are modeled as a correlated set of Gaussian random variables, the limiting distribution of the best channel is a Gumbel distribution.

The challenge is then to determine the set of uncorrelated channels that models a given set of correlated channels. The approach is to select the set of uncorrelated channels such that the minimum MxCL over a two-hop network with the uncorrelated channels closely approximates the minimum MxCL of a two-hop network with correlated channels with $\sigma = 11$ dB and $\rho = 10$ m [29]. To this end, define $U_{Cor}\left(Thresh, M, Range\right)$ to be the probability of finding a two-hop path with $MxCL < Thresh$ when the channels are correlated. For example, in the case of the Single Vertical Column Topology

$$
\begin{aligned}
&U_{Cor}\left(Thresh, M, Range\right) \\
&= P\left(\begin{array}{c}\text{there exist a path from node } \left(0, \left\lfloor \frac{M}{2} \right\rfloor\right) \text{ to node } \left(2, \left\lfloor \frac{M}{2} \right\rfloor\right) \text{ with } MxCL < Thresh \\ \text{where the relay-set has } M \text{ nodes uniformly spaced along a single column}\end{array}\right).
\end{aligned}
$$

Since the relay-set has M nodes, this network has two sets of M correlated channels. For moderate values of M, Monte Carlo simulations can be used to estimate U_{Cor}. Next, define $U_{Uncor}\left(Thresh, \tilde{M}, Range, \tilde{\sigma}\right)$ to be the probability that there exists a two-hop path with $MxCL < Thresh$, where the channels are uncorrelated. In the case of the Single Vertical

Column Topology, this path is through a relay-set made of $\tilde{M}$ nodes uniformly spaced along a single vertical column and with the channel losses (in dB) that are normally distributed with mean 0 and standard deviation $\tilde{\sigma}$. Thus

$$\begin{aligned} & U_{Uncor}\left(Thresh, \tilde{M}, Range, \tilde{\sigma}\right) \\ & = 1 - \prod_{j=-0}^{\tilde{M}-1} \left(1 - \Phi\left(Thresh, 2.7 \times 10 \log_{10}\left(\sqrt{d^2 + \left(j\frac{Range}{\tilde{M}-1} - Range/2\right)^2}\right), \tilde{\sigma}\right)^2\right), \end{aligned}$$

where $\Phi\left(Thresh, \mu, \tilde{\sigma}\right)$ is the probability of a Gaussian random variable with mean μ and standard deviation $\tilde{\sigma}$ being less than $Thresh$. Note that since the above refers to the Single Vertical Column Topology, the source and destination are located at a height of $Range/2$, and, in this case, the height of node $(1, j)$ is $j\frac{Range}{\tilde{M}-1}$, hence the distance from node $(1, j)$ to the source and destination is $\sqrt{d^2 + \left(j\frac{Range}{\tilde{M}-1} - Range/2\right)^2}$.

For a general topology,

$$\begin{aligned} & U_{Uncor}\left(Thresh, \tilde{M}, Range, \tilde{\sigma}\right) \\ & = 1 - \prod_{j=-0}^{\tilde{M}} \left(1 - \Phi\left(Thresh, D\left(\text{Source}, j\right), \tilde{\sigma}\right)\Phi\left(Thresh, D\left(j, \text{Dest}\right), \tilde{\sigma}\right)\right), \end{aligned}$$

where $D\left(\text{Source}, j\right)$ and $D\left(\text{Dest}, j\right)$ are the distances from the source to node $(1, j)$ and from node $(1, j)$ to the destination, respectively, and where the nodes are spaced according to the topology type. In the case of the Random Topology, U_{cor} and U_{Uncor} can be found by averaging over many random topologies.

To determine the number of nodes, $\tilde{M}$, and standard deviation $\tilde{\sigma}$, we solve the following optimization problem

$$\min_{\tilde{M}, \tilde{\sigma}} \max_{Thresh \in [0, \infty]} \left| U_{Uncor}\left(Thresh, \tilde{M}, Range, \tilde{\sigma}\right) - U_{Cor}\left(Thresh, M, Range\right)\right|.$$

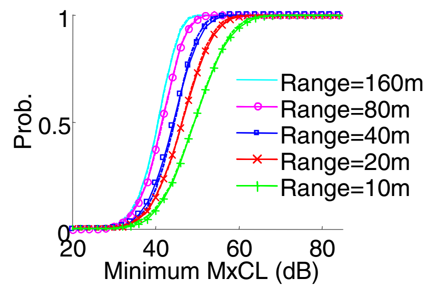

Fig. 14. Two-Hop Performance - Correlated and Fitted Uncorrelated. The above shows the two-hop performance for the single vertical column topology for several ranges of relay-sets. The solid lines show the performance for correlated channels while the dashed lines show the two-hop performance for fitted uncorrelated channels. For small relay-set sizes, some difference between the correlated channel and uncorrelated fit can be detected. However, for larger ranges, the solid and dashed lines are exactly on top of each other. In this example, the nodes are spaced 0.15 m apart and $\rho = 10$ m.

Figure 14 shows U_{Corr} and the best U_{Uncorr} for the Single Vertical Column Topology with various values of $Range$ and with $M = Range/0.15$ m.

C. Improvement of the MxCL When Diversity is Exploited in Dense Networks - Including Path Loss and Channel Correlation

Once the optimal values of $\tilde{M}$ and $\tilde{\sigma}$ are found, a dense relay-set can be approximated with a less dense set, and it is computationally tractable to determine the MxCL following the same approach discussed in Section V-C. By employing these computational techniques, this section examines the impact of channel correlation.

Figure 15 shows the improvement of the MxCL for the Single Vertical Column Topology and the Random Topology for different values of the correlation distance ρ and the path loss exponent α, but with $d = 100$ m. Figure 15 also shows the improvement of the MxCL for the Cluster Topology, which ignores path loss and channel correlation. In Section V-D it was found that if $d = Range$, then the path loss does not impact the MxCL, i.e., the Cluster Topology results in the same MxCL as other topologies. Here, when $d = Range$ (i.e., Figures 15 (a), (c), (d), and (f)), we find that when the node density is low and/or ρ is small, then the impact of channel correlation is insignificant, that is, the improvement of the MxCL matches

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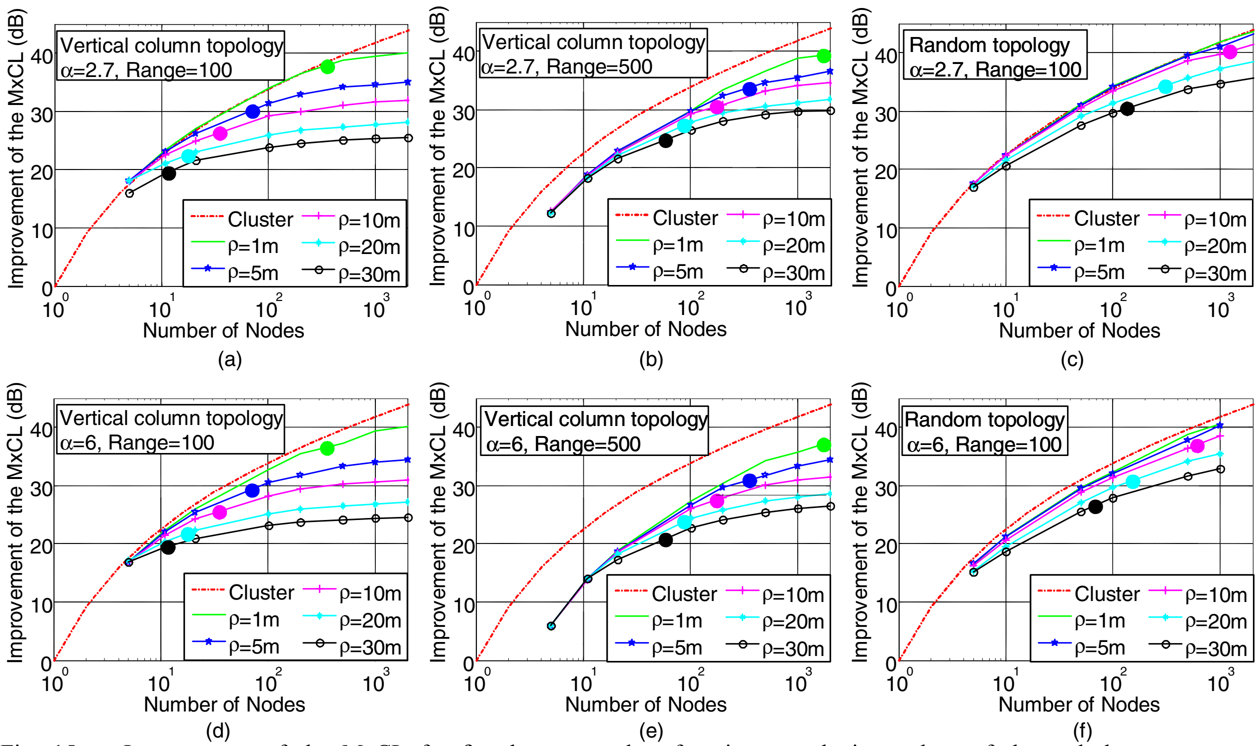

Fig. 15. Improvement of the MxCL for five hop networks of various topologies, values of the path loss exponent, α, and values of the correlation length ρ. The large disks are the points where the numbers of nodes is such that $\exp\left(-\texttt{Minimum}\text{ Internode Distance}/\rho\right) = 0.75$. Also shown is the improvement of the MxCL for the Cluster Topology.

that of the Cluster Topology. However, as the node density increases, the impact of the channel correlation becomes significant. To quantify the impact of channel correlation, we define the `Minimum Internode Distance`. In the case of the Single Vertical Column Topology, let `Minimum Internode Distance`$:= Range/$Number of Nodes, and for the Random Topology, let `Minimum Internode Distance`$:= \sqrt{Range \times d/\text{Number of Nodes}}$. Obviously, as the `Minimum Internode Distance` decreases, the channel correlation becomes more significant. Let $NumNodes^{*}\left(\rho\right)$ be the number of nodes such that $\exp\left(-\texttt{Minimum Internode Distance}/\rho\right) = 0.75$. The disks in Figure 15 indicate $NumNodes^{*}\left(\rho\right)$ and the corresponding improvement of the MxCL. Table I shows the difference between the improvement of the MxCL for $NumNodes^{*}\left(\rho\right)$ when the correlation is ρ and the improvement of the MxCL when there is no correlation (i.e., $\rho = 0$), but there is path loss. Table I shows that the correlation reduces the MxCL by roughly

Topology	Range	α	$\rho = 1$	$\rho = 5$	$\rho = 10$	$\rho = 20$	$\rho = 30$
Single Vertical Column	100	2.7	0.5	2.2	2.9	3.3	3.8
Single Vertical Column	100	6	-	1.9	2.7	3.1	3.2
Single Vertical Column	500	2.7	-	1.5	1.7	1.85	2.5
Single Vertical Column	500	6	-	1.8	2.3	2.6	3.5
Random	100	2.7	-	-	2.1	3.6	4.6
Random	100	6	-	-	2.3	3.1	4.2

TABLE I

THE DIFFERENCE IN THE IMPROVEMENT OF THE MXCL FOR $NumNodes^*(\rho)$ NODES WHEN THE CHANNELS ARE CORRELATED AND THE IMPROVEMENT IN THE MXCL WHEN THE CHANNELS ARE UNCORRELATED. THE MISSING VALUES WERE BEYOND OUR COMPUTATIONAL ABILITIES.

3 dB when the number of nodes is $NumNodes^*(\rho)$. This rough estimate holds for different topologies and for different values of α and ρ.

In summary, the variation of the improvement of the MxCL for different topologies when ρ is small (i.e., when there is little channel correlation) can be predicted from Figure 13. The point where the node density is high enough that channel correlation is significant is predicted by $NumNodes^*(\rho)$. However, Figure 15 only shows the impact of channel correlation for $d =$ 100m. In Section V-D (where path loss was considered, but channel correlation was neglected), the improvement of the MxCL remained constant if the network was scaled, that is, if there exists an s such that a node located at point (x, y) is moved to $(x', y') = (s \times x, s \times y)$, then the improvement of the MxCL is unchanged. However, if the channels are correlated, then from (9), we see that the improvement of the MxCL is unchanged by scaling the network if the correlation length is scaled to $\rho' = \rho/s$. While it is possible to scale a network by physically moving nodes, it is not possible to control the correlation distance. Hence, in order to estimate the improvement of the MxCL under network scaling, other values of ρ must be considered. Figure 15 and Table I, which indicate the performance for a wide range of ρ, can be used for this purpose.

Remark 5: Figure 15 shows huge improvements of the MxCL when $\rho = 10$ m (the value found from experiments) and the number of nodes is large. It is important to note that such

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improvements are dependent on the upper tail of the lognormal distribution. While lognormal shadowing has been widely verified, further work is required to validate that the upper tail obeys the lognormal distribution.

VII. Conclusions

The improvement of the MxCL along a route provided by diversity in a multihop network has been examined. First, a simple topology was examined where the impact of path loss and channel correlation was neglected. In this case, a computationally efficient technique to determined the performance is provided. Furthermore, in this case it is shown that, if the statistics of links are held constant, then, in some settings, the MxCL improves as the path grows longer. Then the impacts of path loss and channel correlation are examined. It is found, for example, that in the random topology with a moderate number of nodes, path loss and channel correlation do not have a significant impact on performance.

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