ORIGINAL RESEARCH

# Chaotic Modeling in Network Spinal Analysis: Nonlinear Canonical Correlation with Alternating Conditional Expectation (ACE): A Preliminary Report.[1]

*Stephan Bohacek,[2] and Edmund Jonckheere, Ph.D.[2]*

**Abstract** — This paper presents a preliminary non-linear mathematical analysis of surface electromyographic (sEMG) signals from a subject receiving Network Spinal Analysis (NSA). The unfiltered sEMG data was collected over a bandwidth of 10-500 Hz and stored on a PC compatible computer. Electrodes were placed at the level of C1/C2, T6, L5, and S2 and voltage signals were recorded during the periods in which the patient was experiencing the "somatopsychic" wave, characteristic of NSA care. The intent of the preliminary study was to initiate mathematical characterization of the wave phenomenon relative to its "chaotic," and/or nonlinear nature. In the present study the linear and nonlinear Canonical Correlation Analyses (CCA) have been used. The latter, nonlinear CCA, is coupled to specific implementation referred to as Alternating Conditional Expectation (ACE). Preliminary findings obtained by comparing canonical correlation coefficients (CCC's) indicate that the ACE nonlinear functions of the sEMG waveform data lead to a smaller expected prediction error than if linear functions are used. In particular, the preliminary observations of larger nonlinear CCC's compared to linear CCC's indicate that there is some nonlinearity in the data representing the "somatopsychic" waveform. Further analysis of linear and nonlinear predictors indicates that 4th order nonlinear predictors perform 20 % better than linear predictors, and 10th order nonlinear predictors perform 30% better than linear predictors. This suggests that the waveform possesses a nonlinear "attractor" with a dimension between 4 and 10. Continued refinement of the ACE algorithm to allow for detection of more nonlinear distortions is expected to further clarify the extent to which the sEMG signal associated with the "somatopsychic" waveform of NSA is differentiated as nonlinear as opposed to random.

## Introduction

Over the past ten years there has been growing interest in modeling experimentally observed time series as nonlinear deterministic or possibly chaotic dynamical processes.[1] For example, the analysis of economic time series[2] (prediction of the stock market), the analysis of geophysical seismographic time series (prediction of earthquakes), and most recently, the virtual explosion of applications related to the analysis of physiological time series[3] (dynamical analysis of heartbeats to predict fibrillation, dynamical analysis of brain waves to predict epileptic seizure, etc.) reflect the wide variety of interests in this area.

The present research fits within this last trend, with the exception that, while many physiological time series have been deemed chaotic by some intuitive criteria, it is the intent here to apply a more rigorous mathematical evaluation of the presence of chaos in the very specific sEMG signal recorded during the "somatopsychic" waveform, characteristic of Network Spinal Analysis (NSA).[4] The "somatopsychic" waveform being studied is observed to undulate primarily between the sacrum and cervical areas of the spine, over a range of amplitudes and frequencies. Whereas these parameters do not appear sequentially predictive in any one subject, there does appear to be a commonality of the distribution of amplitudes and frequencies among the larger population of subjects expressing the "somatopsychic" wave. The present preliminary study presents information acquired to evaluate the nature of this specific waveform as to its linear versus nonlinear character. Since the waveform associated with NSA care has a neuromuscular component, it lends well to measurement by sEMG. Subsequently, the sEMG signals lend well to mathematical analysis.

1. The study was funded by an award from the Association for Network Chiropractic, Inc., Longmont, CO. to the University of Southern California, Dept. of Electrical Engineering - Systems.

2. Department of Electrical Engineering – Systems, University of Southern California, Los Angeles, CA 90089-2563.

*Requests for reprints:* Requests should be made to Stephen Bohacek, Department of Electrical Engineering – Systems. University of Southern California, Los Angeles, CA 90089-2563.

In the present study, caution is being exercised to avoid incorrectly classifying the sEMG signal as nonlinear. This has been reported as a potential problem with the false near neighbor (FNN) approach, as purely random signal has been incorrectly classified as deterministic by this approach.[5] More generally, while the FNN test works well for low order systems, it is not clear whether it works well for systems of higher order.[5] This cautious approach is a reflection of concerns regarding the analysis of chaos in biomedical data, in general. This recent attitude has been triggered by some doubts that have arisen relative to the benchmark study allegedly indicating that the timing between heartbeats is chaotic.[6,7] A point of consideration is that there are many factors that affect heart beats; i.e., psychological factors, breathing, stress associated with being hooked up to an apparatus, etc. There is also some concern that these extraneous factors might have influenced the signal leading to misinterpretation as chaos in the data analysis.

This is not to say that there is no chaos associated with the heart rhythm, but there is some doubt as to whether the data has been correctly collected or analyzed. Moreover, adding to the complexity of evaluating the chaotic nature of heart rhythm, it can also be argued that the heart is naturally subject to psychological and other physiological factors, and that it is hard to conceive heartbeats outside the human body.

While the heart is subject to some voluntary control via breathing, the NSA wave, on the other hand, does not appear to be under any known voluntary control. In fact, if the recipient were to try to interfere with the wave, it would simply cease rather than being modified by voluntary control. Thus, the absence of voluntary control over the NSA sEMG wave signal compared to the considerable voluntary control of factors affecting the electrocardiogram (EKG) signal, renders the former less susceptible to extraneous factors contributing to its incorrect classification.

There has been some dynamical analysis of EEG and EMG signals.[8,9] However, the novelty in the present study is the specificity of the sEMG data, being collected during the administration of NSA care at times when the subject was experiencing the "somatopsychic" wave. Since no previous analysis of this data has been performed, it is of interest to compare these findings with those pertaining to EMG data collected during other physiological events, some of which have been classified as chaotic in nature. Thus, information gained from this on-going study of the "somatopsychic" waveform associated with NSA care is anticipated to be of value to the larger scientific community engaged in the investigation of the chaotic nature of other less repeatable physiological systems. Data derived from this study will also be used to profile any changes which might occur in the sEMG signal patterns relative to different Levels of Care described in the clinical application of NSA.[4] This growing body of information is expected to enhance the clinical application of NSA, as well as serving to further elucidate its underlying mechanisms of action.

## Methods

### *Data Acquisition*

The analyses presented in this preliminary study are based on sEMG data collected during a series of sessions in which a subject, who granted consent to participate in the study, was administered NSA care. The study was conducted in the private office setting with NSA care provided by a certified NSA practitioner.[4]

The data obtained was in the form of raw (unfiltered) sEMG signals in micro-volts. The EMG apparatus, an Insight 7000 (EMG Consultants, 255 W Spring Valley Ave., Maywood, NJ 07607) was modified to research standards by accepting data over a bandwidth of 25 to 500 HZ. Electrodes were applied at the levels of C1/C2, T6, L5 and S2, following the application protocol recommended by EMG Consultants.

The sEMG signal was transported directly to disc and retrieved for analysis on a 330MHz PC compatible computer and HP workstation.

The application of the nonlinear Canonical Correlation Analysis (CCA) was applied through the implementation of a specific algorithm referred to as Alternating Conditional Expectation (ACE).[10]

## Analyses

Two approaches have been utilized in the initial analysis to ascertain if any deterministic nonlinear processes are involved in the sEMG signal. These include the False Near Neighbor approach (FNN),[1] and the linear and nonlinear Canonical Correlation Analyses (CCA), with accompanying implementation of the Alternating Conditional Expectation (ACE) algorithm.[11]

The FNN was utilized initially due to its popularity, although its relevance has been brought into question especially as applied to EMG signals. The CCA, alternatively, is a fundamental tool used by statisticians which has appeared reliable on EMG data.[12] To date, in the present study, this approach has been applied qualitatively. The analysis has centered on the question of whether or not there is some nonlinear dynamical processes in the sEMG signal under study.

The nonlinear CCA has been implemented using two approaches. The first used low order polynomials as simplified models of the nonlinear distortion whereas the second approach used the well-known Alternating Conditional Expectation (ACE). The ACE method provides very explicitly the nonlinear functions that are generating the data. With the ACE method, it is possible to establish the shape of the attractor as well as its topology, etc.

<u>Canonical Variate Analysis</u>

The canonical correlation analysis of an experimentally observed signal

$$\{X(k): k=1, 2, \ldots, K\}$$

is a technique to detect the dynamics involved in the signal and to answer such questions as: "Is this signal random?" "Is it correlated to other factors?" "Are there some deterministic features despite its noisy manifestation?" Hotelling first introduced the linear canonical correlation analysis in biomedical data processing context in 1936.[12]

The canonical correlation analysis has both linear and nonlinear applications. The linear application answers the question, "Can $\{X(k)\}$ be modeled by a linear dynamical process driven by some noise?" In particular, what are the deterministic matrices *F, G, H* and *E* such that

$$\xi(k+1)=F\xi(k)+Gw(k)$$
$$X(k)=H\xi(k)+Ew(k) \qquad (1)$$

where $w(k)$ is considered a "white noise" process, that is, the $w(k)$'s are statistically independent. Here, *E,F, G* and *H* represent the linear and deterministic part of the sequence, while the nonlinear and random parts are combined into *w*. Clearly, the smaller $Gw(k)$ and $Ew(k)$ are, the easier it is to predict $\xi(k)$ and, therefore, X $(k)$. The variable $\xi(k)$ is called the *state;* it is that part of the past $\{\ldots, X(k-2), X(k-1)\}$ necessary to predict the future $\{X(k), X(k+1), \ldots\}$. The dimension of the *state* is known as the *order* of the system. Before any analysis is performed the *order* of the system must be found or estimated. Furthermore, any finite sequence can be approximated by a system of the form (1) with arbitrarily small *w*. However, to produce such an approximation, the *order* of the system may need to be very large. Hence, the goal is to produce a low order model for which $Gw(k)$ and $Ew(k)$ are small.

Alternatively, the nonlinear canonical correlation analysis answers such questions as, "Can $\{X(k)\}$ be modeled as a nonlinear system driven by white noise?" To be specific, what are the deterministic functions *f, g, h* and *e* such that

$$\xi(k+1)=f(\xi(k))+g(w(k))$$
$$X(k)=h(\xi(k))+e(w(k)) \qquad (2)$$

where $w(k)$ is a white noise process? If the underlying system that produced the observed sequence $\{X(k)\}$ is nonlinear, then models of the form (2) will have smaller noise terms $g(w(k))$ and $e(w(k))$ than the corresponding linear model (1). The difficulty with nonlinear analysis is that it is much more involved than the linear one, and to date only simplified versions of the nonlinear canonical correlation analysis are used.

Linear Canonical Correlation Analysis

The linear analysis proceeds as follows: At times, $k$ defines the $L$ (where $L$ stands for "lag") past observations

$$X_-(k)=(X(k),\ldots,X(k-L+1))$$

and the L future observations

$$X_+(k)=(X(k+1),\ldots,X(k+L)).$$

Define the correlation between the past and the future to be the $L \times L$ matrix

$$C_{-+}=\frac{1}{K-2L+1}\sum_{k=L}^{k-L} X_-(k)^T X_+(k)$$

Define the autocorrelation of the past as

$$C_{--}=\frac{1}{K-L+1}\sum_{k=L}^{K} X_-(k)^T X_-(k)$$

and that of the future

$$C_{++}=\frac{1}{K-L+1}\sum_{k=0}^{K-L} X_+(k)^T X_+(k).$$

Define the Cholesky decomposition of $C_{--}$ and $C_{++}$,

$$C_{--}=T_-^T T_-$$
$$C_{++}=T_+^T T_+$$

where the T's are lower triangular matrices.

The canonical correlation matrix C is defined as

$$C := T_-^{-T} C_{-+} T_+^{-1}.$$

This matrix can be decomposed as

$$C=U\Sigma V$$

where U and V are orthogonal matrices and

$$\Sigma=\begin{bmatrix}\sigma_1 & 0 & \ldots & 0\\ 0 & \sigma_2 & \ldots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \ldots & \sigma_L\end{bmatrix}$$

where the $\sigma_i$'s are called canonical correlation coefficients and are ordered so that

$$1 \geq \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_L \geq 0$$

These coefficients are the crucial numerical values in the canonical correlation analysis and are providing much information about the sequence $\{X(k)\}$.

A first interpretation is that the predictability of the future of a sequence given its past (hence a measure of its deterministic as opposed to random aspect) is the past/future mutual information

$$-\frac{1}{2}\log\det(I-\Sigma^2).$$

In practice, no matter how large $L$ is, the canonical correlation coefficients have a break point,

$$\sigma_1 \geq \ldots \geq \sigma_D >> \sigma_{D+1} \geq \ldots \geq \sigma_L.$$

Hence, it can be concluded that only the first D coefficients are important. The rationale for ignoring the tail coefficients $0.1 \approx \sigma_{D+1} \geq \ldots \geq \sigma_L \geq 0$ is that they represent instrumentation and other noises and numerical rounding errors and are too unreliable to take into consideration. Therefore, a $D^{th}$ order model of the form (1) would perform nearly as well as a $L^{th}$ order model.

If the canonical coefficients $\sigma_1 \geq \ldots \geq \sigma_D$ are "large" i.e., close to 1, then the sequence $\{X(k)\}$ is nearly linear and deterministic; that is, it can be accurately modeled with a system of the form (1) with $w(k) = 0$. On the other hand, if the coefficients are small, but not too close to zero, then the sequence is either random or nonlinear; that is, the linear model of the form (1) would have large $Gw(k)$ and $Ew(k)$ terms.

Nonlinear Canonical Variate Analysis

The nonlinear canonical variate analysis[13] is an extension of the linear canonical correlation analysis. If a system is nonlinear, then the canonical correlation coefficients (CCC's) are larger for the nonlinear analysis compared to the linear analysis. Indeed, an increase in the nonlinear CCC's as compared to the linear CCC's is an indicator of nonlinearity. Such an increase is apparent for the sEMG signal of the "somatopsychic" waveform characteristic of NSA care. It can, therefore, be concluded from the present preliminary information, that some type of nonlinearity is present in the sEMG signal.

To perform a complete nonlinear canonical variate analysis (CVA) is rather complex. Instead, the correlation is found only for simple "nonlinearities." In particular, the nonlinear CVA requires the discovery of vector valued functions Φ and Θ such that

$$-\frac{1}{2}\, log\ det\ (I-\Sigma_{\Phi(X_-),\Theta(X_+)})$$

is maximized, where $\Sigma_{\Phi(X_-),\Theta(X_+)}$ denotes the canonical correlation matrix of the signals $\Phi(X_-)$ and $\Theta(X_+)$. This is equivalent to choosing Φ and Θ such that the mean square prediction error

$$E\|\Phi([X(k),\ldots,X(k-L+1)]) - \Theta([X(k+1),\ldots,X(k+L)])\|^2) \qquad (3)$$

is minimized where L is the number of "lags" of the system. As a simplification, the functions Φ are restricted to be simple polynomials of the form

$$\Phi([X(k),\ldots,X(k-L+1)]) = \sum_{i=0}^{L}\alpha_i X(k\text{-}i) + \beta_i X(k-i)X(k-i) + \gamma_i X(k-i)X(k-i-1). \qquad (4)$$

Furthermore, Θ is restricted to be linear, i.e.,

$$\Theta([X(k+1),\ldots,X(k+L)]) = \Sigma\alpha^{+}{}_i X(k+i).$$

Since not all possible functions are allowed, the minimum of (3) might not be achieved. However, (3) might be smaller than in the case where only linear functions are used. Note that if $\beta=\wp=0$ then $\Phi$ is linear, hence the nonlinear CVA encompasses the linear CVA. If the $\Phi$ that minimizes (3) is such that β and ℘ are non-zero, then it can be concluded that the future $\{X(k+1),\ldots,X(k+L)\}$ is related to some nonlinear function of the past $\{X(k),\ldots,X(k-L+1)\}$.

Ace Method

In the continued investigation regarding whether the sEMG signal is due to a nonlinear system as opposed to a random system, it has been found prudent to search for the system generating the data and to evaluate the extent to which it is nonlinear. One reason for not applying tests for nonlinearity (e.g. FNN) is that it is very hard to determine if the results are an artifact of the test or a real product of the data. Furthermore, such difficulties seem to occur with most, if not all, tests for nonlinearity. Alternatively, estimating the nonlinear system has clear results. If one can construct a low order nonlinear system such that its output is the same as the sEMG signal, then clearly the sEMG signal is generated by a low order nonlinear system. If the output of the constructed nonlinear system is the same as the sEMG signal, allowing for some small noise, then the sEMG is mostly nonlinear. In this way, one can precisely gage to what extent the sEMG signal is nonlinear.

Another advantage of directly identifying the system generating the sEMG signal is that a linear system can easily be determined which reflects the best linear system generating the sEMG signal. This linear system then provides a benchmark to be compared with nonlinear systems. However, the disadvantage of identifying the system directly, and the reason why so many tests for nonlinearity have been developed, is that it is computationally difficult to construct these nonlinear systems.

The objective of ACE is to search for a possibly nonlinear system, likely driven by small residual "white noises," that is generating the sEMG signal. In particular, the objective is to find *state* variables $\xi_i, i \leq N$, and possibly nonlinear functions *f, g, h* and *e* such that

$$\begin{bmatrix}\xi_1\ (k+1)\\ \xi_2\ (k+1)\\ \xi_3\ (k+1)\\ \vdots\\ \xi_N\ (k+1)\end{bmatrix} = \begin{bmatrix} f(\xi_1(k),\xi_2(k),\ldots,\xi_N(k))\\ \xi_1(k)\\ \xi_2(k)\\ \vdots\\ \xi_{N-1}(k)\end{bmatrix} + \begin{bmatrix} g(w(k))\\ 0\\ 0\\ \vdots\\ 0\end{bmatrix}$$

$$X(k)=h(\xi(k))+e(w(k))$$

where w is a "white noise", $X$ is the sEMG signal and N is typically the order D where the sequence of nonlinear CCC's shows a break.

A particular approach is to assume that f is linear and h is the projection of the first factor. That is,

$$f(\xi_1(k),\xi_2(k),\ldots,\xi_N(k)) = \sum_{i=1}^{N}\alpha_{i-1}\xi_i(k)$$

and $h(\xi)=\xi_1$. Hence, $\xi_i(k)=X(k-i+1)$. In this case, the sEMG signal is modeled by the Auto Regressive (AR) model

$$X(k+1)=\sum_{i=0}^{N-1}\alpha_i X(k-i). \qquad (5)$$

Such linear predictors are rather simple to determine and provide comparison benchmarks for all other predictors.

Nonlinear predictors are very hard to construct. A well-established approach is the Alternating Conditional Expectation (ACE) algorithm.[11] Although this algorithm does not construct the most general type of nonlinear predictor, it does generate a large class of predictors.

It is assumed that mean $(X) = 0$. The objective of the $N^{th}$ order best linear predictor is to find coefficients $\alpha_i$ that minimize

$$E(\alpha_0 X(k)+\alpha_1 X(k-1)+\alpha_2 X(k-2)+\ldots+\alpha_{N-1}X(k-N+1)-Y(k+1))^2$$

where $Y(k+1)=X(k+1)/\ \text{var}(X)$ and var(X) is the variance of X. The ACE algorithm generalizes this by, instead of searching for coefficients $\alpha_1$, searching for functions $\phi_i$ and $\theta$ (see Figures 2-4) with $\|\theta\|_{L2} = 1$ so as to minimize

$$E(\phi_0(X(k))+\phi_1(X(k-1))+\phi_2(X(k-2))+\ldots$$
$$+\phi_{N-1}(X(k-N+1))-\Theta(Y(k+1)))^2. \quad (6)$$

To link this to the previous nonlinear CCA, the above prediction error is "small" if the nonlinear canonical correlation coefficients between the sets of variables

$$\{X(k),X(k-1),\ldots X(k-N+1)\},\{Y(k+1),Y(k+2),\ldots,Y(k+N)\}$$

are "large," i.e., close to 1.

Instead of searching over all nonlinear distortions of the above two sets of variables, a search is done over a certain class of smooth functions. This class includes linear functions; thus, the ACE algorithm is more general than the best linear predictor and will have a prediction error no greater than that of the best linear predictor. Furthermore, if the ACE algorithm results in a prediction error that is smaller than the prediction error due to the best linear prediction error, it can be assumed that the process generating X is nonlinear.

However, if the ACE algorithm fails to do better than the best linear predictor, it cannot be assumed that the process is linear. Indeed, there are many nonlinear functions that may be generating X and that are not incorporated in the ACE algorithm. For example, functions of the general form

$$f(X(k),X(k-1),\ldots)=\sum_{i,j}\gamma_{i,j}X(k-i)\ln(X(k-j))$$

are not necessarily considered by the ACE algorithm. In particular, only functions of the form

$$\theta^{-1}(\phi_0(X(k))+\phi_1(X(k-1))+\ldots+\phi_{N-1}(X(k-N+1))) \quad (7)$$

are considered. While this is a large class of functions, it does not generate all possible functions. Furthermore, generating these nonlinear functions $\phi_1$ and $\theta$ takes considerable computational effort. Thus, it is difficult to check high order functions.

Since the ACE algorithm generates functions of a single variable, these functions can easily be plotted. The best linear predictor can be viewed as also generating functions $\phi_i$, but the $\phi_i$'s are restricted to be linear. Thus, not only the prediction errors can be compared, but also the shape of the linear functions generated by the best linear predictor and the possibly nonlinear functions generated by the ACE algorithm can be compared.

## Preliminary Results

### *Canonical Correlation Analysis*

By observing the variation in the linear CCC's compared to the nonlinear CCC's it can be determined whether the nonlinear functions of the form (4) lead to a smaller expected prediction error (3) than if linear functions are used. In particular, if the nonlinear CCC's are larger than the linear CCC's it can be concluded that there is some nonlinearity present in the system. Figure 1 shows the linear and nonlinear CCC's of burst of sEMG activity labeled as Burst C, a particular burst of sEMG activity. The information presented in this figure indicates the presence of some nonlinearity in the sEMG data collected during the NSA "somatopsychic" wave.

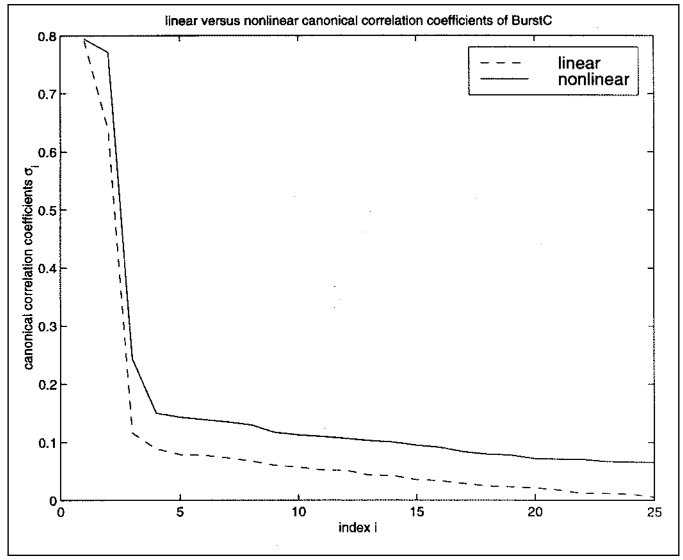


**Figure 1. The linear and nonlinear CCC's for the sEMG of the waveform associated with NSA are shown. The key difference between the plots is that the second coefficient is much larger in the case of the nonlinear canonical correlation. This is an indicator that some nonlinearity is present.**

### *The ACE Method*

The first tests were on a particular burst of sEMG activity labeled as "Burst C." Fourth order predictors (N=4) were computed. The mean square prediction error for the best linear predictor was 0.42, whereas the prediction error for the ACE method was 0.33. This implies that the ACE method predicts about 20% better than linear prediction.

Tenth order predictors (N=10) were also found. The results were similar to the fourth order predictors. The linear prediction error was 0.41 and the ACE prediction error was 0.29 - a 30% improvement. Furthermore, linear prediction did not improve as the order increased, whereas the ACE method appeared to be improving. However, the improvement was slow. These findings corroborate the assumptions of the simplified nonlinear CCA. Specifically, both approaches indicate that the dimension of the attractor is somewhere between 4 and 10.

There is no direct relationship between high order systems and complexity. For example, the chaotic Lorenz map, which is related to the weather, is only of order 3, and is highly complicated. However, systems of high order (e.g. the stock market) also tend to have complicated dynamics. Modeling high order systems is very difficult. Clearly, the waveforms of the NSA sEMG data are complicated. It is hoped that the order of the system that describes the waveforms will be relatively low order. Since the order 10 predictor performs well, it appears that a significant part of the waveform is generated by a low order nonlinear dynamical system.

Figures 2-4 depict the nonlinear functions $\phi_i$ with $i$=1,…,10 and $\theta$ generated by a tenth order ACE algorithm and the linear functions generated by the best linear predictor. The figures reveal that some of the functions, $\phi_1$, $\phi_2$ and $\theta$ are nearly linear with some saturation. Additionally, functions $\phi_1$ and $\phi_2$ appear to be the most significant since they range from -2 to 2 and -5 to 5, respectively. The other functions $\phi_1$ with $i$=2,…,10 appear very nonlinear. These plots also include the linear functions generated by the best linear predictor. Note that the linear trend of the nonlinear functions roughly coincides with these linear functions. This similarity is strong for the most significant functions $\phi_9$ and $\phi_{10}$.

## Discussion and Tentative Conclusions

Further study will be necessary to clarify the extent to which the sEMG signal associated with the waveforms of NSA is due to a nonlinear versus random source. The findings that the fourth order ACE predictor performs 20% better than the fourth order linear predictor and the tenth order ACE predictor performs 30% better than the tenth order linear predictor are encouraging. Moreover, the structure of the prediction error will require further investigation. The histograms of the linear and nonlinear prediction errors look very similar (Figure 5). It is known that the maximum of the canonical correlation between two sets of variables is achieved when the nonlinearly distorted variables are jointly Gaussian.[14] Hence, it appears that to insure that the best possible nonlinear predictor has been developed, the histogram of the prediction error should be Gaussian. However, it is not clear at this point in the study whether a good, but not quite optimal, predictor would yield a nearly Gaussian error.

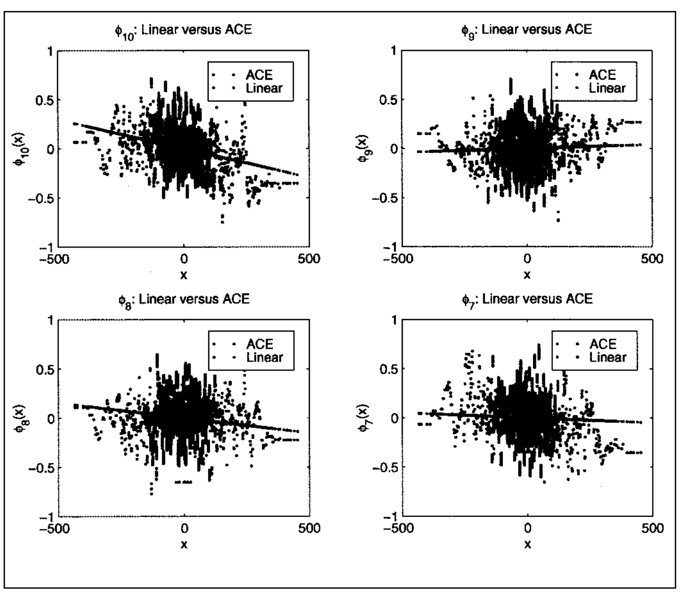


**Figure 2. The nonlinear functions $\phi_1$ for $7 \leq i \leq 10$, generated by the ACE algorithm as defined by equation (6) are shown. For comparison, the linear functions used in the least mean square linear predictor (5) are shown. Note that the functions generated from the sEMG data by the ACE algorithm are nonlinear.**

It is suspected that the following features of the ACE procedure may have to be refined to provide the clearest analysis:

1. The function $\theta$ has been introduced by statisticians to allow for more freedom in the modeling and to link the ACE algorithm to the nonlinear canonical correlation analysis. It is customary to invert the function $\theta$ to obtain $Y(k+1)$ as a nonlinear function of the past as in equation (7). However, applied to the present sEMG signal, $\theta$ appears to saturate and

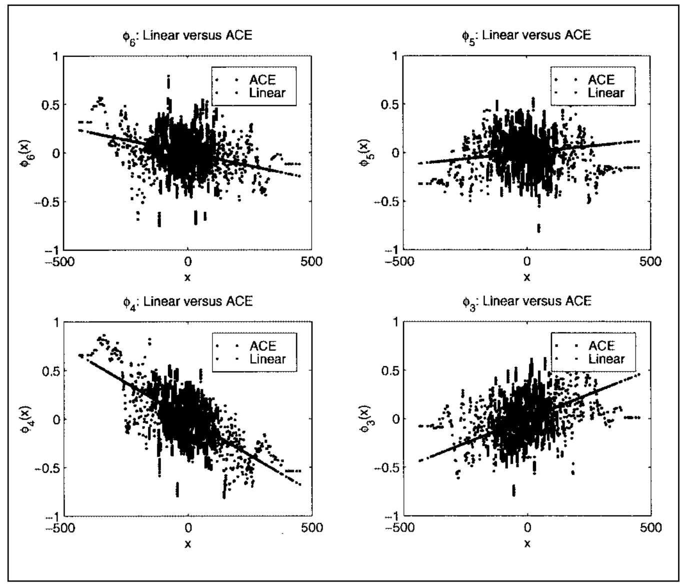


**Figure 3. More nonlinear functions, $\phi_1$ for $3 \leq i \leq 7$, are shown along with their linear counterparts. As in Figure 2, the functions generated from the sEMG data by the ACE algorithm are nonlinear.**

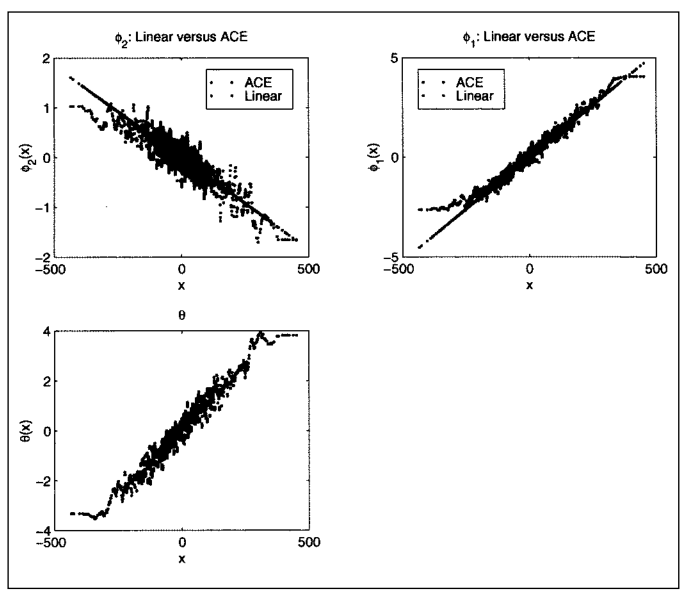


**Figure 4. The nonlinear functions $\phi_1$, $\phi_2$ and $\theta$ are shown. These functions are used for nonlinear prediction so as to minimize (6). The linear functions used in the best linear predictor (5) are also shown. Note that these functions follow the strong linear trend. However, there is sufficient deviation from the linear trend to lower the prediction error from the linear case of 0.41 to the nonlinear case of 0.29.**

is, hence, not an invertible function. The particular way the function is presented in this sEMG analysis requires re-interpretation of its lack of invertibility relative to the degree of singularity of the nonlinear system. A singular nonlinear system is a system of the form

$$F(\xi(k+1),\xi(k))=0$$

whereas a regular system is of the form

$$\xi(k+1)=f(\xi(k))$$

In this sEMG case, the function $F$ appears to be

$$F(\xi(k+1),\xi(k))\equiv\theta\left(\frac{\xi_1(k+1)}{\mathrm{var}\xi}\right)-\sum_{i=0}^{N-1}\phi_i(\xi_{i+1}(k))$$

If 6 is a pure limiter that saturates between the values $\theta_\star < \theta^*$, then it would be a case similar to the Lorenz attractor, where the dynamical system flips back and forth between two components of the attractor, which in this case would be the hypersurfaces $\sum_{i=0}^{N-1}\phi_i(X(k-i))=\theta_\star$ and $\sum_{i=0}^{N-1}\phi_i(X(k-i))=\theta^*$.

However, it appears that most of the motion is concentrated not on the saturating part of 6 but rather in between. It appears, therefore, that the motion is concentrated on that part of the *state* space $\Re^N$ bounded by the two hypersurfaces; that is, the space defined by $\theta_\star\leq\sum_{i=0}^{N-1}\phi_i(X(k-i))\leq\theta^*$

2. If 6 is interpreted as the degree of singularity of the system, it turns out that the ACE algorithm bears significant limitation as to the range of allowable nonlinearities in the sense that it does not allow cross coupling of the form $X(k-i)X(k-j)$.

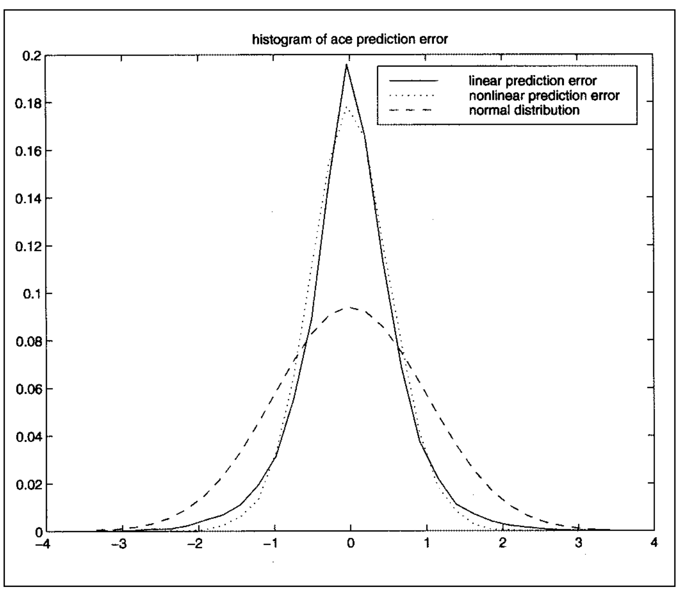


**Figure 5. The error histograms of both the ACE and best linear methods are shown along with a histogram of a normally distributed error. Ideally, the histogram of the prediction error should be normally distributed. It is shown that the ACE prediction error is minimally more normally distributed than the best linear method.**

Therefore, there is a need to adapt the ACE algorithm to the sEMG analysis. In particular, given the interpretation of the lack of invertibility of 6 to be a measure of the degree of singularity, it appears to be important to extend the ACE procedure to allow for more nonlinear distortions. Hence, it may be necessary to develop the following extrapolation of ACE:

a. An $N^{th}$ order ACE prediction error $E_0$ is found.

b. The ACE algorithm is used to predict $E_0$ with the cross term $X(k)X(k-1)$. Thus a new prediction error $E_1$ is found.

c. In the same manner, the cross terms $X(k)X(k-i)$ are used to predict $E_{i-1}$ for $i\leq N$

d. The same procedure is repeated for $X(k)X(k-i)X(k-j)$.

Before deciding whether chaos is present in an experimentally observed time series, the preliminary consideration should be whether the sequence is, for the most part, a manifestation of random or nonlinear phenomena. As far as the sEMG signal in the present study is concerned, it is apparent that most of the sequence can be justified by nonlinear rather than linear or stochastic phenomena. To conclude that the sEMG signal is chaotic will require a definition of chaos. In so far as it has been argued that a time series is chaotic whenever it can be explained by nonlinearities despite its external random appearance, the sEMG signal herein studied fits the definition of chaos. However, the most recent trend is that the concept of chaos is relevant to the theory of dynamical systems and as such requires more than just nonlinearity. In particular, the definition would require such dynamical concepts as existence of non-periodic recurrent points, transitivity, etc. From this point of view, further study will be required to demonstrate that the sEMG data is predominantly chaotic in nature.

However, the most important achievement is that the nonlinear phenomena, chaotic or not, underlying the sEMG have at this juncture of study been identified and an "attractor" has been found. Perhaps the most important issues will be whether there is some difference among the attractors of all three levels of NSA care, and to what extent the attractor(s) bear similarities with those of other physiological events, including classical and other forms of muscular activity. Moreover, it will be important to further investigate the uniformity of these findings among a spectrum of individuals undergoing NSA care, as well as evaluating data derived from sEMG electrodes being placed at different anatomical locations relative to the spine.

## Acknowledgments

The authors would like to acknowledge EMG Consultants, Inc., 255 W. Spring Valley Ave., Maywood, NJ

07607, for the generous gift of the Insight 7000 unit. Thanks are also extended to Mr. Lee Brody for technical expertise in configuring the Insight 7000 to the requested research requirements.

## References

1. Abarbanel HDJ. Analysis of observed chaotic data. 1996. New York. Springer- Verlag.
2. Chavas JP and Holt M. Nonlinear dynamics and economic instabilities in the optimal management of biological populations. Journal of Agricultural and Resource Econ.1992; 20(2): 231-248.
3. Elbert T, Ray WJ, Kowalik ZJ, Skinner JE, et al. Chaos and physiology: Deterministic chaos in excitable cell assemblies. Physiological Reviews 1994; 74(1): 1-40.
4. Epstein D. Network spinal analysis: A system of health care delivery within the subluxation-based chiropractic model. Journal of Vertebral Subluxation Research 1996; 1(1): 51-59.
5. Fredkin DR, and Rice JA. Method of false nearest neighbors - a cautionary note. Physiol Rev. E. 1995; 51: 2950 - 2954.
6. Garfinkel A, Spano M, Ditto WL, and Weiss JN. Controlling cardiac chaos. Science 1992; 257: 1230 - 1236.
7. Skinner JE, Goldberger AL, Mayer-Kress G, and Ideker RE. Chaos in the heart: implications for clinical cardiology. Bio/Technology 1990; 8: 1018 - 1024.
8. Baboyantz A. Some remarks on nonlinear data analysis of physiological time series. In: Abraham NB, Albano AM, Passamante A, and Rapp PE (Eds.) Measures of complexity and chaos. 1989. New York, Plenum Press: 51-62.
9. Frank GW, Lookman T, and Nerenberg MAH. Chaotic time series analysis using short and noisy data sets: Application to a clinical epilepsy seizure. In Abraham N, Albano A, Passamante A, and Rapp P (Eds). Measures of complexity and chaos. 1989; New York, Plenum Press.
10. Gallez D, and Babloyantz A. Predictability of human egg: A dynamic approach. Biol. Cybern. 1991; 64: 381-391.
11. Breiman L, and Friedman JH. Estimating optimal transformations for multiple regression and transformation. Journal of the American Statistical Assoc. 1985; 80: 580-619.
12. Hotelling H. Relations between two sets of variates. Biometrika 1936; 28: 321-377.
13. Larimore, WD. Development of statistical methods using predictive inference and entropy. Technical report, Final Report for Air Force Office of Scientific Res, 1986.
14. Wu BF. Identification and control of chaotic processes - The Komogorov-Sinai Entropy Approach. 1992; Ph.D. Thesis, University of Southern California, Los Angeles, California.