\[
\ln P_1 - \ln \sigma_1 - \frac{(T - \mu_1)^2}{2\sigma_1^2} - \ln P_2 + \ln \sigma_2 + \frac{(T - \mu_2)^2}{2\sigma_2^2} = 0
\]
\[
\ln \frac{P_1}{P_2} + \ln \frac{\sigma_1^2}{\sigma_2^2} \Bigg( \frac{1}{2\sigma_1^2}(T^2 - 2\mu_1 T + \mu_1^2) + \frac{1}{2\sigma_2^2}(T^2 - 2\mu_2 T + \mu_2^2) \Bigg) = 0
\]
\[
\ln \frac{\sigma_2 P_1}{\sigma_1 P_2} + T^2 \Bigg( \frac{1}{2\sigma_2^2} - \frac{1}{2\sigma_1^2} \Bigg) + T \Bigg( \frac{\mu_1}{\sigma_1^2} - \frac{\mu_2}{\sigma_2^2} \Bigg) + \Bigg( \frac{\mu_2^2}{2\sigma_2^2} - \frac{\mu_1^2}{2\sigma_1^2} \Bigg) = 0.
\]

From this expression we get
\[
AT^2 + BT + C = 0
\]
with
\[
A = (\sigma_1^2 - \sigma_2^2)
\]
\[
B = 2(\sigma_2^2 \mu_1 - \sigma_1^2 \mu_2)
\]
and
\[
C = \sigma_1^2 \mu_2^2 - \sigma_2^2 \mu_1^2 + 2\sigma_1^2 \sigma_2^2 \ln \frac{\sigma_2 P_1}{\sigma_1 P_2}.
\]

**Problem 10.25**

If \(\sigma_1 = \sigma_2 = \sigma\), then \(A = 0\) in Eq. (10.3-12) and we have to solve the equation
\[
BT + C = 0
\]
with
\[
B = 2\sigma^2 (\mu_1 - \mu_2)
\]
and
\[
C = \sigma^2 (\mu_2^2 - \mu_1^2) + 2\sigma^4 \ln \frac{P_1}{P_2}.
\]

Substituting and cancelling terms gives
\[
2(\mu_1 - \mu_2)T - (\mu_1 + \mu_2)(\mu_1 - \mu_2) + 2\sigma^2 \ln \frac{P_1}{P_2} = 0
\]
or
\[
T = \frac{\mu_1 + \mu_2}{2} + \frac{\sigma^2}{\mu_1 - \mu_2} \ln \frac{P_1}{P_2}.
\]

**Problem 10.26**

The simplest solution is to use the given means and standard deviations to form two Gaussian probability density functions, and then to use the optimum thresholding approach discussed in Section 10.3.5 (in particular, see Eqs. (10.3-11) through (10.3-13)). The probabilities \(P_1\) and \(P_2\) can be estimated by visual analysis of the images (i.e., by determining the relative areas of the image occupied by objects and background). It is clear by looking at the image that the probability of occurrence of object points is less than that of background points. Alternatively, an automatic estimate can be obtained by