CORRECTIONS TO PROBLEM SOLUTIONS

PROBLEM 4.4: The second equation in the problem statement should be revised to

\[ h(x, y) = A2\pi\sigma^2 e^{-2\pi^2\sigma^2(x^2 + y^2)} \]

We want to show that the inverse Fourier transform of \( A e^{-(u^2 + v^2)/2\sigma^2} \) is equal to this function. Rather than doing it as shown in the original problem solution, it will be clearer if we start with one variable and show that, if

\[ H(u) = e^{-u^2/2\sigma^2} \]

then

\[
 h(x) = 3^{-1}[H(u)] \\
= \int_{-\infty}^{\infty} e^{-u^2/2\sigma^2} e^{i2\pi ux} du \\
= \sqrt{2\pi \sigma} e^{-\pi^2 x^2/\sigma^2}.
\]

We can express the integral in the preceding equation as

\[
 h(x) = \int_{-\infty}^{\infty} e^{-1/2\sigma^2 [x^2 - 4\pi^2 u^2]} du.
\]

We now make use of the identity

\[
 e^{-1/2 \left( 2\pi^2 x^2 + 2\pi^2 y^2 \right)} e^{-1/2 \left( 2\pi^2 x^2 + 2\pi^2 y^2 \right)} = 1.
\]

Inserting this identity into the preceding integral yields,

\[
 h(x) = e^{-1/2 \left( 2\pi^2 x^2 + 2\pi^2 y^2 \right)} \int_{-\infty}^{\infty} e^{-1/2 \left( 2\pi^2 x^2 + 2\pi^2 y^2 \right)} du \\
= e^{-1/2 \left( 2\pi^2 x^2 + 2\pi^2 y^2 \right)} \int_{-\infty}^{\infty} e^{-1/2 \left( 2\pi^2 x^2 + 2\pi^2 y^2 \right)} du.
\]

Next we make the change of variables \( r = u - j2\pi\sigma^2 x \). Then \( dr = du \) and the preceding integral becomes

\[
 h(x) = e^{-1/2 \left( 2\pi^2 x^2 + 2\pi^2 y^2 \right)} \int_{-\infty}^{\infty} e^{-r^2/2\sigma^2} dr.
\]

Finally, we multiply and divide the right side of this equation by \( \sqrt{2\pi \sigma} \) and obtain

\[
 h(x) = \sqrt{2\pi \sigma} e^{-\pi^2 x^2/\sigma^2} \left[ \int_{-\infty}^{\infty} e^{-r^2/2\sigma^2} dr \right].
\]

The expression inside the brackets is recognized as a Gaussian probability density function, whose value from \(-\infty\) to \(\infty\) is 1. Therefore,

\[
 h(x) = \sqrt{2\pi \sigma} e^{-\pi^2 x^2/\sigma^2}.
\]

With this result as background, we are now ready to show that

\[
 h(x, y) = 3^{-1}[A e^{-(u^2 + v^2)/2\sigma^2}] \\
= A2\pi\sigma^2 e^{-2\pi^2\sigma^2(x^2 + y^2)}.
\]

By substituting directly into the definition of the inverse Fourier transform we have:

\[
 h(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A e^{-(u^2 + v^2)/2\sigma^2} e^{i2\pi(ux + vy)} dudv \\
= \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} A e^{-u^2/2\sigma^2 + j2\pi uy} du \right] e^{-v^2/2\sigma^2 + j2\pi vy} dv.
\]

The integral inside the brackets is recognized from the previous discussion to be equal to \( A\sqrt{2\pi \sigma} e^{-\pi^2 x^2/\sigma^2} \). Then, the preceding integral becomes
\[
\begin{align*}
    h(x, y) &= A\sqrt{2\pi\sigma} e^{-2\pi^2\sigma^2} \int_{-\infty}^{\infty} e^{-\frac{y^2}{2\sigma^2}} dy.
\end{align*}
\]

We now recognize the remaining integral to be equal to \(\sqrt{2\pi\sigma} e^{-2\pi^2\sigma^2}\), from which we have the final result:

\[
\begin{align*}
    h(x, y) &= (A\sqrt{2\pi\sigma} e^{-2\pi^2\sigma^2}) (A\sqrt{2\pi\sigma} e^{-2\pi^2\sigma^2}) \\
    &= A2\pi\sigma^2 e^{-2\pi^2\sigma^2(x^2+y^2)}.
\end{align*}
\]

**PROBLEM 4.15(a):** There is a discrepancy in sign between the problem statement and the solution. To be consistent with the problem statement, the solution should start with

\[
g(x, y) = f(x + 1, y) - f(x, y) + f(x, y + 1) - f(x, y).
\]

In practice, one sees both formulations used interchangeably since ultimately we use squares or absolute values, which are independent of the sign used in the definition of the derivative.

**PROBLEM 5.3:** The images shown in the solution of this problem are incorrect. They should be identical to the ones in PROBLEM 5.5.

**PROBLEM 6.16(a):** 5th line: Delete the words “the entire”. 6th line: Change 255/360 to 360/255.

**PROBLEM 7.9.** (a) Equation (7.1-28) defines the 2 \times 2 Haar transformation matrix as

\[
H_2 = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix}.
\]

Thus, using Eq. (7.1-24), we get

\[
T = HFH^T = \left(\frac{1}{\sqrt{2}}\right)^T \begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix} \begin{bmatrix}
3 & -1 \\
6 & 2
\end{bmatrix} \begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix} = \begin{bmatrix}
5 & 4 \\
-3 & 0
\end{bmatrix}.
\]

(b) First, compute

\[
H_2^{-1} = \begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
\]

such that

\[
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
ob & 1
\end{bmatrix}.
\]

Solving this matrix equation yields

\[
H_2^{-1} = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix} = H_2^T = H_2.
\]