1. Let \( f_x(t) \) be symmetric about 0. Prove that \( \mu \) is the expected value of a sample distributed according to \( f_{x-\mu}(t) \).

2. The complimentary cumulative distribution function is defined as \( Q_x(x) = 1 - F_x(x) \), or more explicitly in the zero mean, unit variance Gaussian distribution case as

\[
Q_x(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp \left( \frac{-t^2}{2} \right) \, dt.
\]

Show that

\[
Q_x(x) \approx \frac{1}{\sqrt{2\pi x}} \exp \left( \frac{-t^2}{2} \right) \, dt.
\]

Hint: use integration by parts on \( Q_x(x) = \int_x^\infty \frac{1}{\sqrt{2\pi t}} \exp \left( \frac{-t^2}{2} \right) \, dt \). Also explain why the approximation improves as \( x \) increases.

3. The probability density function for a two dimensional random vector is defined by

\[
f_x(x) = \begin{cases} \text{Ax}_1^2x_2 & \text{if } x_1, x_2 \geq 0 \text{ and } x_1 + x_2 \leq 1 \\ 0 & \text{otherwise} \end{cases}
\]

(a) Determine \( F_x(x) \) and the value of \( A \).

(b) Determine the marginal density \( f_{x_2}(x) \).

(c) Are \( f_{x_1}(x) \) and \( f_{x_2}(x) \) independent? Show why or why not.

4. Consider the two independent marginal distributions

\[
f_{x_1}(x) = \begin{cases} 1 & 0 \leq x_1 \leq 1 \\ 0 & \text{otherwise} \end{cases}
\]

\[
f_{x_2}(x) = \begin{cases} 2x & 0 \leq x_2 \leq 1 \\ 0 & \text{otherwise} \end{cases}
\]

Let \( A \) be the event \( x_1 \leq x_2 \).

(a) Find and sketch \( f_x(x) \).

(b) Determine \( Pr\{A\} \).

(c) Determine \( f_{x|A}(x|A) \). Are the components independent, i.e., are \( f_{x_1|A}(x|A) \) and \( f_{x_2|A}(x|A) \) independent?

5. The entropy \( \mathcal{H} \) for a random vector is defined as \(-E\{\ln f_x(x)\}\). Show that for the complex Gaussian case

\[
\mathcal{H} = N(1 + \ln \pi) + \ln |C_x|.
\]

Determine the corresponding expression when the vector is real.

6. Let

\[
x = 3u - 4v \\
y = 2u + v
\]

where \( u \) and \( v \) are unit mean, unit variance, uncorrelated Gaussian random variables.
(a) Determine the means and variances of $x$ and $y$.
(b) Determine the joint density of $x$ and $y$.
(c) Determine the conditional density of $y$ given $x$.

7. Consider the orthogonal transformation of the correlated zero mean random variables $x_1$ and $x_2$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Note $E\{x_1^2\} = \sigma_1^2$, $E\{x_2^2\} = \sigma_2^2$, and $E\{x_1x_2\} = \rho \sigma_1 \sigma_2$. Determine the angle $\theta$ such that $y_1$ and $y_2$ are uncorrelated.

8. The covariance matrix and mean vector for a real Gaussian density are

$$C_x = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

and

$$m_x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(a) Determine the eigenvalues and eigenvectors.
(b) Generate a mesh plot of the distribution using MATLAB.
(c) Change the off-diagonal values to $-0.5$ and repeat (a) and (b).

9. Let $\{x_k(n)\}_{k=1}^{K}$ be i.i.d. zero mean, unit variance uniformly distributed random variables and set

$$y_K(n) = \sum_{k=1}^{K} x_k(n).$$

(a) Determine and plot the pdf of $y_K(n)$ for $K = 2, 3, 4$.
(b) Compare the pdf’s to the Gaussian density.
(c) Perform the comparison experimentally using MATLAB. That is, generate $K$ sequences of $n = 1, 2, \ldots, N$ uniformly distributed samples. Add the sequences and plot the resulting distribution (histogram). Fit the results to a Gaussian distribution for various $K$ and $N$. 

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