Problem 5.7

(a) The biorthogonal signals are defined as the negatives of orthogonal signals. Consider for example the two orthogonal signals $s_1(t)$ and $s_2(t)$ defined as follows:

\[ s_1(t) = \sqrt{E}\phi_1(t) \]
\[ s_2(t) = \sqrt{E}\phi_2(t) \]

where $\phi_1(t)$ and $\phi_2(t)$ are orthonormal basis functions. The biorthogonal signals are given by $-s_1(t)$ and $-s_2(t)$, which are respectively expressed in terms of the basis functions as $\sqrt{E}\phi_1(t)$ and $-\sqrt{E}\phi_2(t)$. Hence, the inclusion of these two biorthogonal signals leaves the dimensionality of the signal-space diagram unchanged. This result holds for the general case of $M$ orthogonal signals.

(b) The signal-space diagram for the biorthogonal signals corresponding to those shown in Fig. P5.5 is as shown in Fig. 1a. Incorporating this diagram with that of the solution to Problem 5.5, we get the 4-signal constellation shown in Fig. 1b.

![Figure 1](image)

**Figure 1**

Problem 5.8

(a) A pair of signals $s_i(t)$ and $s_k(t)$, belonging to an $N$-dimensional signal space, can be represented as linear combinations of $N$ orthonormal basis functions. We thus write

\[ s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \quad 0 \leq t \leq T \]
\[ i = 1, 2 \]

(1)

where the coefficients of the expansion are defined by