Lecture 10
FIR Filtering Intro

READING ASSIGNMENTS
- This Lecture:
  - Chapter 5, Sects. 5-1, 5-2 and 5-3 (partial)
- Other Reading:
  - Recitation: Ch. 5, Sects 5-4, 5-6, 5-7 and 5-8
  - CONVOLUTION
  - Next Lecture: Ch 5, Sects. 5-3, 5-5 and 5-6

LECTURE OBJECTIVES
- INTRODUCE FILTERING IDEA
  - Weighted Average
  - Running Average
- FINITE IMPULSE RESPONSE FILTERS
  - FIR Filters
  - Show how to compute the output y[n] from the input signal, x[n]

DIGITAL FILTERING
- CONCENTRATE on the COMPUTER
  - PROCESSING ALGORITHMS
  - SOFTWARE (MATLAB)
  - HARDWARE: DSP chips, VLSI
- DSP: DIGITAL SIGNAL PROCESSING
The TMS32010, 1983
First PC plug-in board from Atlanta Signal Processors Inc.

Rockland Digital Filter, 1971
For the price of a small house, you could have one of these.

Digital Cell Phone (ca. 2000)
Free (?) with 2 year contract

DISCRETE-TIME SYSTEM

\[ x[n] \rightarrow \text{COMPUTER} \rightarrow y[n] \]

- OPERATE on \( x[n] \) to get \( y[n] \)
- WANT a GENERAL CLASS of SYSTEMS
  - ANALYZE the SYSTEM
    - TOOLS: TIME-DOMAIN & FREQUENCY-DOMAIN
  - SYNTHESIZE the SYSTEM
**D-T SYSTEM EXAMPLES**

- **EXAMPLES:**
  - **POINTWISE OPERATORS**
    - SQUARING: \( y[n] = (x[n])^2 \)
  - **RUNNING AVERAGE**
    - RULE: "the output at time \( n \) is the average of three consecutive input values"

- **SYSTEM:**
  - \( x[n] \) \( \rightarrow \) \( y[n] \)

**DISCRETE-TIME SIGNAL**

- \( x[n] \) is a LIST of NUMBERS
- INDEXED by "\( n \)"

**3-PT AVERAGE SYSTEM**

- **ADD 3 CONSECUTIVE NUMBERS**
- Do this for each "\( n \)"

Make a TABLE

The following input–output equation:

\[
y[n] = \frac{1}{3}(x[n] + x[n+1] + x[n+2])
\]

<table>
<thead>
<tr>
<th>( n )</th>
<th>( n &lt; -2 )</th>
<th>(-2)</th>
<th>(-1)</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( 2 )</th>
<th>( 3 )</th>
<th>( 4 )</th>
<th>( 5 )</th>
<th>( n &gt; 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x[n] )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( y[n] )</td>
<td>0</td>
<td>( \frac{2}{3} )</td>
<td>2</td>
<td>4</td>
<td>( \frac{14}{3} )</td>
<td>4</td>
<td>2</td>
<td>( \frac{5}{3} )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\( n=0 \)

\( y[0] = \frac{1}{3}(x[0] + x[1] + x[2]) \)

\( n=1 \)

\( y[1] = \frac{1}{3}(x[1] + x[2] + x[3]) \)

**INPUT SIGNAL**

\[
y[n] = \frac{1}{3}(x[n] + x[n+1] + x[n+2])
\]

**OUTPUT SIGNAL**

\[
y[n] = \frac{1}{3}(x[n] + x[n+1] + x[n+2])
\]
PAST, PRESENT, FUTURE

Sec. 5.2 The Running Average Filter

FIR Filtering
(a weighted sum over past, present, and future points)

Figure 5.4 The running-average filter calculation at time index \( n \) uses values within a sliding window (shaded). Dark shading indicates the future \( (\ell > n) \); light shading, the past \( (\ell < n) \).

ANOTHER 3-pt AVERAGER

- Uses “PAST” VALUES of \( x[n] \)
- IMPORTANT IF “\( n \)” represents REAL TIME
- WHEN \( x[n] \) & \( y[n] \) ARE STREAMS

\[
y[n] = \frac{1}{3} (x[n] + x[n-1] + x[n-2])
\]

GENERAL FIR FILTER

- FILTER COEFFICIENTS \( \{b_k\} \)
- DEFINE THE FILTER

\[
y[n] = \sum_{k=0}^{M} b_k x[n-k]
\]

For example, \( b_k = \{3, -1, 2, 1\} \)

\[
y[n] = 3x[n] - x[n-1] + 2x[n-2] + x[n-3]
\]

GENERAL FIR FILTER

- FILTER COEFFICIENTS \( \{b_k\} \)

\[
y[n] = \sum_{k=0}^{M} b_k x[n-k]
\]

- FILTER ORDER is \( M \)
- FILTER LENGTH is \( L = M+1 \)
- NUMBER of FILTER COEFFS is \( L \)
GENERAL FIR FILTER

*SLIDE a WINDOW across x[n]*

\[ y[n] = \sum_{k=0}^{M} b_k x[n-k] \]

- M-th Order FIR Filter Operation (Causal)
- Weighted Sum over M + 1 points
- Running onto the Data
- Zero Output

SPECIAL INPUT SIGNALS

* x[n] = SINUSOID
* x[n] has only one NON-ZERO VALUE

\[ \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \]

UNIT IMPULSE SIGNAL δ[n]

\[
\begin{array}{cccccccccc}
 n & \ldots & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & \ldots \\
\delta[n] & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
\delta[n-3] & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \ldots \\
\end{array}
\]

δ[n] is NON-ZERO when its argument is equal to ZERO

FILTERED STOCK SIGNAL

INPUT

OUTPUT

UNIT IMPULSE SIGNAL δ[n - 3]

Figure 5.7 Shifted impulse sequence, δ[n - 3].
MATH FORMULA for $x[n]$

- Use SHIFTED IMPULSES to write $x[n]$
  
  $$x[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$$

SUM of SHIFTED IMPULSES

<table>
<thead>
<tr>
<th>$n$</th>
<th>...</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
<th>$4$</th>
<th>$5$</th>
<th>$6$</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\delta[n]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$4\delta[n-1]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$6\delta[n-2]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$4\delta[n-3]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$2\delta[n-4]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$$x[n] = \sum_{k} x[k]\delta[n-k]$$

This formula ALWAYS works

$$x[n] = \ldots + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \ldots \quad (5.3.6)$$

4-pt AVERAGER

- CAUSAL SYSTEM: USE PAST VALUES
  
  $$y[n] = \frac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

- INPUT = UNIT IMPULSE SIGNAL = $\delta[n]$
  
  $$x[n] = \delta[n]$$

  $$y[n] = \frac{1}{4}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n-3]$$

- OUTPUT is called "IMPULSE RESPONSE"

  $$h[n] = \{\ldots, 0, 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, \ldots\}$$

4-pt Avg Impulse Response

- $\delta[n]$ "READS OUT" the FILTER COEFFICIENTS

  $$h[n] = \{\ldots, 0, 0, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, \ldots\}$$

- "h" in $h[n]$ denotes Impulse Response

- NON-ZERO When window overlaps $\delta[n]$
FIR IMPULSE RESPONSE

- Convolution = Filter Definition
- Filter Coeffs = Impulse Response

\[
y[n] = \sum_{k=0}^{M} b[k] x[n-k] \quad y[n] = \sum_{k=0}^{M} h[k] x[n-k]
\]

FILTERING EXAMPLE

- 7-point AVERAGER
  - Removes cosine
  - By making its amplitude (A) smaller

\[
y_7[n] = \sum_{k=0}^{6} \left(\frac{1}{7}\right) x[n-k]
\]

- 3-point AVERAGER
  - Changes A slightly

\[
y_3[n] = \sum_{k=0}^{2} \left(\frac{1}{3}\right) x[n-k]
\]

3-pt AVG EXAMPLE

Input: \( x[n] = (1.02)^n + \cos(2\pi n / 8 + \pi / 4) \) for \( 0 \leq n \leq 40 \)

7-pt FIR EXAMPLE (AVG)

Input: \( x[n] = (1.02)^n + \cos(2\pi n / 8 + \pi / 4) \) for \( 0 \leq n \leq 40 \)