ELEG404/604: Digital Imaging & Photography

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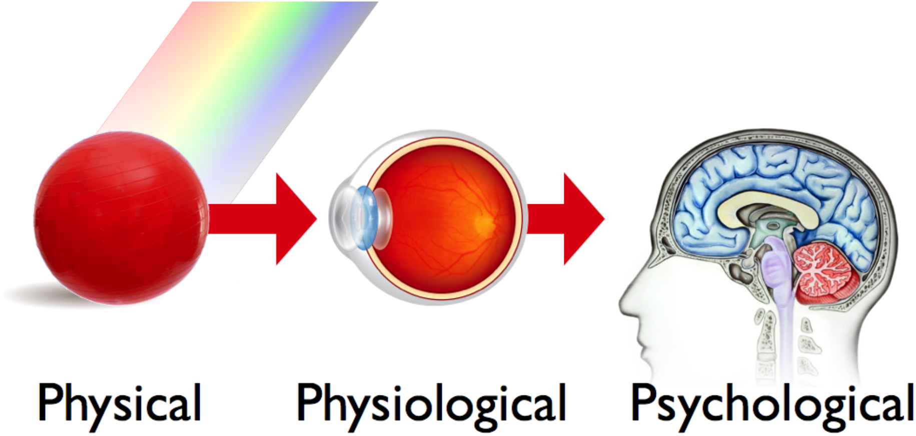
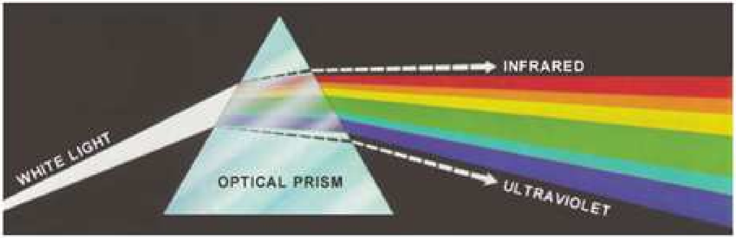
Chapter IX

Color Perception



Color Fundamentals



- The visible light spectrum is continuous

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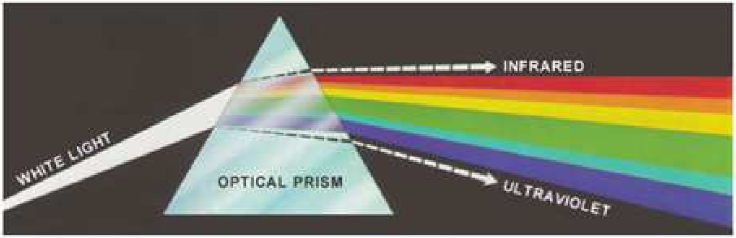

- The visible light spectrum is continuous
- Six broad regions:
  - Violet, blue, green, yellow, orange and red

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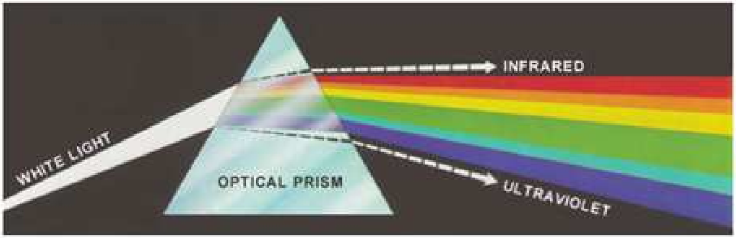
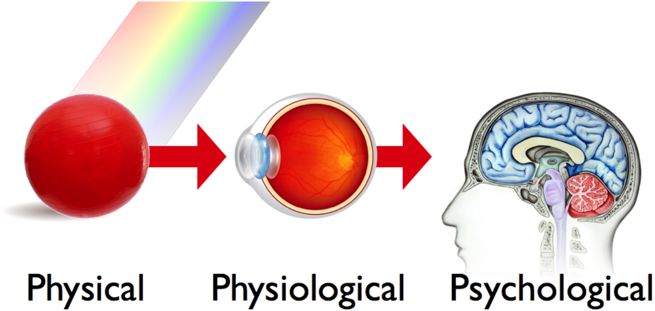


- The visible light spectrum is continuous
- Six broad regions:
  - Violet, blue, green, yellow, orange and red
- Achromatic light is void of color
  - Characterization: intensity (gray level)

Color Perception

- Object color depends on what wavelength it reflects



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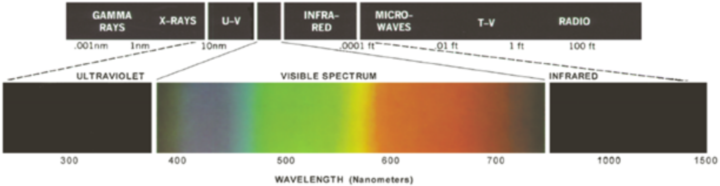
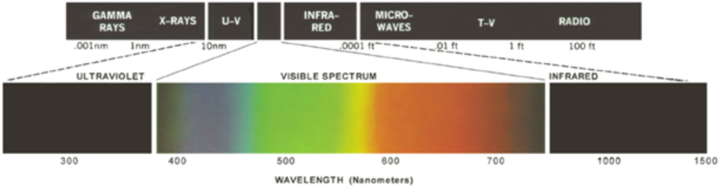


Same luminance but varying brightness

- Chromatic light spectrum: 400-700nm

Color Fundamentals

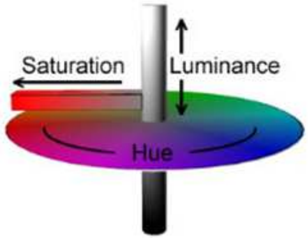


Same luminance but varying brightness

- Chromatic light spectrum: 400-700nm
- Quality of chromatic light:
  - Radiance-total energy that flows from a light source
  - Luminance-amount of energy an observer perceives from a light source (lumens)
  - Brightness-subjective descriptor of intensity

Color Attributes

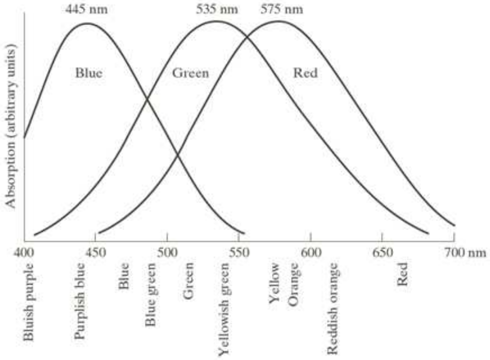
- Brightness: perception of intensity
- Hue: an attribute associated with the dominant wavelength (color)
  - The color of an object determines its hue



- Saturation: relative purity, or the amount of white light mixed with a hue
  - Pure spectrum colors are fully saturated, *e.g.*, red
  - Saturation is inversely proportional to the amount of white light in a color
- Chromaticity: hue and saturation together
  - A color may be characterized by its brightness and chromaticity

Color Vision Response

- Cone response
  - 6-7 million receptors
  - Tristimulus model
  - Red sensitive: 65%
  - Green sensitive: 33%
  - Blue sensitive: 2%–most sensitive receptors



Color Vision Response

- Primary colors: red (R), green (G), blue (B)

$$R(\lambda) = \int_0^\infty C(\lambda) R_S(\lambda) d\lambda$$
$$G(\lambda) = \int_0^\infty C(\lambda) G_S(\lambda) d\lambda$$
$$B(\lambda) = \int_0^\infty C(\lambda) B_S(\lambda) d\lambda$$

where $C(\lambda)$ is the spectral distribution of light incident on the retina and R_s,G_s and B_s are the sensitivity of the cones.

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- Two different spectra could produce the same cone response and therefore represent the same to the human eye.

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Metamerism

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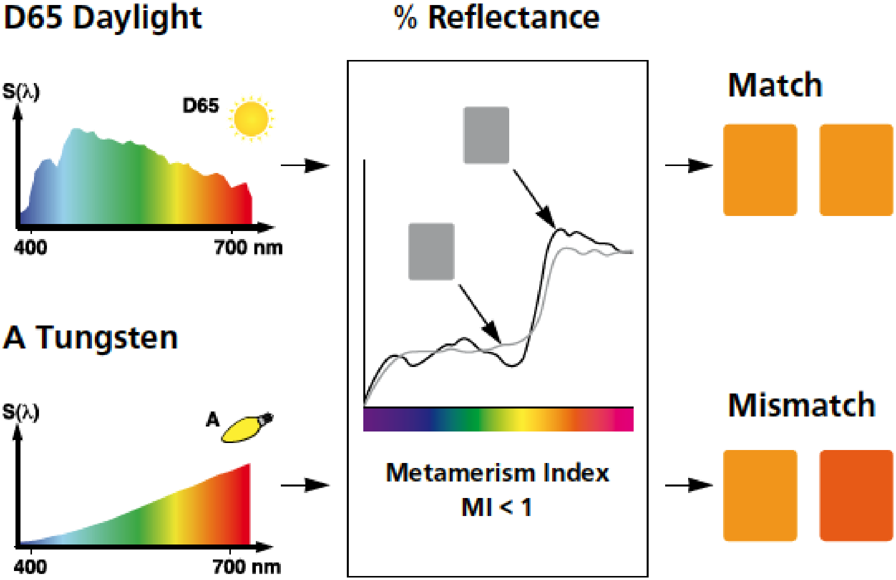
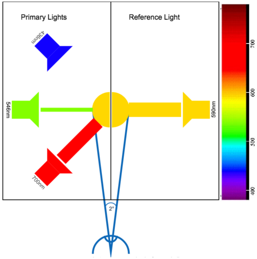
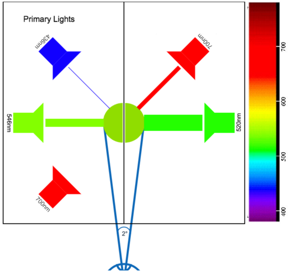
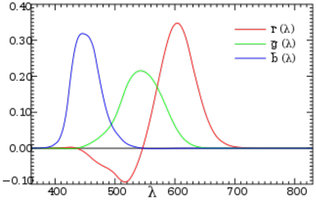

D65 Daylight
S(λ)
D65
400
700 nm
A Tungsten
S(λ)
A
400
700 nm
% Reflectance
Metamerism Index
MI < 1
Match
Mismatch

Color Matching

- International Commission on Illumination (CIE) standard definitions:
  - Blue (435.8 nm), Green (546.1 nm), Red (700 nm)
- Defined in 1931, it doesn't really match human perception. It is based on experimental data.

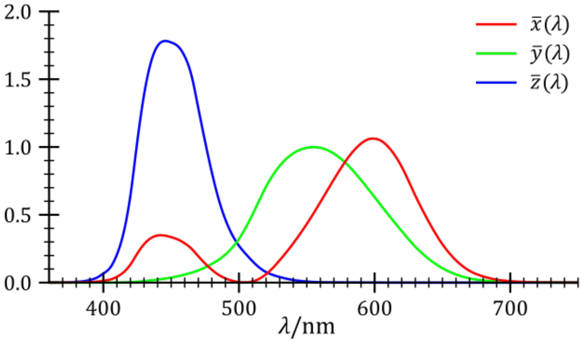






CIE XYZ System

- Hypothetical primary sources such that all the tristimulus values are positive
- $Y \equiv$luminance
- Convenient for colormetric calculations



Tristimulus Representation

- Tristimulus values: X, Y, Z
- Trichromatic coefficients:

$$x = \frac{X}{X+Y+Z} \quad y = \frac{Y}{X+Y+Z} \quad z = \frac{Z}{X+Y+Z}$$

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then

$$x+y+z=1$$

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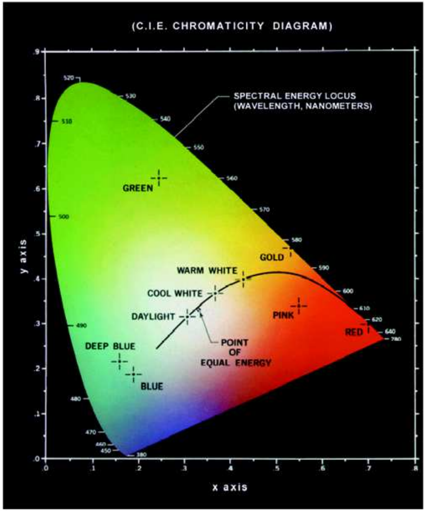
then

$$x + y + z = 1$$

- Alternate approach: chromaticity diagram
  - Gives color composition as a function of x and y
  - Solve for z according to the above expression
  - Projects 3–D color space on to two dimensions

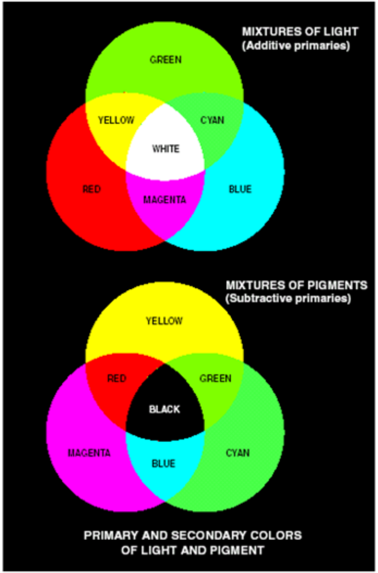
Chromaticity Diagram

- Pure colors are on the boundary
  - Fully saturated
- Interior points are mixtures
  - A line between two colors indicates all possible mixtures of two colors
- *Color gamut:* triangle defined by three colors
  - Three color mixtures are restricted to the gamut
  - No three-color gamut completely encloses the chromaticity diagram



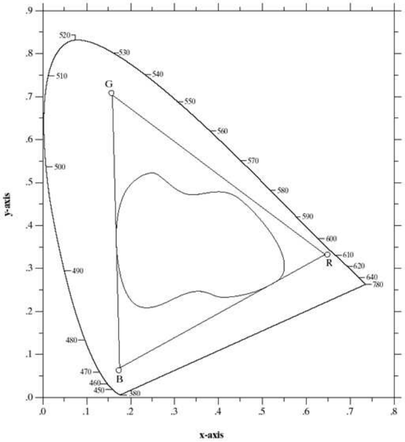
Primary and Secondary Colors

- Primary colors of light:
  - Red, green and blue
- Add primary colors to obtain secondary colors of light:
  - Magenta, cyan and yellow
- Primary colors of pigments–absorbs (subtracts) a primary color of light and reflects (transmits) the other two
  - Magenta absorbs green, cyan absorbs red, and yellow absorbs blue
  - Secondary pigments: red, green and blue



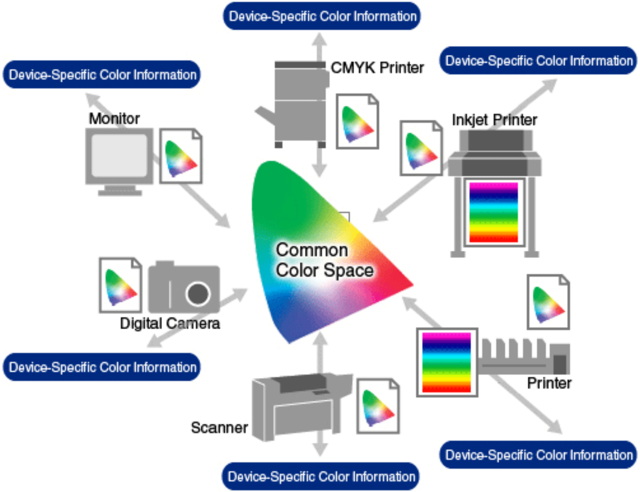
Color Gamut Examples

- RGB monitor color gamut
  - Regular (triangular) shape
  - Based on three highly controllable light primaries
- Printing device color gamut
  - Combination of additive and subtracted color mixing
  - Difficult control process
- Neither gamut includes all colors–monitor is better



Color Spaces

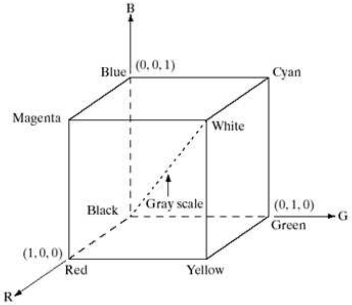
- Hardware-oriented
  - RGB (monitors and cameras)
  - CMY - CMYK (printers)
- Application-oriented
  - Perception-Based (HSI, HSL, HSV)
  - Adequate color spaces in which distances model color mismatches (Lab, Luv)

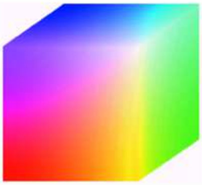


The RGB Color Model (Space)

RGB is the most widely used hardware-oriented color space

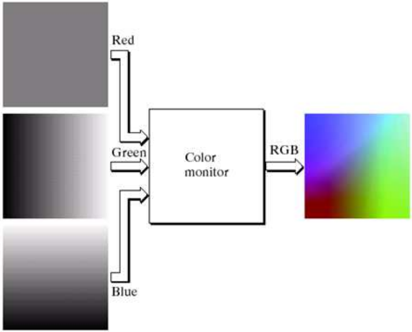
- Graphics boards, monitors, cameras, etc
- Normalized RGB values
- Grayscale is a diagonal line through the cube
- Quantization determines color depth
  - Full-color: 24 bit representations (16,77,216 colors)

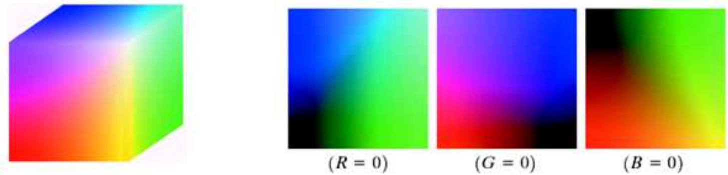




RGB Color Image Generation

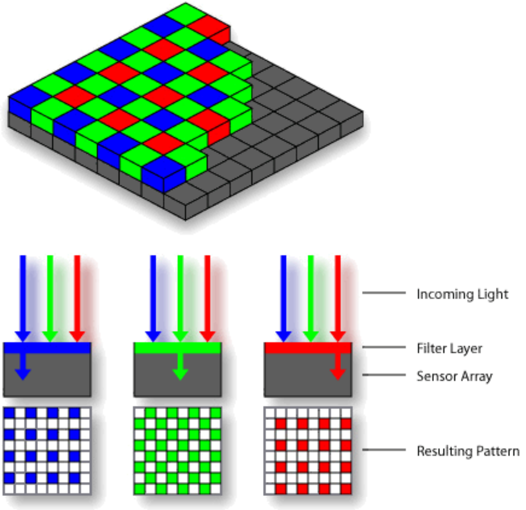
- Monochrome images represent each color component
- Hyperplane examples:
  - Fix one dimension
  - Example shows three hidden sides of the color cube





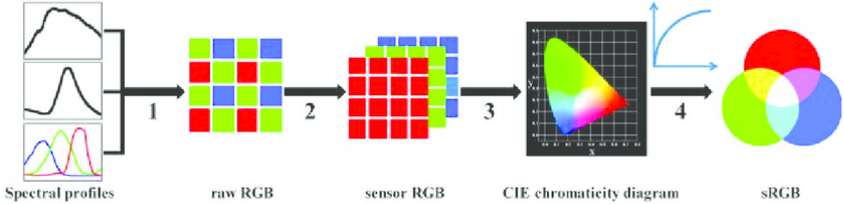
RGB Color Image Generation

- Acquisition process: reverse operation
  - Filter light to obtain RGB components
- The data acquired by the sensor is in the color space of the camera.



Acquisition of Color Images

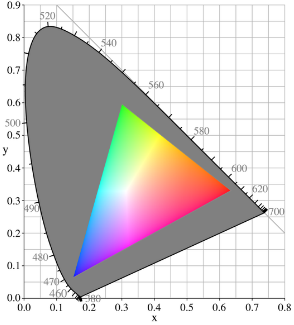
- Sensor color filter array data
- White Balance
- Demosaicking
- Color transformation to unrendered color space
- Color transformation to rendered color space



CIE XYZ Color Space to sRGB

Linear transformation given by

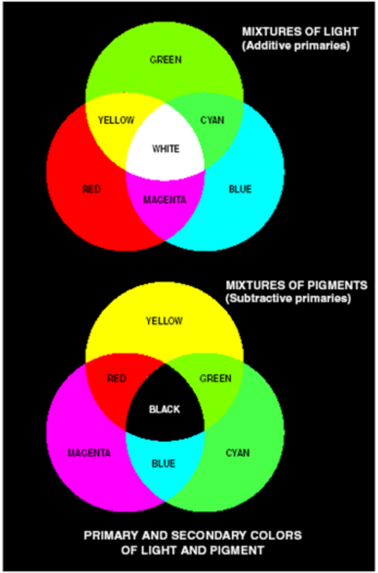
$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 3.24 & -1.54 & -0.50 \\ -0.97 & 1.88 & 0.04 \\ 0.06 & -0.20 & 1.06 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



The CMY and CMYK Color Spaces

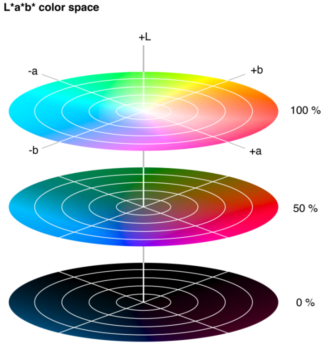
- CMY: cyan, magenta and yellow
- CMYK: adds black
  - Black is difficult (and costly) to reproduce with CMY
  - Four color printing
- Subtracted primaries are widely used in printing

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$



Lab Color Space

- CIELAB is used extensively in imaging
- Transforms to and from CIELAB to other color spaces are commonly employed.
- $L^* \equiv$brightness, $a^* \equiv$red-green, $b^* \equiv$yellow-blue



$L^*a^*b^*$ Color Space

$$L^* = 25\left(\frac{100Y}{Y_0}\right)^{1/3} - 16,\ 1 \le 100Y \le 100$$

$$a^* = 500\left[\left(\frac{X}{X_0}\right)^{1/3} - \left(\frac{X}{X_0}\right)^{1/3}\right]$$

$$b^* = 200\left[\left(\frac{Y}{Y_0}\right)^{1/3} - \left(\frac{Z}{Z_0}\right)^{1/3}\right]$$

- X_0, Y_0, Z_0 tristimulus values of reference white

$L^*a^*b^*$ Color Space

- Radial distance serve as measure of perceived chroma.

$$C_{ab} = \sqrt{a^{*2} + b^{*2}}$$

- The angular position as perceived hue

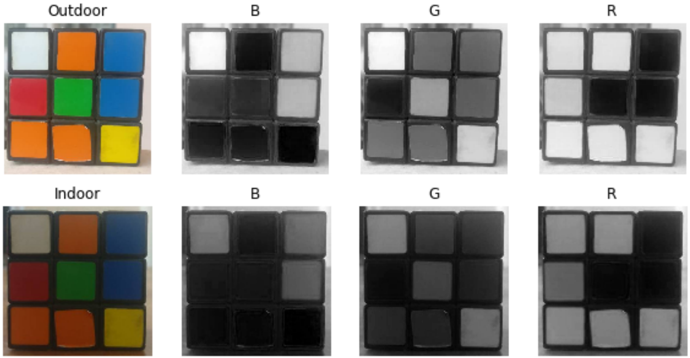
$$h_{ab} = \tan^{-1}\left(\frac{a^*}{b^*}\right)$$

- The perceived color difference is measured by the Euclidean distance

$$\Delta E_{ab} = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2}$$

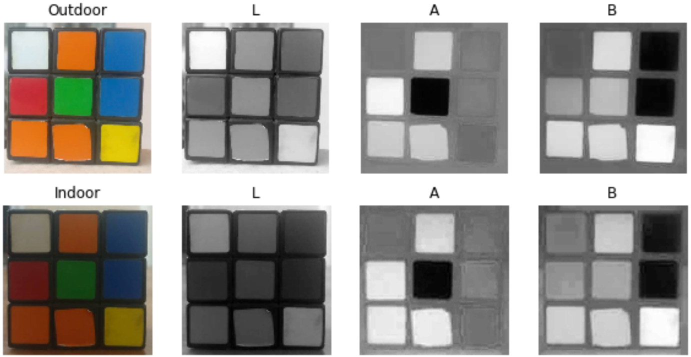
- A ΔE_{ab} value of around 2.3 correspond to a Just Noticeable Difference.

RGB vs $L^*a^*b^*$



- Significant perceptual non-uniformity
- Mixing of chrominance and luminance.

RGB vs $L^*a^*b^*$



- Perceptually uniform color space which approximates how we perceive color.
- Separates the luminance and chrominance components into different channels.
- Changes in illumination mostly affects the L component.

The HSI Color Space

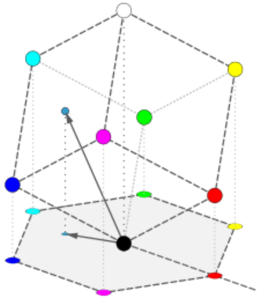
- Hue, saturation, intensity: human perceptual descriptions of color
- Decouples intensity (gray level) from hue and saturation

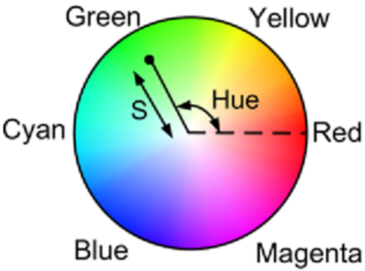
a b c

FIGURE 7.37 HSI components of the RGB color image in Fig. 7.36(a). (a) Hue. (b) Saturation. (c) Intensity.

The HSI Color Space

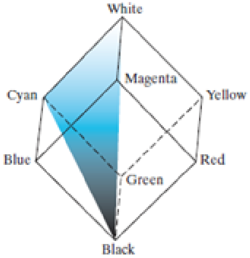
- Rotate RGB cube so intensity is the vertical axis
  - The intensity component of any color is its vertical component
  - Saturation: distance from vertical axis
    - Zero saturation: colors (gray values) on the vertical axis
    - Fully saturated: pure colors on the cube boundaries
  - Hue: primary color indicated as an angle of rotation

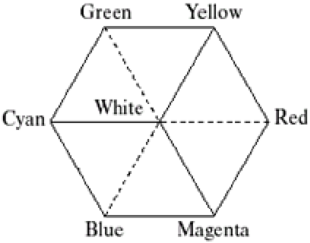


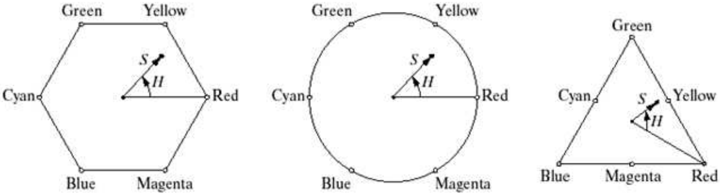


The HSI Color Space

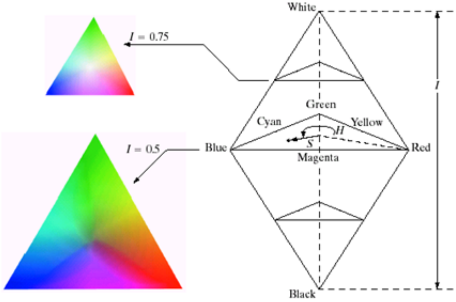
- View the HSI space from top down
  - Slicing plane perpendicular to intensity
- Intensity: height of slicing plane
- Saturation: distance from center
- Hue: rotation angle from red
- Natural shape: hexagon

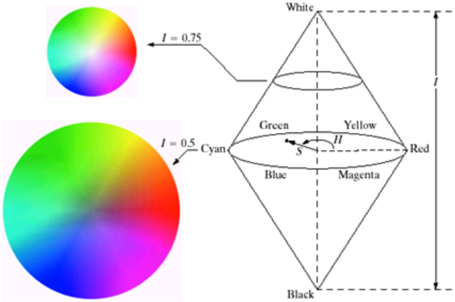






Common HSI representations





RGB to HSI Conversion

$$H = \begin{cases} \theta & \text{if } B \leq G \\ 360 - \theta & \text{if } B > G \end{cases}$$

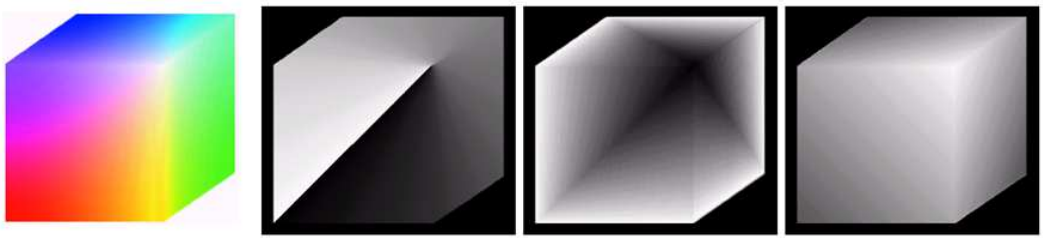
$$\theta = \cos^{-1}\left\{ \frac{[(R-G)+(R-B)]/2}{[(R-G)^2+(R-B)(G-B)]^{1/2}} \right\}$$

$$S = 1 - \frac{3}{R+G+B}[\min(R,G,B)]$$

$$I = \frac{1}{3}(R+G+B)$$

- Result for normalized (circular) representation
- Take care to note which HSI representation is being used
- HSI to RGB conversion depends on hue region

HSI Component Example



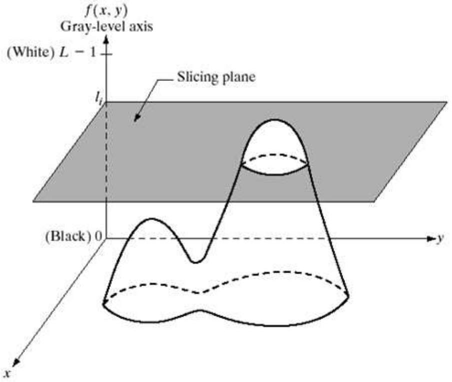
- HSI representation of the color cube
  - Normalized values represented as gray values
  - Only values on surface cube shown
- Explain:
  - Sharp transition in hue
  - Dark and light corners in saturation
  - Uniform intensity

Pseudocolor Image Processing

- Assigning colors to gray values yields Pseudocolor (false color) images
- Assignment criteria is application-specific
- Intensity (density) slicing
  - Assign colors based on gray value relation to slicing plane

$$f(x,y) = c_k, \text{ if } f(x,y) \in I_k$$

where c_k is the color associated with the kth intensity interval I_k.



Density Slicing Example

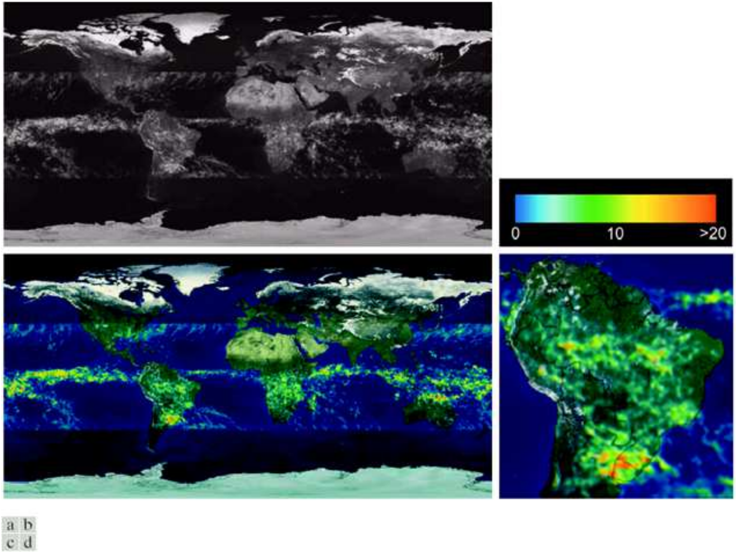
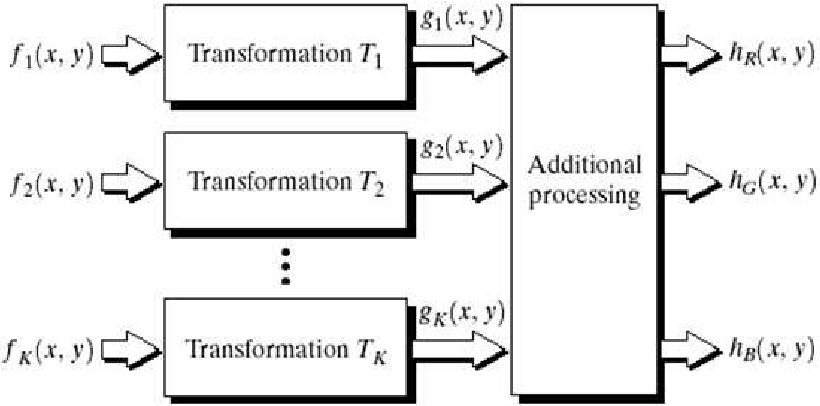


FIGURE (a) Gray-scale image in which intensity (in the lighter horizontal band shown) corresponds to average monthly rainfall. (b) Colors assigned to intensity values. (c) Color-coded image. (d) Zoom of the South America region. (Courtesy of NASA.)

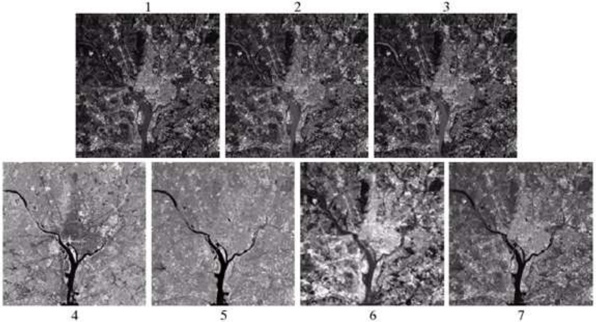
Multispectral Extensions



- Pseudocolor is often used in the visualization of multispectral images
  - Example of satellite images: visible spectrum, infrared, radio waves, etc.
  - Transformations are application and spectral band dependent

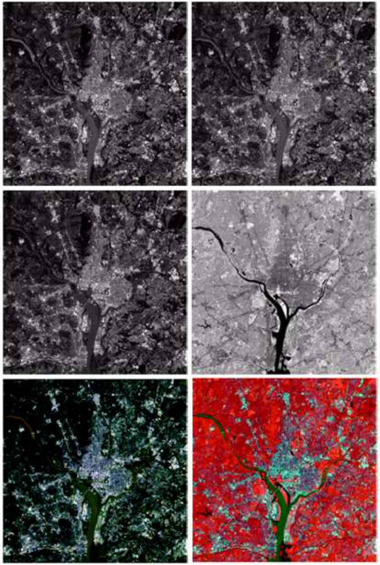
Washington DC LANDSAT Example (I)

Band No.	Name	Wavelength (μm)	Characteristics and Uses
1	Visible blue	0.45–0.52	Maximum water penetration
2	Visible green	0.52–0.60	Good for measuring plant vigor
3	Visible red	0.63–0.69	Vegetation discrimination
4	Near infrared	0.76–0.90	Biomass and shoreline mapping
5	Middle infrared	1.55–1.75	Moisture content of soil and vegetation
6	Thermal infrared	10.4–12.5	Soil moisture; thermal mapping
7	Middle infrared	2.08–2.35	Mineral mapping



Wash. DC LANDSAT Example (I)

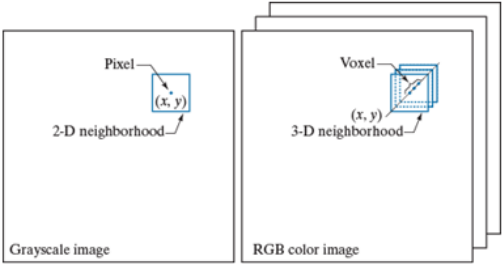
- Images in bands 1-4
- Bottom left: color composite image using
  - Band 1 (visible blue) as blue
  - Band 2 (visible green) as green
  - Band 3 (visible red) as red
  - Result is difficult to analyze
- Bottom right: color composite image using
  - Bands 1 and 2 as above
  - Band 4 (near infrared) as red
  - Better distinguishes between biomass (red dominated) and man-made structures



Basics of Full-Color Image Processing

- Per-component-image processing
- Vector-based processing

$$\mathbf{c}(x,y) = \begin{bmatrix} c_R(x,y) \\ c_G(x,y) \\ c_B(x,y) \end{bmatrix} = \begin{bmatrix} R(x,y) \\ G(x,y) \\ B(x,y) \end{bmatrix}$$



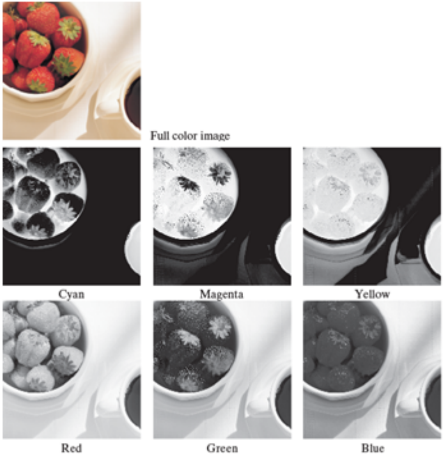
a b

FIGURE 7.27 Spatial neighborhoods for grayscale and RGB color images. Observe in (b) that a *single* pair of spatial coordinates, (x, y), addresses the same spatial location in all three images.

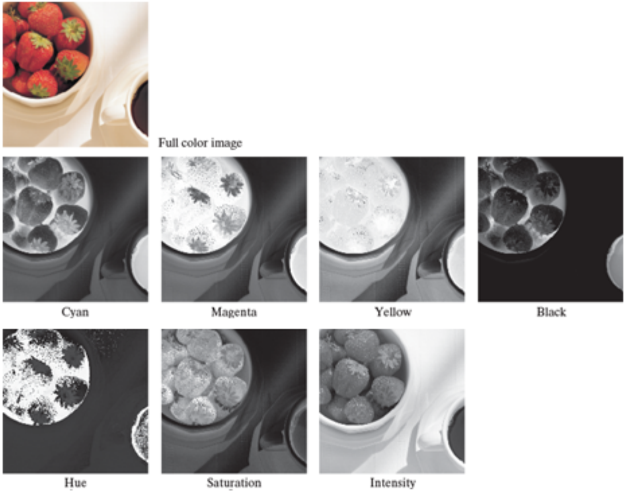
Color Transformations

$$s_i = T_i(r_i) \quad i = 1, 2, ..., n$$

- $n \rightarrow$ number of component images
- $T_i \rightarrow$ set of transformation or color mapping functions
- Performed in any color model
- Some are better suited to specific models



Color Transformations



Modifying the intensity of a full-color image

$$s_i = T_i(r_i) \quad i = 1, 2, ..., n$$

- HSI color space

$$\begin{aligned} s_1 &= r_1 \\ s_2 &= r_2 \\ s_3 &= kr_3 \end{aligned}$$

- RGB color space

$$s_i = kr_i \quad i = 1, 2, 3$$

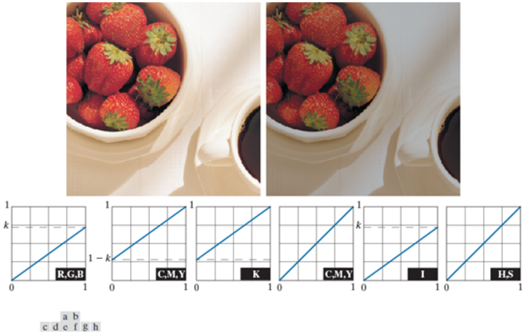


FIGURE 7.29 Adjusting the intensity of an image using color transformations. (a) Original image. (b) Result of decreasing its intensity by 30% (i.e., letting $k = 0.7$). (c) The required RGB mapping function. (d)–(e) The required CMYK mapping functions. (f) The required CMY mapping function. (g)–(h) The required HSI mapping functions. (Original image courtesy of MedData Interactive.)

Modifying the intensity of a full-color image

$$s_i = T_i(r_i) \quad i = 1, 2, ..., n$$

- CMY color space

$$s_i = kr_i + (1-k) \quad i = 1, 2, 3$$

- CMYK color space

$$s_i = \begin{cases} r_i & i = 1, 2, 3 \\ kr_i + (1-k) & i = 4 \end{cases}$$

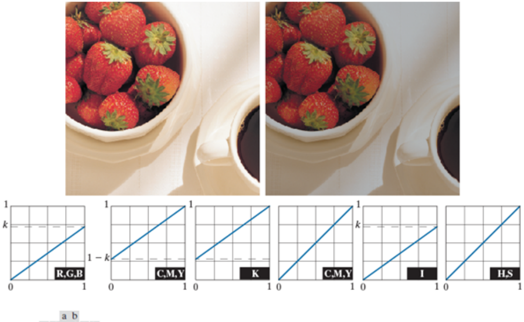
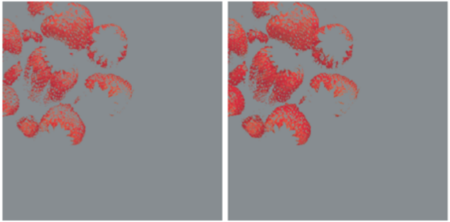


FIGURE 7.29 Adjusting the intensity of an image using color transformations. (a) Original image. (b) Result of decreasing its intensity by 30% (i.e., letting $k = 0.7$). (c) The required RGB mapping function. (d)–(e) The required CMYK mapping functions. (f) The required CMY mapping function. (g)–(h) The required HSI mapping functions. (Original image courtesy of MedData Interactive.)

Color Slicing

- Stand out a color of interest from the background.
- Use the region defined as a mask for further processing, e.g. segmentation.
- Vector-based processing

$$s_i = \begin{cases} 0.5 & \text{IF } \left[|r_j - a_j| > \frac{W}{2} \right]_{\text{ANY } 1 \le j \le n} \\ r_i & \text{OTHERWISE} \end{cases} \qquad i = 1, 2, \ldots, n$$



a b

FIGURE 7.32 Color-slicing transformations that detect (a) reds within an RGB cube of width $W = 0.2549$ centered at $(0.6863, 0.1608, 0.1922)$, and (b) reds within an RGB sphere of radius 0.1765 centered at the same point. Pixels outside the cube and sphere were replaced by color $(0.5, 0.5, 0.5)$.

Tonal Corrections

- Stand out a color of interest from the background.
- Equally distribution of the intensities of a color image is desired.
- Adjust image's brightness and contrast.
- S-shape curve is used for boosting contrast.
- Power-law transformations.

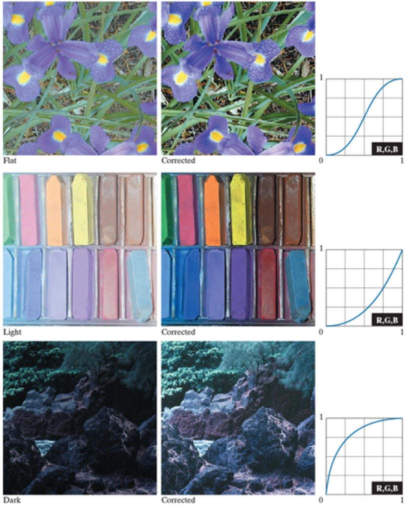


FIGURE 7.33 Tonal corrections for flat, light (high key), and dark (low key) color images. Adjusting the red, green, and blue components equally does not always alter the image hues significantly.

Color Balancing

- White balance is used to correct the effect of light.

$$s_i = \frac{W_s}{W_r} r_i$$

where W_s represents a destination color and W_r represents a source color.

- Skin tones are also used for color balance.

FIGURE 7.34 Color balancing a CMYK image.

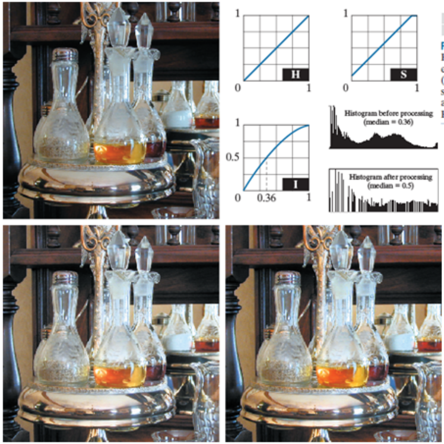
Histogram Processing of Color Images

- Transformation that produces an image with a uniform histogram.

$$s_k = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$k = 0, 1, 2, \ldots, L-1$$

- Would it be wise to equalize color components independently?
- Alter colors of the image.
- HSI color space is ideal.



a b
c d

FIGURE 7.35 Histogram equalization (followed by saturation adjustment) in the HSI color space.

Color Image Smoothing

- Per-component-image processing
- Vector-based processing

$$\bar{\mathbf{c}}(x,y) = \begin{bmatrix} \frac{1}{K}\sum_{(s,t)\in S_{xy}} R(s,t) \\ \frac{1}{K}\sum_{(s,t)\in S_{xy}} G(s,t) \\ \frac{1}{K}\sum_{(s,t)\in S_{xy}} B(s,t) \end{bmatrix}$$

a b c

FIGURE 7.38 Image smoothing with a 5×5 averaging kernel. (a) Result of processing each RGB component image. (b) Result of processing the intensity component of the HSI image and converting to RGB. (c) Difference between the two results.

Color Image Sharpening

$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

- Laplacian of each component image

$$\nabla^2[\mathbf{c}(x,y)] = \begin{bmatrix} \nabla^2 R(x,y) \\ \nabla^2 G(x,y) \\ \nabla^2 B(x,y) \end{bmatrix}$$

$$s_i(x,y) = r_i(x,y) + \nabla^2[r_i(x,y)]$$

a b c

FIGURE 7.39 Image sharpening using the Laplacian. (a) Result of processing each RGB channel. (b) Result of processing the HSI intensity component and converting to RGB. (c) Difference between the two results.

Segmentation

- Partitioning an image into a collection of regions or objects based on:
  - Discontinuities (edge-based).
  - Similarity (predefined criteria).

a b c
d e f

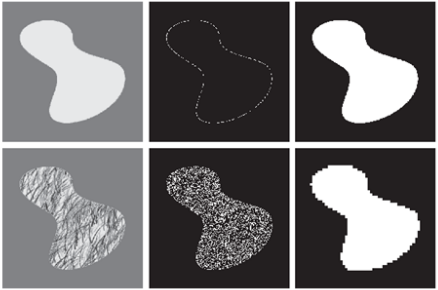


FIGURE 10.1 (a) Image of a constant intensity region. (b) Boundary based on intensity discontinuities. (c) Result of segmentation. (d) Image of a texture region. (e) Result of intensity discontinuity computations (note the large number of small edges). (f) Result of segmentation based on region properties.

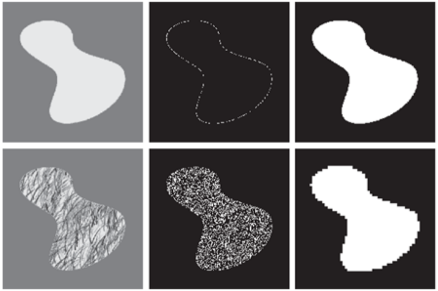
Segmentation

- R represent the entire region occupied by the image.

$$\bigcup_{i=1}^{n} R_i = R$$
$$R_i \text{ is a connected set, for } i = 0, 1, 2, \ldots, n$$
$$R_i \cap R_j = \oslash \text{ for all } i \text{ and } j, i \neq j$$
$$Q(R_i) = \text{TRUE for } i = 0, 1, 2, \ldots, n$$
$$Q(R_i \cup R_j) = \text{ FALSE for any adjacent region } R_i \text{ and } R_j$$

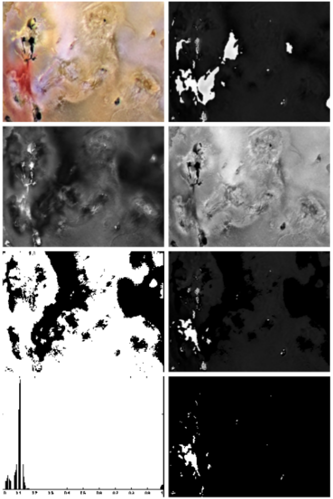
a b c
d e f

FIGURE 10.1 (a) Image of a constant intensity region. (b) Boundary based on intensity discontinuities. (c) Result of segmentation. (d) Image of a texture region. (e) Result of intensity discontinuity computations (note the large number of small edges). (f) Result of segmentation based on region properties.



Segmentation in HSI Color Space

- Saturation is used as a mask.
- Intensity is not used for segmentation of color images.
- Range of hue values of the regions of interest is used as descriptor.
- The product of the mask with the hue is determined.
- The segmented image is obtained by thresholding this product.



a b c d e f g h

FIGURE 7.40 Image segmentation in HSI space. (a) Original. (b) Hue. (c) Saturation. (d) Intensity. (e) Binary saturation mask (black = 0). (f) Product of (b) and (e). (g) Histogram of (f). (h) Segmentation of red components from (a).

Segmentation in RGB Color Space

- Samples of color of interest (**a**).
- Classify each pixel in the image (**z**).

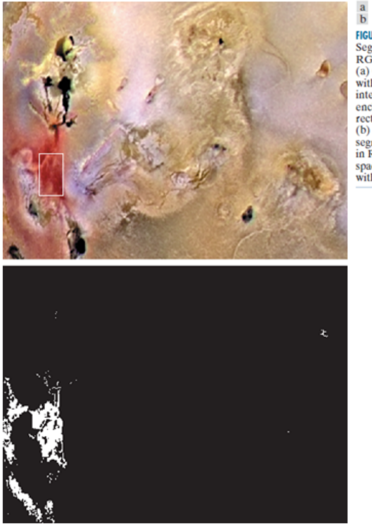


FIGURE 7.42 Segmentation in RGB space. (a) Original image with colors of interest shown enclosed by a rectangle. (b) Result of segmentation in RGB vector space. Compare with Fig. 7.40(h).

Segmentation in RGB Space

- Euclidean distance to measure of similarity

$$D(\mathbf{z},\mathbf{a}) = \|\mathbf{z}-\mathbf{a}\|$$

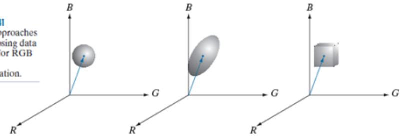
$$D(\mathbf{z},\mathbf{a}) = [(\mathbf{z}-\mathbf{a})^T(\mathbf{z}-\mathbf{a})]^{1/2}$$

$$D(\mathbf{z},\mathbf{a}) = [(\mathbf{z}-\mathbf{a})^T C^{-1}(\mathbf{z}-\mathbf{a})]^{1/2}$$

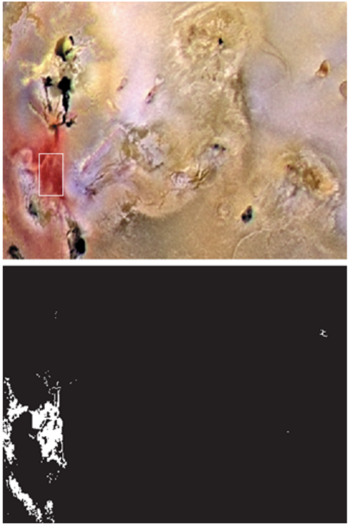
C is the covariance matrix of the samples.

a b c

FIGURE 7.41 Three approaches for enclosing data regions for RGB vector segmentation.



Segmentation in RGB Color Space



a
b

FIGURE 7.42 Segmentation in RGB space. (a) Original image with colors of interest shown enclosed by a rectangle. (b) Result of segmentation in RGB vector space. Compare with Fig. 7.40(h).

Segmentation in Lab Color Space

fabric

- How many colors can you distinguish from the background?

Segmentation in Lab Color Space

- How many colors can you distinguish from the background?
- Six: red, green, purple, yellow, and magenta

Segmentation in Lab Color Space - Nearest Neighbor Classification

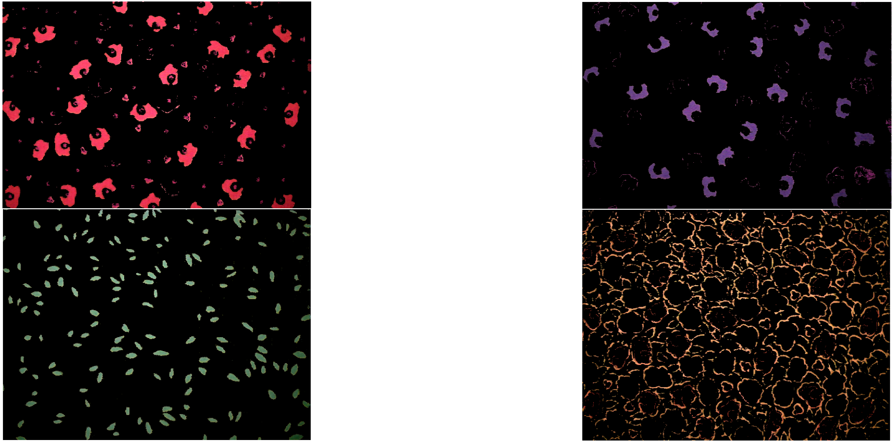
- Uniform changes of components in the Lab color space correspond to uniform changes in perceived color.
- Perceived color differences are measured by Euclidean distances.
- Segmentation can be performed by means of clustering.

There are K clusters with sample mean $\mathbf{a}_k$.

$$D_k(\mathbf{z}, \mathbf{a}) = \|\mathbf{z} - \mathbf{a}_k\| \qquad k = 1, 2, ..., N$$

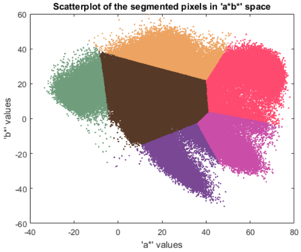
Every pixel is assigned to the class that minimizes the color difference.

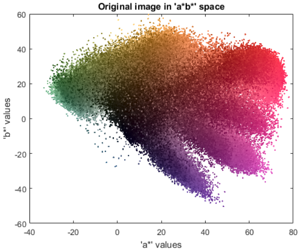
Segmentation in Lab Color Space

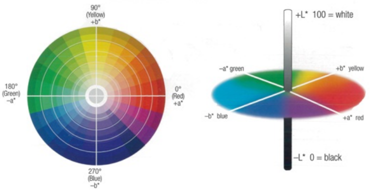


https://www.mathworks.com/help/images/examples/color-based-segmentation-using-the-l-a-b-color-space.html

Segmentation in Lab Color Space



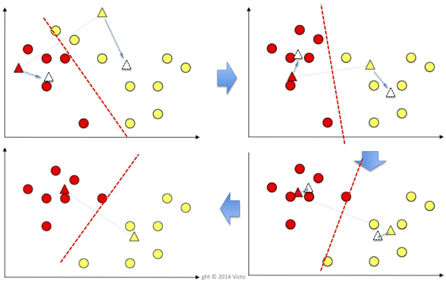




Segmentation using k-means clustering

- Partition of a set Q, of observations into a specified number, k, of clusters.
- Assign to the cluster with the nearest mean.
- Iterative procedure.

K-means clustering example



Segmentation using k-means clustering

- Let **Z** be the color pixel dataset of the form

$$\mathbf{Z} = \{\mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_Q\}$$

where $\mathbf{z} \in R^n$

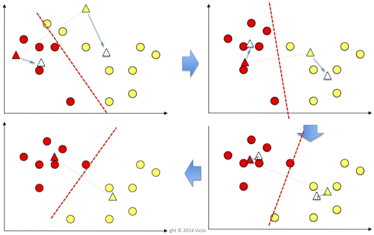
- We want to classify the data into k disjoint sets of the form

$$C = \{C_1, C2, ..., C_k\}$$

such that the criterion of optimality is satisfied

$$\arg\min_{C} = \left(\sum_{i=1}^{k} \sum_{\mathbf{z} \in C_i} \|\mathbf{z} - \mathbf{m}_i\|^2 \right)$$

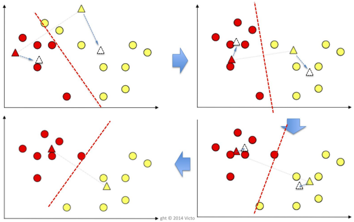
K-means clustering example



k-means Algorithm

- Initialize the algorithm: $\mathbf{m}_i(1),\ i = 1, 2, ..., k$

K-means clustering example



- Assign samples to clusters whose mean is the closest.

$$\mathbf{z}_q \to C_i \qquad \text{IF } \|\mathbf{z}_q - \mathbf{m}_i\|^2$$

$$j = 1, 2, ..., k (j \neq i); \quad q = 1, 2, ..., Q$$

- Update the clusters' means

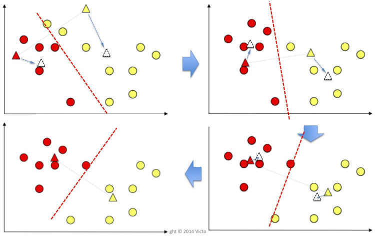
$$\mathbf{m}_i = \frac{1}{C_i} \sum_{\mathbf{z} \in C_i} \mathbf{z} \quad i = 1, 2, ..., k$$

where $|C_i|$ is the number of samples in cluster set C_i.

k-means Algorithm

- Compute residual error, E, as the sum of the k Euclidean norms of the differences between the mean vectors in the current and previous steps. Stop if $E \leq T$, where T is a specified threshold.

K-means clustering example



Segmentation using k-means clustering in different Color Spaces

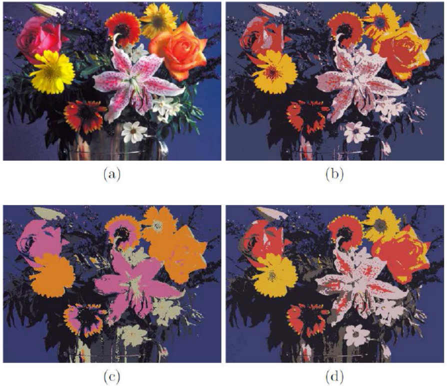


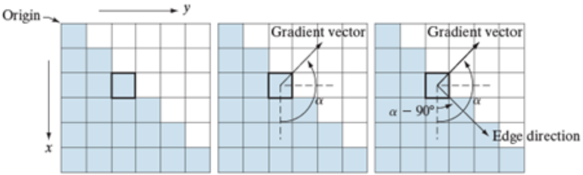
Figure 5.7 (a) Original color image. Results of segmentation using the k-means clustering algorithm in three different color spaces: (b) $sRGB$, (c) HSV, and (d) $L^*a^*b^*$. In each case, the image has been segmented into six regions. The color assigned to each region is the final centroid of the color values in the corresponding region in the original image after application of the k-means algorithm.

Component-based Color Edge Detection

$$\nabla f(x,y) = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \qquad M(x,y) = \sqrt{g_x^2 + g_y^2} \qquad \alpha(x,y) = \tan^{-1} = \left[\frac{g_y}{g_x}\right]$$

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Prewitt Operators



a b c

FIGURE 10.12 Using the gradient to determine edge strength and direction at a point. Note that the edge direction is perpendicular to the direction of the gradient vector at the point where the gradient is computed. Each square represents one pixel. (Recall from Fig. 2.19 that the origin of our coordinate system is at the top, left.)

Component-based Color Edge Detection

a b c d
e f g h

FIGURE 7.43 (a)–(c) R, G, and B component images, and (d) resulting RGB color image. (e)–(g) R, G, and B component images, and (h) resulting RGB color image.

Color Edge Detection

- One of the ways that the concept of a gradient can be extend to vector functions.

$$\mathbf{u} = \frac{\partial R}{\partial x}\mathbf{r} + \frac{\partial G}{\partial x}\mathbf{g} + \frac{\partial B}{\partial x}\mathbf{b}$$

$$\mathbf{v} = \frac{\partial R}{\partial y}\mathbf{r} + \frac{\partial G}{\partial y}\mathbf{g} + \frac{\partial B}{\partial y}\mathbf{b}$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0		0	−1
0	1		1	0

Roberts

−1	−1	−1		−1	0	1
0	0	0		−1	0	1
1	1	1		−1	0	1

Prewitt

−1	−2	−1		−1	0	1
0	0	0		−2	0	2
1	2	1		−1	0	1

Sobel

a
b c
d e
f g

FIGURE 10.14 A 3×3 region of an image (the z's are intensity values), and various kernels used to compute the gradient at the point labeled z_5.

Color Edge Detection

$$g_{xx} = \mathbf{u} \cdot \mathbf{u} = \mathbf{u}^T \mathbf{u} = \left| \frac{\partial R}{\partial x} \right|^2 + \left| \frac{\partial G}{\partial x} \right|^2 + \left| \frac{\partial B}{\partial x} \right|^2$$

$$g_{yy} = \mathbf{v} \cdot \mathbf{v} = \mathbf{v}^T \mathbf{v} = \left| \frac{\partial R}{\partial y} \right|^2 + \left| \frac{\partial G}{\partial y} \right|^2 + \left| \frac{\partial B}{\partial y} \right|^2$$

$$g_{xy} = \mathbf{u} \cdot \mathbf{v} = \mathbf{u}^T \mathbf{v} = \frac{\partial R}{\partial x}\frac{\partial R}{\partial y} + \frac{\partial G}{\partial x}\frac{\partial G}{\partial y} + \frac{\partial B}{\partial x}\frac{\partial B}{\partial y}$$

The direction and value of maximum rate of change of $\mathbf{c}(x, y)$ is given by

$$\theta(x, y) = \frac{1}{2} \tan^{-1} \left[\frac{2g_{xy}}{g_{xx} - g_{yy}} \right]$$

$$F_\theta(x, y) = \left\{ \frac{1}{2} [(g_{xx} + g_{yy}) + (g_{xx} - g_{yy}) \cos 2\theta(x, y) + 2g_{xy} \sin 2\theta(x, y)] \right\}^{\frac{1}{2}}$$

Color Edge Detection

a b
c d

FIGURE 7.44
(a) RGB image. (b) Gradient computed in RGB color vector space. (c) Gradient image formed by the elementwise sum of three individual gradient images, each computed using the Sobel operators. (d) Difference between (b) and (c).

Noise in Color Images

a b
c d

FIGURE 7.46 (a)–(c) Red, green, and blue 8-bit component images corrupted by additive Gaussian noise of mean 0 and standard deviation of 28 intensity levels. (d) Resulting RGB image. [Compare (d) with Fig. 7.44(a).]

Noise in Color Images

a b c

FIGURE 7.47 HSI components of the noisy color image in Fig. 7.46(d). (a) Hue. (b) Saturation. (c) Intensity.

Noise in Color Images

FIGURE 7.48 (a) RGB image with green plane corrupted by salt-and-pepper noise. (b) Hue component of HSI image. (c) Saturation component. (d) Intensity component.