

ELEG404/604: Digital Imaging & Photography

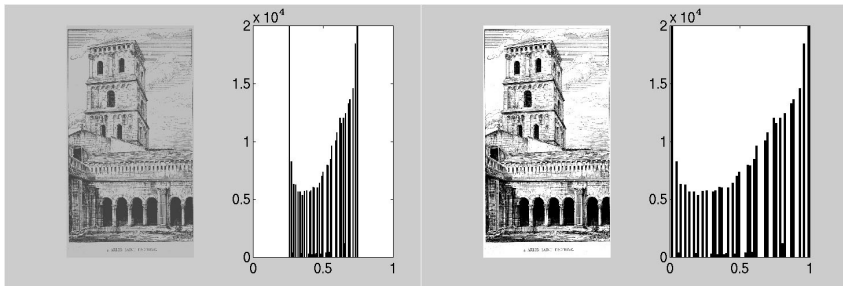
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Chapter II

Image Enhancement in the Spatial Domain

- ▶ Algorithms for improving the visual appearance of images
 - ▶ Gamma correction
 - ▶ Contrast improvements
 - ▶ Histogram equalization
 - ▶ Noise reduction
 - ▶ Image sharpening
- ▶ Optimality is in the eye of the observer: Ad hoc.



What is a histogram

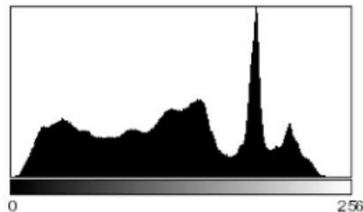
Each individual histogram entry is defined as

$$h(i) = \text{Number of pixels in } l \text{ with the intensity value } i$$

for all $0 \leq i < K$, that is

$$h(i) = \text{card} \{ (u, v) \mid I(u, v) = i \}$$

$h(0)$ is the number of pixels with the value 0, $h(1)$ the number of pixels with the value 1, and so forth.



Count: 1920000
Mean: 118.848
StdDev: 59.179

Min: 0
Max: 251
Mode: 184 (30513)

Histograms

Three very different images with identical histograms.

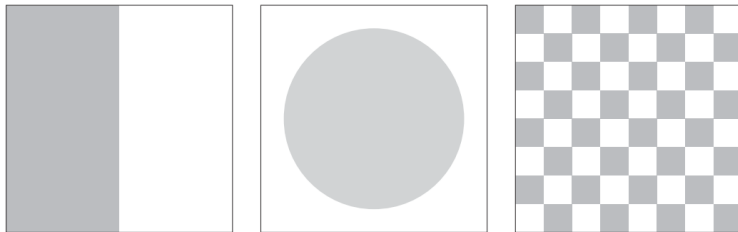
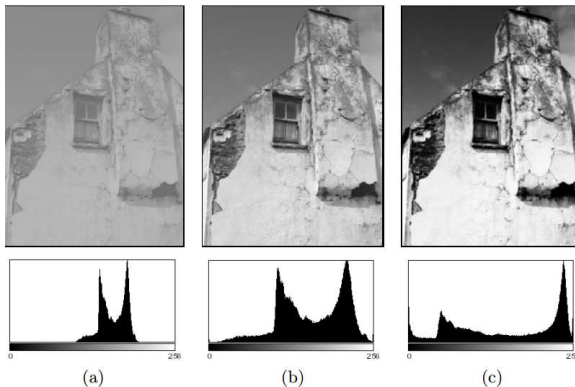


Image Acquisition- Contrast

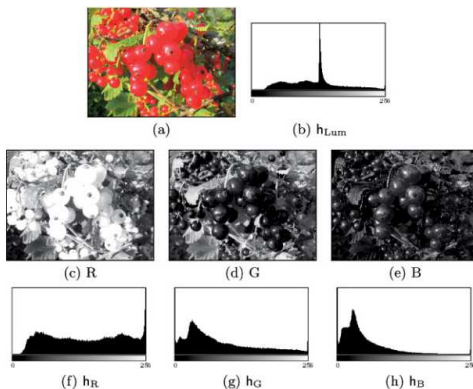
- ▶ Range of intensity values effectively used.
- ▶ Difference between the image's maximum and minimum pixel values.
- ▶ A full-contrast image makes effective use of the entire range of available intensity values from $a = a_{min} \cdots a_{max} = 0 \cdots K - 1$ (black to white).



(a) Low contrast, (b) Normal contrast, (c) High contrast

Color Image Histograms

Histograms of an RGB color image: (a) original image, (b) luminance histogram h_{Lum} , (c–e) RGB color components, and associated component histograms h_R, h_G, h_B (f–h). All three color channels have saturation problems. The spike in the distribution resulting from this is found in the middle of the luminance histogram (b).



Spatial Domain Processing

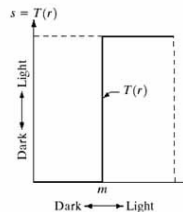
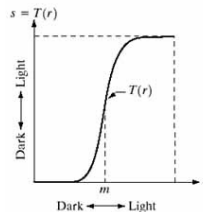
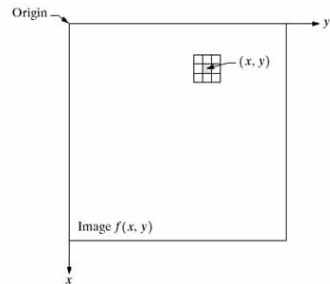
- ▶ Utilize neighborhood operations

$$g(x, y) = T[f(x, y)]$$

- ▶ Simple case: point operations

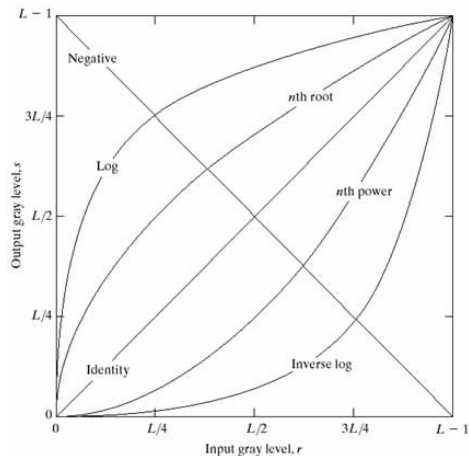
$$s = T(r)$$

- ▶ Contrast stretching
- ▶ Thresholding



Gray Level Transformation Curves

- More general transformations
 - Log
 - Inverse Log
 - n^{th} power
 - n^{th} root
- Used to map narrow dark (log/root) or bright (inverse log/power) range to a greater dynamic range

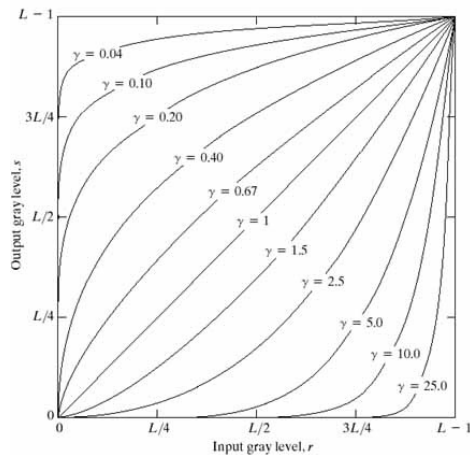


Power-Law (Gamma) Transformations

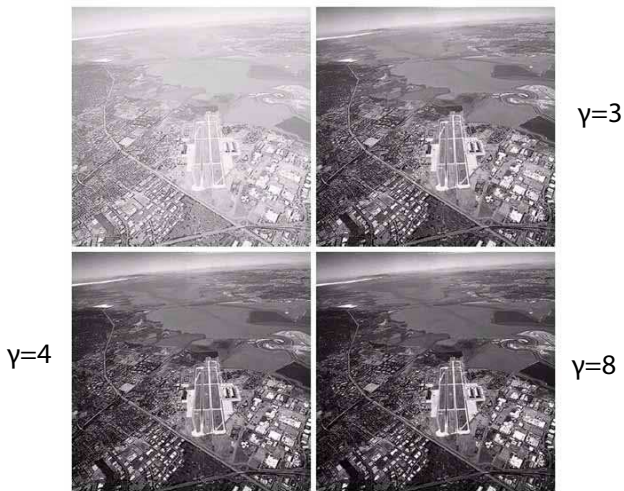
- ▶ Power-law transformation:

$$s = cr^\gamma$$

- ▶ Many devices require gamma correction
 - ▶ CRTs have power function intensity-to-voltage responses
 - ▶ Monitor specific
 - ▶ Applied in color planes



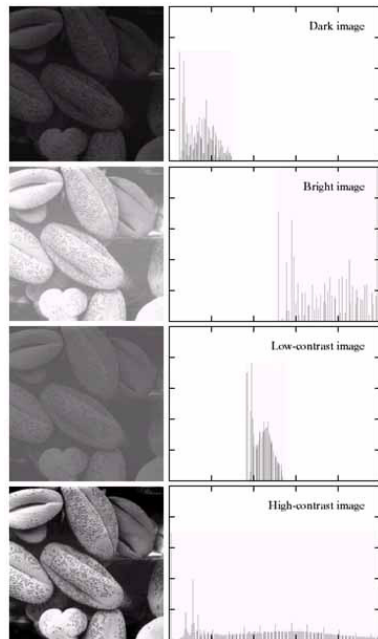
Gamma Correction Example - Aerial



Top Left: aerial image. Top right, Bottom left and bottom right: results of applying Gamma correction with $c = 1$ and $\gamma = 3, 4, 5$ respectively.

Histogram Processing

- ▶ Light, dark, and low contrast images have concentrated histograms
- ▶ Images with uniform histograms
 - ▶ Contain the full range of gray values
 - ▶ Have high contrast
 - ▶ Better general visual appearance



Histogram Equalization

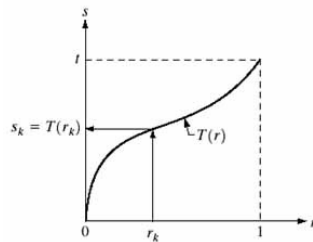
- ▶ We focus on the (normalized) scalar mapping

$$s = T(r) \quad 0 \leq r \leq 1$$

where the following are satisfied:

- ▶ $T(r)$ is single-valued and monotonically increasing in $[0,1]$
- ▶ $0 \leq T(r) \leq 1$ for $0 \leq r \leq 1$
- ▶ The single-valued condition allows the inverse transformation to be defined

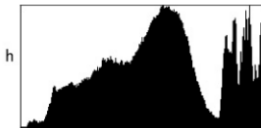
$$r = T^{-1}(s) \quad 0 \leq s \leq 1$$



Probability Density Function

- ▶ Let the PDF of r be the $p_r(r)$
- ▶ The CDF is $P_r(r) = \int_0^r p_r(w)dw$
 - ▶ Note CDFs are monotonically increasing and have range $[0,1]$
- ▶ Defined the RV $s = T(r)$
 - ▶ The PDF of a RV function is

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right|$$



CDF Distribution

- ▶ Set $T(r) = P_r(r)$

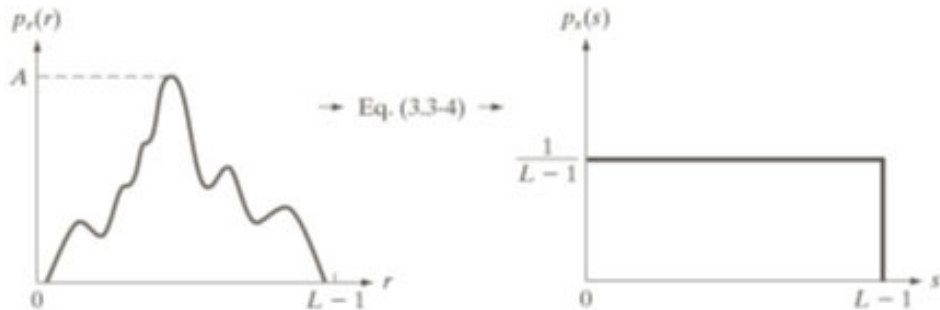
$$\begin{aligned}\frac{ds}{dr} &= \frac{dT(r)}{dr} \\ \frac{d[\int_0^r p_r(w)dw]}{dr} &= p_r(r)\end{aligned}$$

- ▶ Thus the PDF of s is

$$\begin{aligned}p_s(s) &= p_r(r) \left| \frac{dr}{ds} \right| \\ p_r(r) \left| \frac{1}{p_r(r)} \right| &= 1\end{aligned}$$

- ▶ The CDF is uniformly distributed.

Histogram Equalization



a b

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r . The resulting intensities, s , have a uniform PDF, independently of the form of the PDF of the r 's.

Histogram Equalization

A 3-bit image ($L = 8$) of size 64×64 ($MN = 4096$) has the intensity distribution in table.

Intensity levels are integers in $[0, L - 1] = [0, 7]$.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1
Intensity
distribution and
histogram values
for a 3-bit,
 64×64 digital
image.

Histogram Equalization

Values of the histogram equalization are obtained using:

$$s_0 = T(r_0) = 7 \sum_{j=0}^0 P_r(r_j) = 7p_r(r_0) = 1.33$$

Similarly,

$$s_1 = T(r_1) = 7 \sum_{j=0}^1 P_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08$$

and $s_2 = 4.55$, $s_3 = 5.67$, $s_4 = 6.23$, $s_5 = 6.65$, $s_6 = 6.86$, $s_7 = 7.00$

The s values still have fractions because they were generated by summing probability values, so we round them to the nearest integer:

$$s_0 = 1.33 \rightarrow 1$$

$$s_1 = 3.08 \rightarrow 3$$

$$s_2 = 4.55 \rightarrow 5$$

$$s_3 = 5.67 \rightarrow 6$$

$$s_4 = 6.23 \rightarrow 6$$

$$s_5 = 6.65 \rightarrow 7$$

$$s_6 = 6.86 \rightarrow 7$$

$$s_7 = 7.00 \rightarrow 7$$

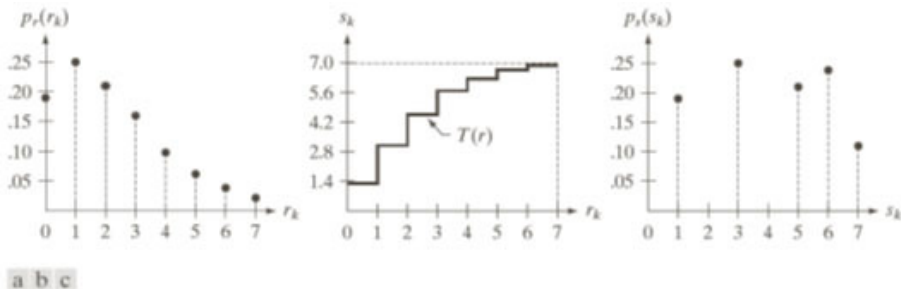
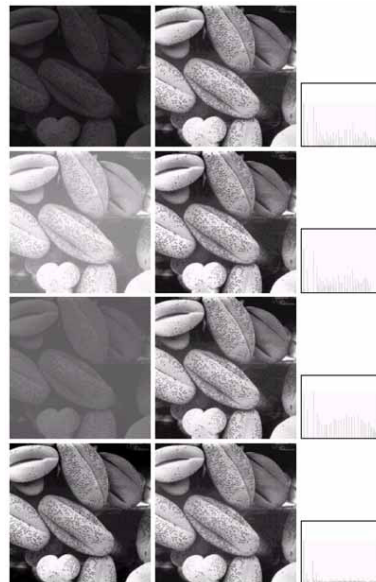


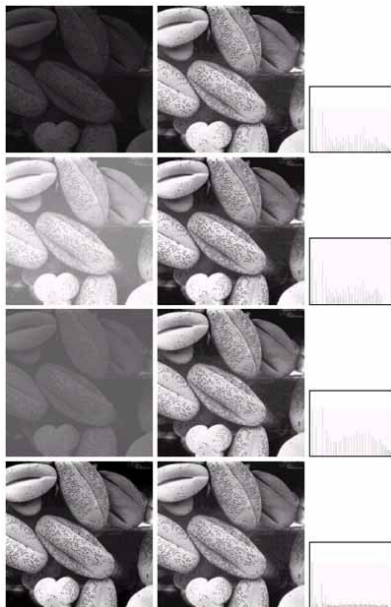
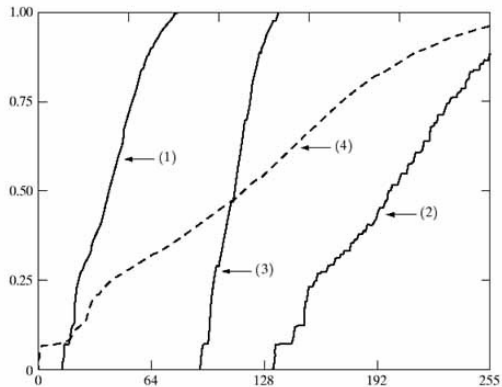
FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Histogram Equalization

- ▶ CDF mapping of gray values
 - ▶ Yields uniformed histogram
 - ▶ Simple, parameter-free
- ▶ Discrete case
 - ▶ Results not strictly uniform
 - ▶ Implementation issues



Histogram EQ Mappings



Histogram Equalization Result

Image



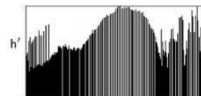
(a)

(b)

Histograms



(c)



(d)

Cumulative
Histograms



(e)



(f)

Original image and corresponding histograms. Notice that new peaks are created in the resulting histogram h (d) by merging original histogram cells.

Filters

What is a filter?

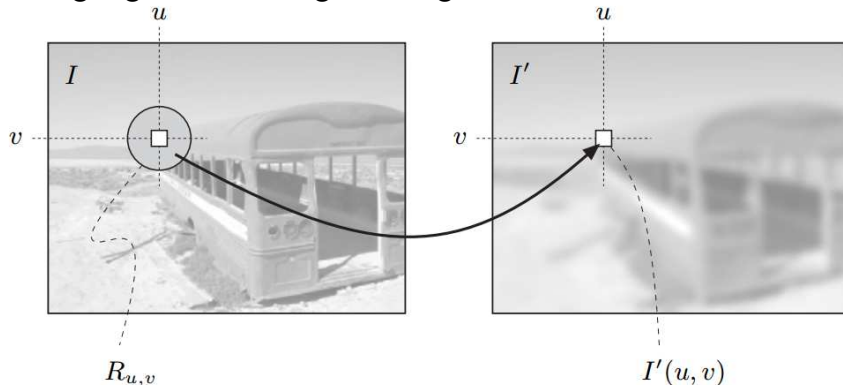
Main difference between filters and point operations is that filters generally use more than one pixel to compute each new pixel input.



No point operation can blur or sharpen an image.

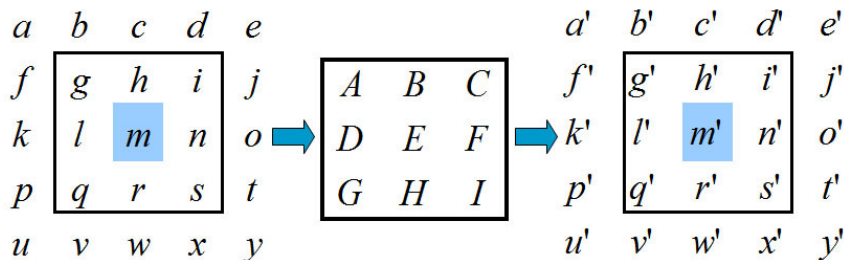
Filters

1 Each new pixel value $I'(x,y)$ is computed as a function of the pixels in a corresponding region in the original image I .



Filtering

To sharpen or filter an image that was taken out-of-focus and is blurred, each pixel is replaced with a linear combination of its neighbors.



$$m' = A \cdot g + B \cdot h + C \cdot i + D \cdot l + \dots$$
$$\dots + E \cdot m + F \cdot n + G \cdot q + H \cdot r + I \cdot s$$

Filtering

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>
<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>
<i>p</i>	<i>q</i>	<i>r</i>	<i>s</i>	<i>t</i>
<i>u</i>	<i>v</i>	<i>w</i>	<i>x</i>	<i>y</i>



<i>a'</i>	<i>b'</i>	<i>c'</i>	<i>d'</i>	<i>e'</i>
<i>f'</i>	<i>g'</i>	<i>h'</i>	<i>i'</i>	<i>j'</i>
<i>k'</i>	<i>l'</i>	<i>m'</i>	<i>n'</i>	<i>o'</i>
<i>p'</i>	<i>q'</i>	<i>r'</i>	<i>s'</i>	<i>t'</i>
<i>u'</i>	<i>v'</i>	<i>w'</i>	<i>x'</i>	<i>y'</i>

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>
<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>
<i>p</i>	<i>q</i>	<i>r</i>	<i>s</i>	<i>t</i>
<i>u</i>	<i>v</i>	<i>w</i>	<i>x</i>	<i>y</i>



<i>a'</i>	<i>b'</i>	<i>c'</i>	<i>d'</i>	<i>e'</i>
<i>f'</i>	<i>g'</i>	<i>h'</i>	<i>i'</i>	<i>j'</i>
<i>k'</i>	<i>l'</i>	<i>m'</i>	<i>n'</i>	<i>o'</i>
<i>p'</i>	<i>q'</i>	<i>r'</i>	<i>s'</i>	<i>t'</i>
<i>u'</i>	<i>v'</i>	<i>w'</i>	<i>x'</i>	<i>y'</i>

Simple Smoothing Masks

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \quad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

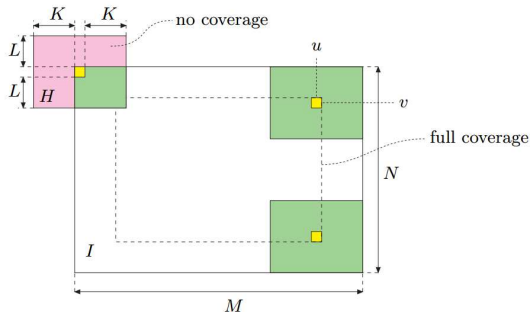
- ▶ Simplest linear filter: spatial average: reduces noise, but introduces blurring
- ▶ Distance weight samples
 - ▶ Centrally located samples are more important
 - ▶ Reduces blurring (somewhat)
 - ▶ Example above: simple integer arithmetic
- ▶ Alternative approach: utilize spectral characteristics

$$\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$



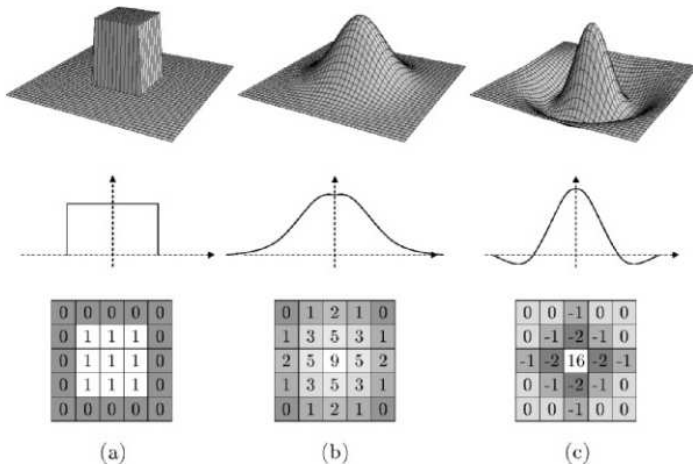
Border geometry

Filter can be applied only at locations (x, y) where the filter matrix H of size $(2K + 1) \times (2L + 1)$ is fully contained in the image.



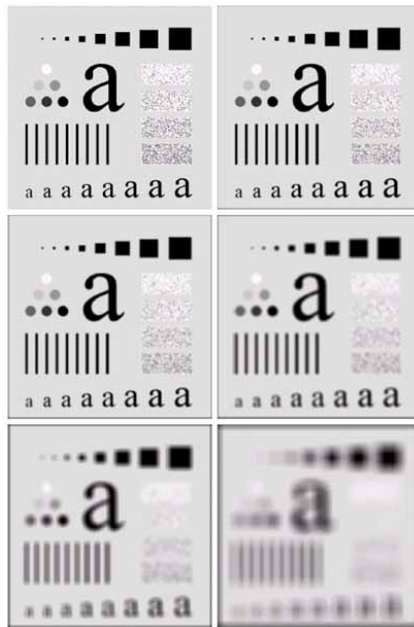
Linear Filters

Examples of linear filters. The “box” filter (a) and the Gauss filter (b) are both smoothing filters with all-positive coefficients. The “Laplace” or “Mexican hat” filter (c) is a difference filter.



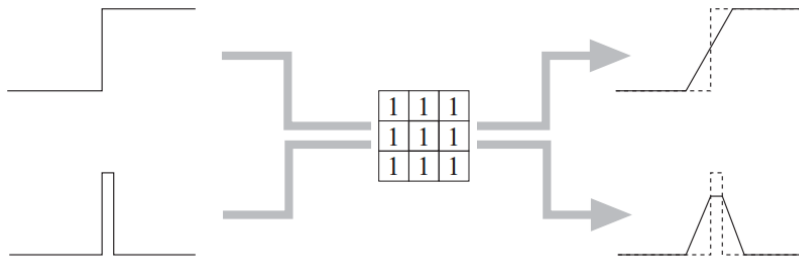
Smoothing Example

- ▶ Square averaging filter results
- ▶ Window sizes: 3, 5, 9, 15, and 35



NonLinear Filters

Any image structure is blurred by a linear filter. Image structures such as edges (top) or thin lines (bottom) are widened, and the local contrast is reduced.



Order-Statistic Filters

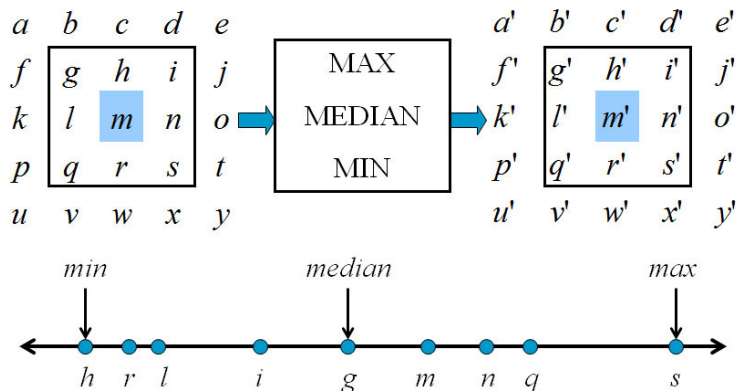
- ▶ Linear (weighted sum) filters
 - ▶ Blur edges and details
 - ▶ Are susceptible to outliers
- ▶ Order-statistic filters (nonlinear)
 - ▶ Preserve edges and details
 - ▶ Are less susceptible to outliers
- ▶ Spatially ordered samples: z_1, z_2, \dots, z_N
- ▶ Rank ordered samples: $z_{(1)}, z_{(2)}, \dots, z_{(N)}$

$$z_{(1)} \leq z_{(2)} \leq \dots \leq z_{(N)}$$
$$MED[z_1, z_2, \dots, z_N] = z_{((N+1)/2)}$$

Selection-type filter

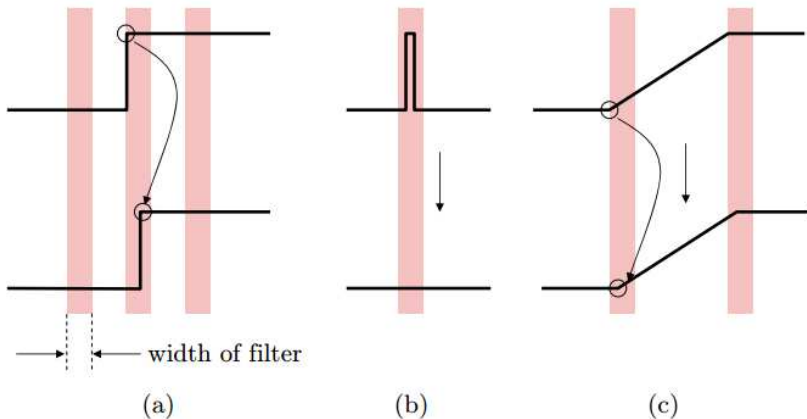
Filtering

Sometimes it is very useful to apply a non-linear filter such as the median.

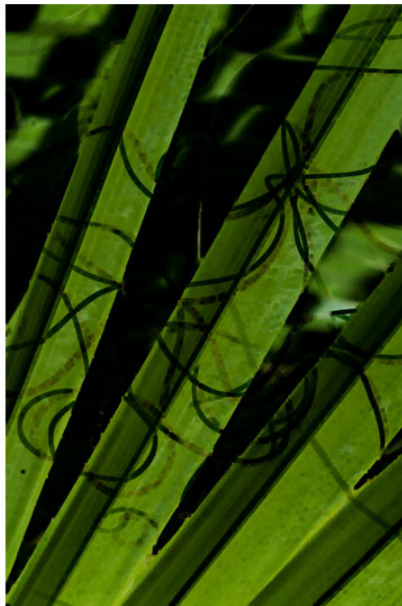
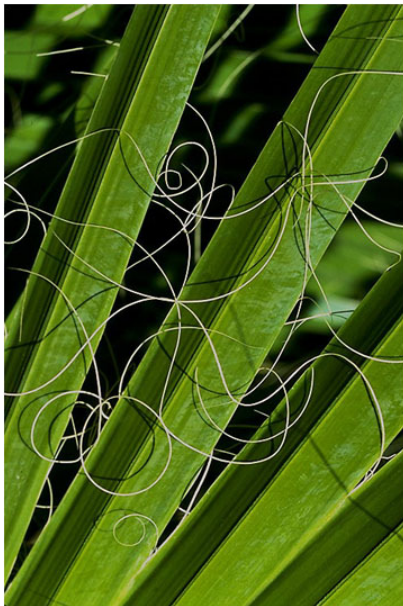


Effects of a minimum filter on various local signal structures

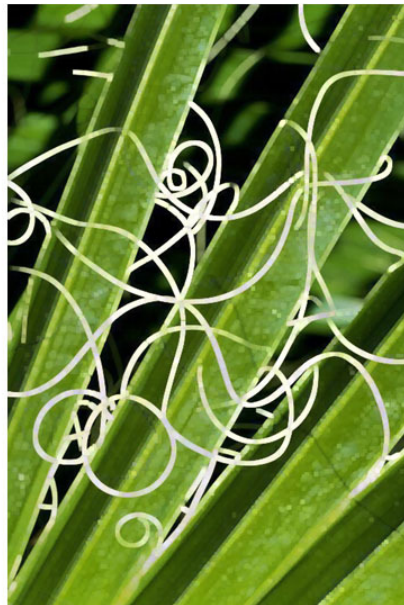
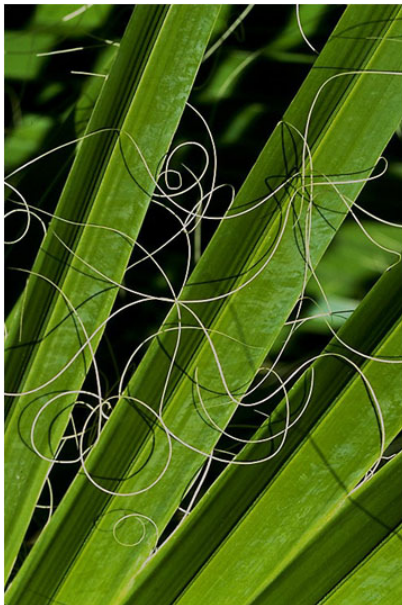
Original signal (top) and result after filtering (bottom), where color bars indicate the extent of the filter. Step edge (a) and linear ramp (c) are shifted to the right. Narrow pulse (b) is removed.



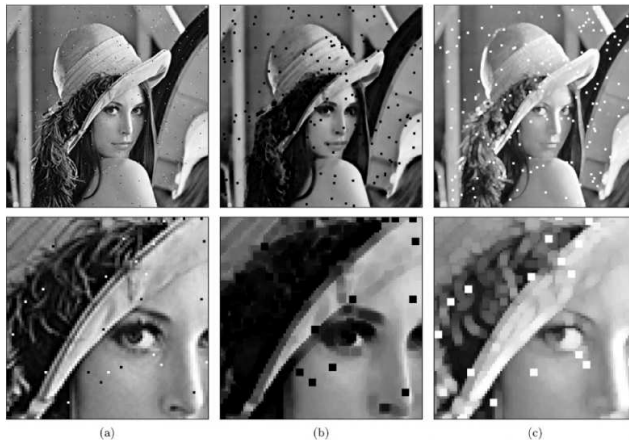
Minimum Filter



Maximum Filter



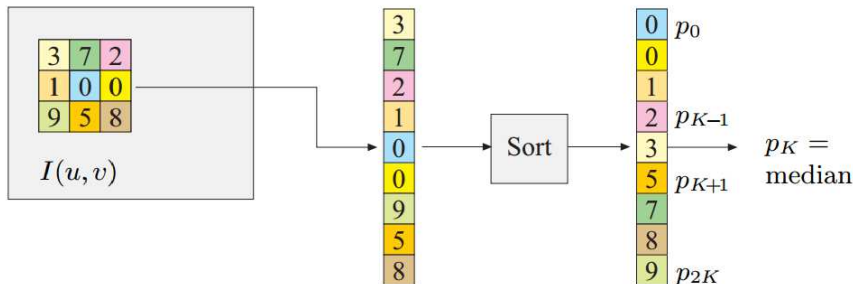
Minimum and Maximum filters applied to a grayscale image



The original image is corrupted with "salt and pepper" noise (a). The 3×3 pixel minimum filter eliminates the bright dots and widens all dark image structures (b). The maximum filter shows the exact opposite effects (c).

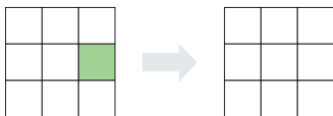
Median Filter

Computation of a 3×3 median filter. The nine pixel values extracted from the window are sorted and the center value is the median

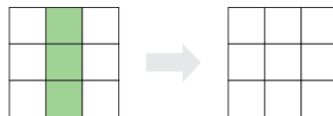


Effects of Median Filter

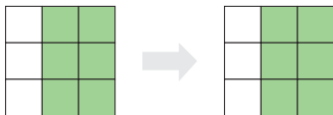
Effects of a 3×3 median filter on two-dimensional structures. Isolated dots are eliminated (a), as are thin lines (b). Step edge remains unchanged (c), corner is rounded off (d).



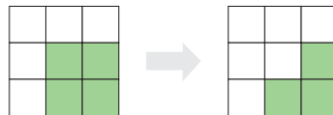
(a)



(b)

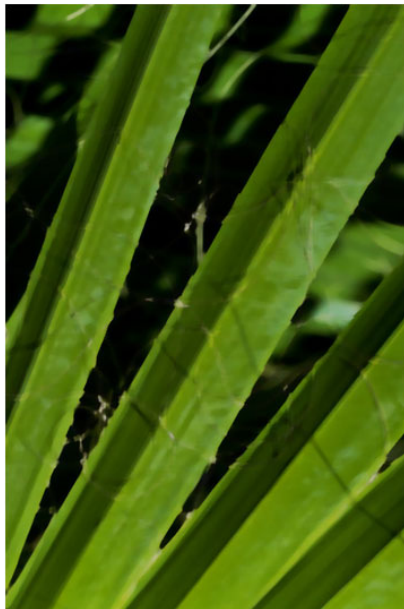
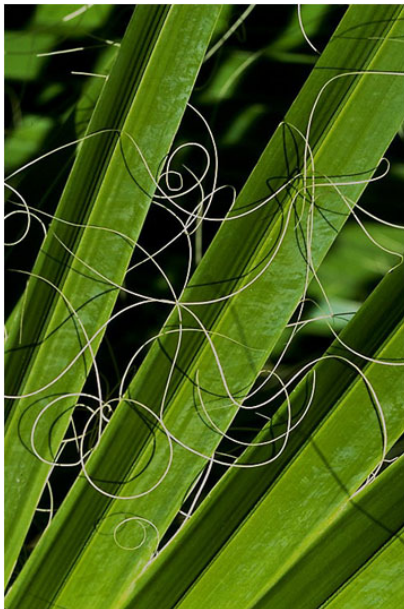


(c)



(d)

Median Filter



Linear smoothing filter vs median filter



(a) Original image corrupted with "salt-and-pepper" noise. (b) Linear 3×3 pixel box filter (b) reduces noise but the entire image is blurred. (c) The median filter eliminates the noise dots and keeps the structures largely intact.

Weighted Median Filters (WMF)

The median filter can be generalized to weighted order statistic filters.

Given $x(n) = [x_1, \dots, x_N]$, and a set of weights w_1, \dots, w_N , the WMF output is given by

$$y(n) = \text{MEDIAN} [x_1 \diamond w_1, x_2 \diamond w_2, \dots, x_N \diamond w_N] \quad (1)$$

where $x_i \diamond w_i$ denotes replication:

$$x_i \diamond w_i = \overbrace{x_i, x_i, \dots, x_i}^{w_i \text{ times}} \quad (2)$$

and $w_i \in \mathbb{Z}^+$

Center Weighted Median Filter (CWMF)

A special case is the center weighted median filter (CWMF)

$$y(n) = \text{MEDIAN}[x_1, x_2, \dots, x_{c-1}, x_c \diamond w_c, x_{c+1}, \dots, x_N] \quad (3)$$

where $c = (N + 1)/2 = N_1 + 1$ is the index of the center sample.

For $w_c = 1$, the CWMF reduces to a median filter.

For $w_c \geq N$ the CWMF is an identity filter.

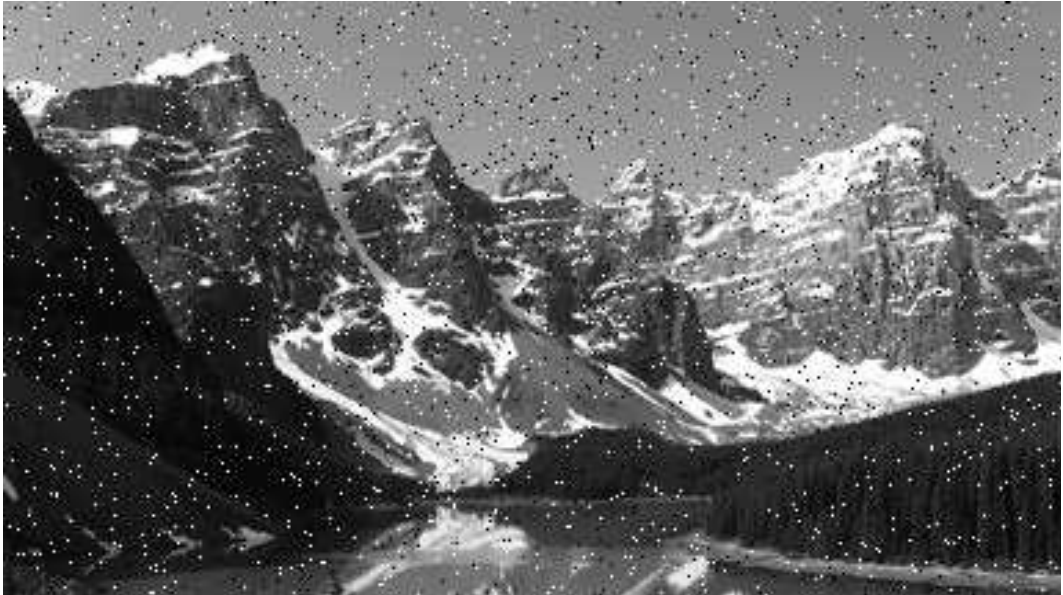
By varying the parameter

$$\begin{array}{ccc} \text{more} & \text{smoothing} & \text{less} \\ 1 & \xleftarrow{\quad w_c \quad} & N \end{array}$$

Original



Salt and pepper noise



5×5 window - $w_c = 1$



5×5 window - $w_c = 3$



5×5 window - $w_c = 5$



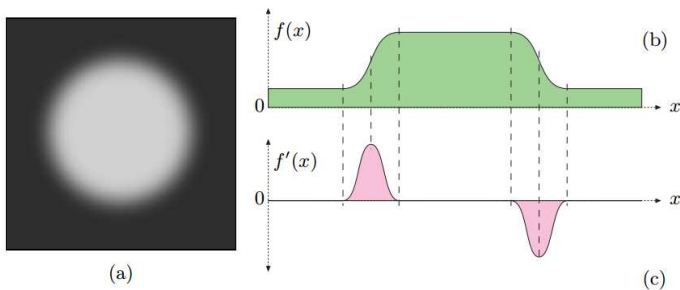
Sharpening Filters

- ▶ Objective: Highlight and enhance fine detail
 - ▶ Details may have been blurred in acquisition process
- ▶ Method: utilize first- and second-order derivative
 - ▶ Derivatives identify signal changes (details/features)
- ▶ First-derivative requirements:
 - ▶ Zero in flat regions
 - ▶ Nonzero along ramps
- ▶ Second-derivative requirements:
 - ▶ Zero in flat regions
 - ▶ Zero along ramps of constant slope

First derivative in one dimension

Original image (a), horizontal intensity profile $f(x)$ along the center image line (b), and first derivative $f'(x)$ (c).

$$\frac{df}{dx}(x) \approx \frac{f(x+1) - f(x-1)}{2} = 0.5 \cdot (f(x+1) - f(x-1)).$$



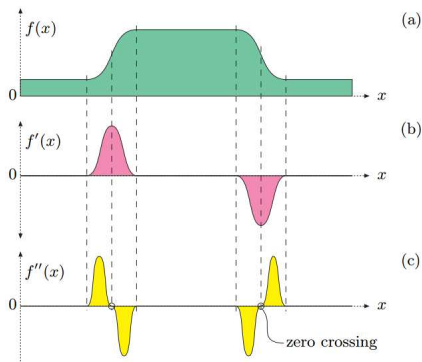
Second Derivative

- Utilize difference equations: First derivative:

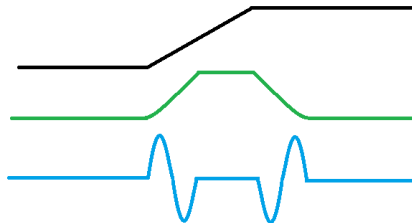
$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

Second derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$



Derivative Observations



- ▶ First-order derivatives generate thick edges
- ▶ Second-order derivatives
 - ▶ Have stronger response to details
 - ▶ Produce a double response at step changes
 - ▶ Order of response strength. Point, line, step.
- ▶ Second-order derivative is therefore preferred for enhancement
 - ▶ Use isotropic (rotation invariant) formulation

Partial Derivatives and the Gradient

Derivative of a multidimensional function taken along one of its coordinates
partial derivative;

$$\frac{\partial I}{\partial x}(x, y) \text{ and } \frac{\partial I}{\partial y}(x, y)$$

The gradient function is

$$\nabla I(x, y) = \begin{bmatrix} \frac{\partial I}{\partial x}(x, y) \\ \frac{\partial I}{\partial y}(x, y) \end{bmatrix}$$

The *magnitude* of the gradient,

$$|\nabla I|(x, y) = \sqrt{\left(\frac{\partial I}{\partial x}(x, y)\right)^2 + \left(\frac{\partial I}{\partial y}(x, y)\right)^2}$$

Derivative Filters

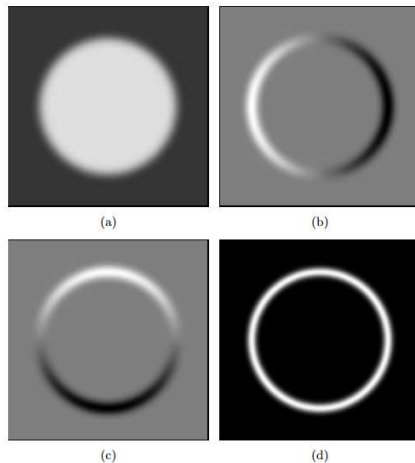
The gradient approximation of the horizontal derivatives is the coefficient matrix:

$$H_x^D = [-0.5 \quad \mathbf{0} \quad 0.5] = 0.5 \cdot [-1 \quad \mathbf{0} \quad 1]$$

where the coefficients -0.5 and $+0.5$ correspond to the image elements $I(x-1, y)$ and $I(x+1, y)$. Vertical component of the gradient

$$H_y^D = \begin{bmatrix} -0.5 \\ \mathbf{0} \\ 0.5 \end{bmatrix} = .5 \cdot \begin{bmatrix} -1 \\ \mathbf{0} \\ 1 \end{bmatrix}$$

Partial Derivatives of a Two Dimensional Function



(a) Synthetic image, (b) first derivatives in the horizontal direction $\partial I / \partial x$. (c) vertical direction $\partial I / \partial y$; (d) magnitude of the gradient $|\nabla I|$ (d).

Edge Operators-Variou Approaches

- ▶ Prewitt and Sobel Operators: Classic methods that differ in the filters they use:
 - ▶ Prewitt: Computes an average gradient across 3 neighboring lines or columns respectively

$$H_x^P = \begin{pmatrix} -1 & 0 & 1 \\ -1 & \mathbf{0} & 1 \\ -1 & 0 & 1 \end{pmatrix} \quad H_y^P = \begin{pmatrix} -1 & -1 & -1 \\ 0 & \mathbf{0} & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

- ▶ Sobel: The smoothing part assigns higher weight to the current center line and column, respectively:

$$H_x^S = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix} \quad H_y^S = \begin{pmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

Edge Strength and Orientation

Denote the scaled filter results as:

$$D_x(x, y) = H_x * I \text{ and } D_y(x, y) = H_y * I$$

In both cases, the local edge strength $E(x, y)$ is defined as the gradient magnitude

$$\sqrt{(D_x(x, y))^2 + (D_y(x, y))^2}$$

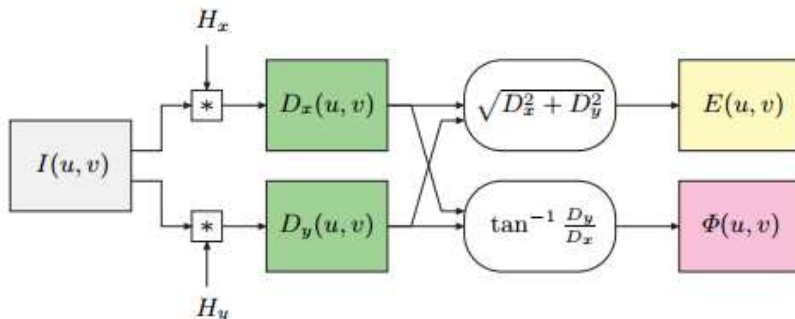
and the local edge orientation angle $\phi(x, y)$ is

$$\phi(x, y) = \arctan\left(\frac{D_y(x, y)}{D_x(x, y)}\right)$$

An improved version of the Sobel filter:

$$H_x^{S'} = \frac{1}{32} \begin{pmatrix} -3 & 0 & 3 \\ -10 & 0 & 10 \\ -3 & 0 & 3 \end{pmatrix} \quad H_y^{S'} = \frac{1}{32} \begin{pmatrix} -3 & -10 & -3 \\ 0 & 0 & 0 \\ 3 & 10 & 3 \end{pmatrix}$$

Edge Operators



Process of gradient-based edge extraction. Linear filters H_x and H_y produce two gradient images, D_x and D_y . They are used to compute the edge strength E and orientation ϕ for each image position (x, y) .

Laplacian Edge Operator

Problem with edge operators based on first derivatives; edges are as wide as the underlying intensity transition. Alternative makes use of the second derivatives.

Laplacian is the simplest isotropic derivative:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$



$$\nabla^2 f \approx [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

- ▶ Isotropic to 90° rotations
- ▶ Add diagonal derivatives to make it 45° isotropic

Mask Implementations and Enhancement

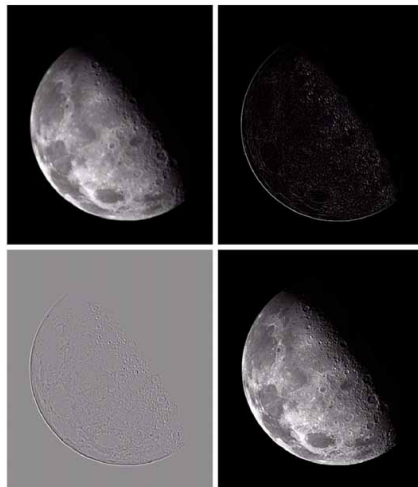
- ▶ Similar definition produces a sign change
- ▶ Enhancement adds (subtracts) derivative and observed image

0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1

0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{if the center is negative} \\ f(x,y) + \nabla^2 f(x,y) & \text{if the center is positive} \end{cases}$$

Example



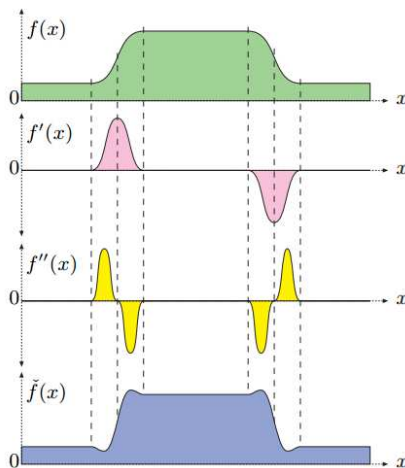
Top Left: image of the north pole of the moon. Top Right: Laplacian-filtered image.
Bottom Left: Laplacian image scaled for display purposes. Bottom Right: Image enhanced

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & +5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$



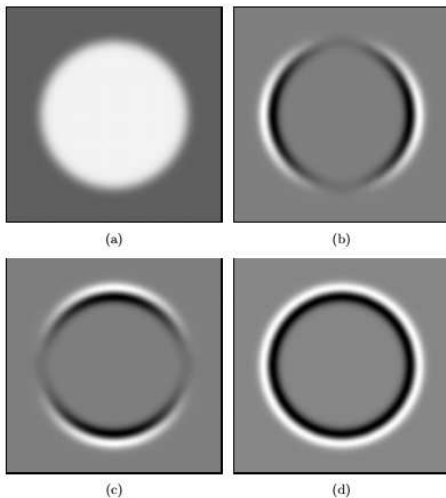
Edge Sharpening with second derivative

Original intensity function $f(x)$, first derivative $f'(x)$, second derivative $f''(x)$, and sharpened intensity function $\check{f}(x) = f(x) - w \cdot f''(x)$



Results of Laplace filter H^L

(a) Image $I(a)$, (b) second partial derivative $\partial^2 I / \partial^2 u$ in the horizontal direction, (c) second partial derivative $\partial^2 I / \partial^2 v$ in the vertical direction, and (d) Laplace filter $\nabla^2 I(u, v)$.

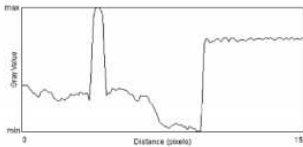


Edge Sharpening with the Laplace filter

Original image and marked line (a, b), result of Laplace filter H^L (c, d), and sharpened image (e, f).



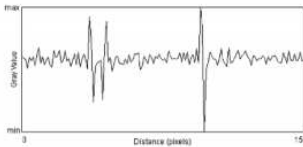
(a)



(b)



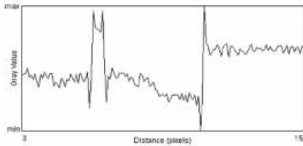
(c)



(d)



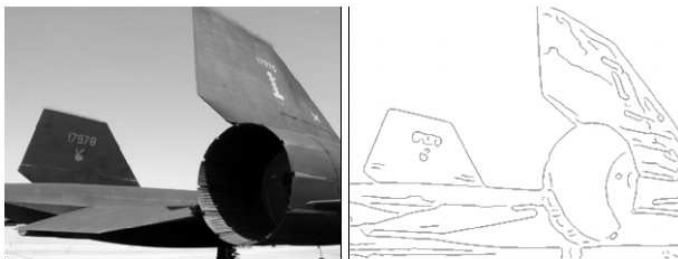
(e)



(f)

Edges and Contours

- ▶ Edges and contours, are of high importance for the visual perception and interpretation of images.
- ▶ Edges: local intensity changes distinctly along a particular orientation. The amount of change with respect to spatial distance is the first derivative of a function.

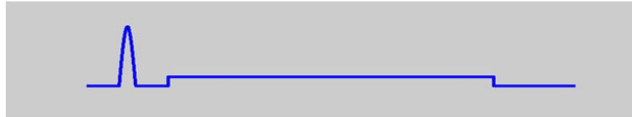


Edge Detection

$$f(x)$$



$$|df(x)|$$



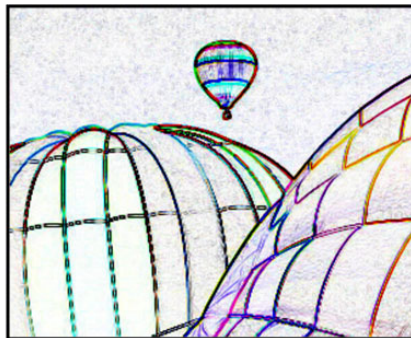
$$|df(x)|$$



Edge Detection

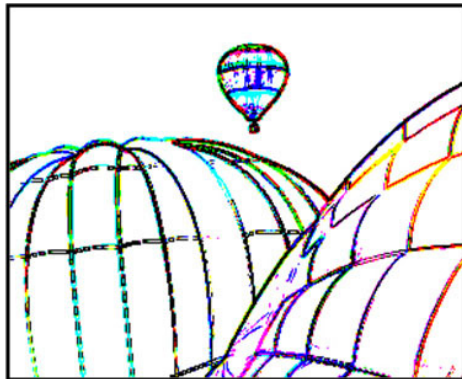
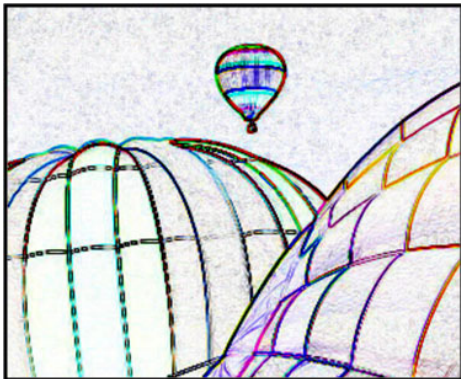
The derivative of a digital image can be found by applying the linear filter.

$$\begin{bmatrix} -1 & +2 & -1 \\ +2 & -4 & +2 \\ -1 & +2 & -1 \end{bmatrix}$$

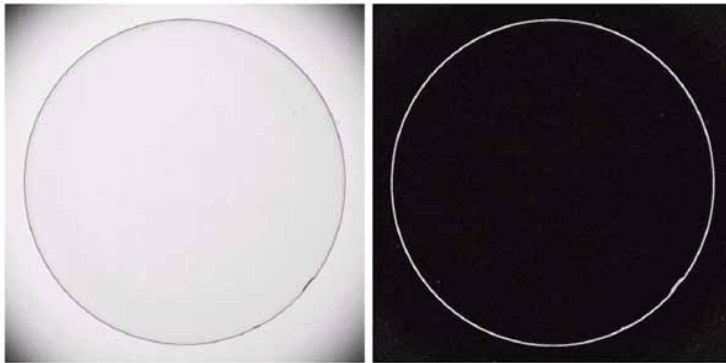


Edge Detection

The edges are then thinned by applying a threshold.



Sobel Example



Left: Optical image of contact lens. Right: Sobel gradient. Common application: edge detection

- ▶ Threshold sobel output
- ▶ Binary edge mask

Comparison of Edge Operators



Original



Roberts



Prewitt



Sobel



Laplacian of Gaussian



Canny ($\sigma = 1.0$)