Kernel Landmarks: An Empirical Statistical Approach to Detect Covariate Shift

- We propose an alternative solution to kernel max-slicing
- Each data point (landmark) defines a witness function
- The landmark which identifies the largest discrepancy between the distribution is chosen
- Our approach detects class-based covariate shift
- It identifies instances from minority class based on witness functions
- The landmark-based kernel max-slicing is much simpler to compute than the kernel max-slicing

What is the goal?

Divergence measures for interpreting and minimizing discrepancies between data distributions.

- Probability measures are \( \mu, \nu \in P(X) \)
- Covariate shift: When the testing cases are not class-balanced
  - Detect: Divergence between train and test
  - Identify: Classes for witness’s top-K training set examples

Two-sample Tests Using Kernel Divergences

Maximum mean discrepancy (MMD)

\[ \text{MMD}^2(\mu, \nu) = \sup_{p,q \in [0,1]} \mathbb{E}_{X \sim p}[\ell(X) - \ell(Y)] = \mathbb{E}_{X \sim \mu}[\ell(X)] - \mathbb{E}_{Y \sim \nu}[\ell(Y)] \]

The max-sliced kernel Wasserstein-2 (W2)

\[ W_{2,\ell}^2(\mu, \nu) = \sup_{p,q \in [0,1]} \mathbb{E}_{X \sim p}[\ell(X) - \ell(Y)] \]

Empirical measures formed from two samples: \( \mu = \sum_{i=1}^{m} \delta_{x_i} \) and \( \nu = \sum_{i=1}^{n} \delta_{y_i} \)

\[ W_{2,\ell}^2(\mu, \nu) = \min_{i,j} \mathbb{E}_{X \sim p} \mathbb{E}_{Y \sim q} [\ell(X) - \ell(Y)] \]

Saddlepoint optimization problem: evaluation requires \( O(N\log N) \)

Identify the Class Imbalance with Witness Function

Precision of the witness function in detecting underrepresented classes

CIFAR-10 (Inception Codes w/ linear kernel)

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<table>
<thead>
<tr>
<th>Minority class:</th>
<th>L-W2, Prec@5 = 0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MMD: Prec@5 = 0.6</td>
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MNIST (raw pixels with Gaussian kernel median distance for kernel) in “5” is underrepresented digit

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The implementation of our approach and demos can be found by running GitHub code.